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# Three dimensional stability analysis of reinforced slopes

## Analyse de Stabilité Tridimensionnelle des Remblais en Terre Armée

F. Askari

*Geotechnical Engineering Dept., International Institute of Earthquake Engineering and Seismology, Tehran, Iran*

O. Farzaneh

*Civil Engineering Department, School of Engineering, University of Tehran, Tehran, Iran*

H. Mohamadzadeh

*Civil Engineering Department, Iran University of Science and Technology, Tehran, Iran*

### ABSTRACT

The kinematic limit analysis approach is applied to study the three dimensional stability of the reinforced slopes. The amount of reinforcement necessary to prevent collapse of a slope is determined based on a collapse mechanism including simultaneous failure of whole reinforcements. The transitional failure mechanism consists of several hexagonal blocks and a pentagonal one at the base of mechanism. All planes between blocks are supposed to be horizontal. In this paper, the 3D stability of slopes with geosynthetic layers is analyzed. Each block consists of one reinforcement layers intersecting by back and lateral planes of the block. The geosynthetics are supposed to collapse in these intersections when the mechanism begins to be unstable. The reinforcement is uniformly distributed through the height of the slope. Based on these assumptions, calculations were performed and several results were gained that some of them are presented in order to illustrate the influence of different parameters such as three dimensional parameters and soil characteristics on reinforced slope stability.

### RÉSUMÉ

L'approche par l'extérieure de l'Analyse Limite est appliquée pour l'étude de la stabilité tridimensionnelle des remblais de terre armée. Le mécanisme de rupture supposé consiste en plusieurs blocs hexagonaux et un bloc pentagonal de base. Les surfaces de discontinuité de vitesse entre les blocs sont horizontales, ce qui simplifie considérablement les calculs. Les couches de l'armatures sont supposées uniformément réparties en hauteur du remblais. Les résultats obtenus sont comparés avec ceux d'autres chercheurs dans les cas bidimensionnels. Pour les cas tri dimensionnels, les résistances requises de l'armature, en fonction de la géométrie du remblais et de l'angle de frottement intérieure du sol, sont obtenues.

Keywords : Reinforced slopes, Collapse mechanism, Kinematic limit analysis, 3D slope stability, Horizontal blocks

## 1 INTRODUCTION

Stability analysis of reinforced slopes has been the subject of research for the last three decades using analytical and numerical techniques, and reinforcement of soil structures such as slopes and walls has become an accepted engineering practice. Analytical approaches are widely used to analyze the stability of reinforced slopes. The most common analytical method used for slope designing is the 'limit equilibrium' method. Ling et al. have proposed some seismic design procedures for geosynthetic-reinforced soil structures based on pseudo-static limit equilibrium. In their study, internal and external stability analyses have been conducted to determine the required strength and length of reinforcement (Ling et al. 1997). Nouri et al. have proposed a failure mechanism consisting of horizontal sliding blocks based on limit equilibrium method to determine the required strength and length of reinforcement (Nouri et al. 2005).

Michalowski has applied the kinematic theorem of limit analysis to calculate the strength and length of reinforcement necessary to prevent earth structures from collapse, under the assumption that the reinforcement strength is uniformly distributed through the slope height or linearly increasing with depth. He has determined the required strength of reinforcement using rotational log-spiral mechanism and the length of

reinforcement using both rotational and direct sliding mechanisms (Michalowski 1997, 1998).

Stability of reinforced slopes is often analyzed by methods based on two-dimensional models, assuming plane strain condition. However, the failure regions of reinforced slopes usually have finite dimensions. Therefore three-dimensional approaches are more appropriate to analyze such stability problems.

Ignoring three dimensional condition effects on the slope stability usually leads to conservative answers. This condition may lead to unsafe answers. For example, if one neglect the 3D effects in back analyses of reinforcement layers tension forces, the back calculated forces will be too low.

In the present study, the kinematic theorem of limit analysis is applied to calculate the strength of reinforcements required to ensure the three dimensional stability of slopes reinforced with geosynthetic layers. The failure mechanism is divided into a number of horizontal slices containing reinforcements, which do not intersect the reinforcements. Therefore the reinforcements have no direct influence on the inter-slice forces. The possible failure mechanisms with different shapes are considered, and analytical expressions are derived to obtain the required strength of reinforcement to prevent slope from collapse. Comparisons of obtained results in this study to those existing in literature for 2D stability analysis are shown, and the most significant differences are indicated. Results for 3D stability analysis are

also presented in order to illustrate the effect of soil characteristics and the geometrical parameters of slopes on the stability of reinforced earth structures.

## 2 METHOD OF ANALYSIS

The 3D failure mechanism, used in this study is shown in Fig. 1. This mechanism consists of rigid transitional horizontal blocks separated by planar velocity discontinuity surfaces. The width of failure mechanism is limited to lateral surfaces that have orientation  $\xi$  with respect to the vertical symmetry plane. The lateral planes must be defined so that the compatibility conditions between velocity discontinuity surfaces can be satisfied. The applied mechanism consists of several hexagonal blocks and one pentagonal block at the bottom of mechanism. The planar velocity discontinuity surfaces can be defined by points  $A_{1(i)}$  to  $A_{4(i)}$ .

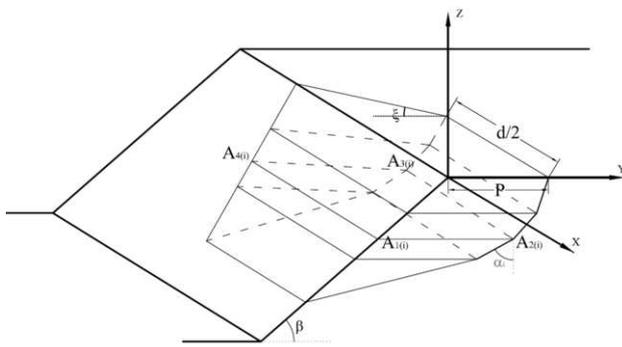


Figure 1 3D transitional multi-block mechanism

The amount of ultimate tensile strength in the intersections of the reinforcement layers and lateral surfaces can be simply determined. In Fig. 2 a single reinforcement layer is shown in plan. This layer has three rupture boundaries which are the intersections of reinforcement with two lateral and one bottom slip surfaces. The distribution of tensile stress along the reinforcement intersection with these surfaces is illustrated on Fig. 2.

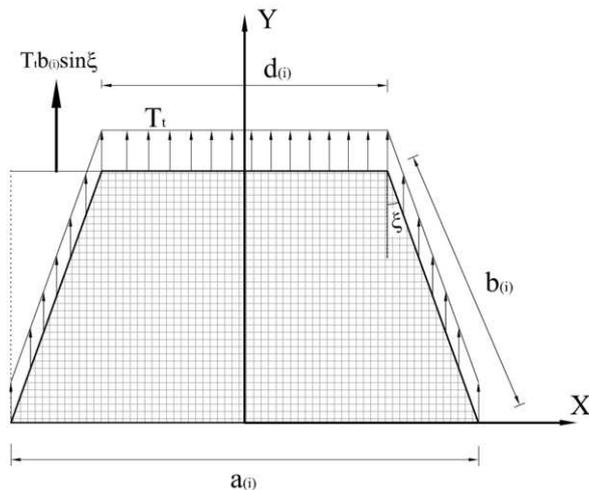


Figure 2 Tensile strength distribution over reinforcement velocity discontinuity boundaries

With respect to Fig. 2 the tensile force of (i)<sup>th</sup> reinforcement layer in Y direction which resists against slope instability, can be obtained using:

$$T_{(i)} = d_{(i)}T_t + 2b_{(i)}T_t \sin \xi = T_t a_{(i)} \quad (1)$$

where  $T_t$  is the tensile strength of the single reinforcement layer per unit width.  $a_{(i)}$ ,  $b_{(i)}$ ,  $d_{(i)}$  are geometric parameters of (i)<sup>th</sup> reinforcement layer and  $\xi$  is a geometric parameter of failure mechanism.

The transitional multi-block mechanism has been applied to obtain the requirements of reinforcement (Fig. 3). In this failure mode, the (i)<sup>th</sup> rigid block including one reinforcement layer, slides over the failure surface and lower block with velocities  $V_i$  and  $[V_i]$  respectively which are shown in Fig. 4.

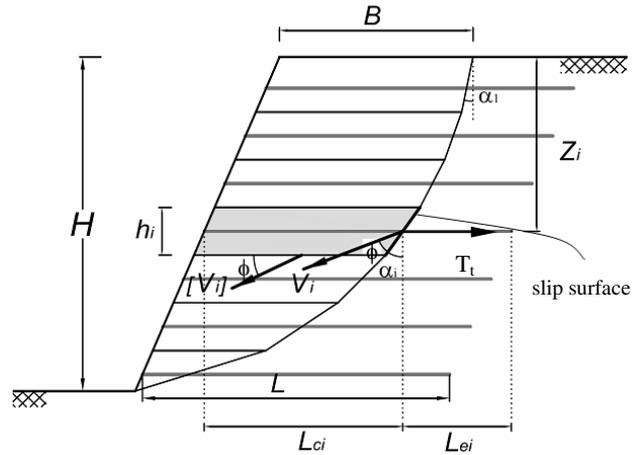


Figure 3 transitional multi-block mechanism

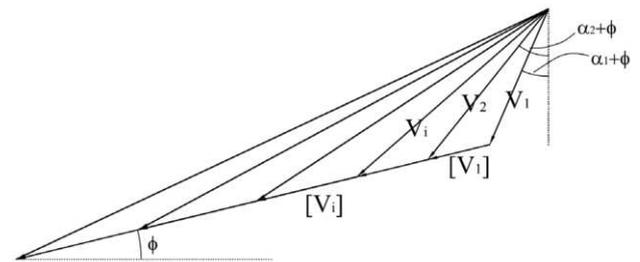


Figure 4 Velocity hodograph for transitional multi-block mechanism

The amount of absolute velocity of (i)<sup>th</sup> horizontal block ( $V_i$ ) and relative velocity of (i+1)<sup>th</sup> block with respect to (i)<sup>th</sup> one ( $[V_i]$ ) can be obtained as follows:

$$\begin{cases} V_{i+1} = \frac{\cos(\alpha_i + 2\phi)}{\cos(\alpha_{i+1} + 2\phi)} V_i \\ [V_i] = \frac{\cos(\alpha_i + 2\phi) \sin(\alpha_{i+1} - \alpha_i)}{\cos(\alpha_{i+1} + 2\phi) \cos(\alpha_i + 2\phi)} V_i \end{cases} \quad (2)$$

Ignoring the reinforcement mass, the work rate of block's weight can be obtained as

$$W_\gamma = \sum_{i=1}^n w_i V_i \cos(\alpha_i + \varphi) \quad (3)$$

where  $w_i$  is the weight of the ( $i$ )<sup>th</sup> block,  $V_i$  is the absolute velocity of the ( $i$ )<sup>th</sup> horizontal block and  $\alpha_i$  and  $\varphi$  are identified at Fig. 3.

For calculation of required reinforcement strength, it is assumed that the length of reinforcement layers is long enough so that in the unstable conditions, the rupture collapse occurs at the whole reinforcement layers. Based on this assumption, the sum of energy rate dissipated by reinforcements tensile forces, can be shown as follow

$$\dot{D} = \sum_{i=1}^n T_{(i)} V_i \sin(\alpha_i + \varphi) = \sum_{i=1}^n T_i a_{(i)} V_i \sin(\alpha_i + \varphi) \quad (4)$$

where  $n$  is the number of blocks. Since the amount of reinforcement is assumed to be distributed uniformly through the slope height, the average strength of the reinforcement per unit height can be presented as

$$k_t = \frac{nT_t}{H} \quad (5)$$

Following expression can be obtained by substitution the expression  $T_t$  in Eq. (4) with the expression  $\frac{H}{n}k_t$ , as a result of Eq. (5),

$$\dot{D} = \frac{H}{n} k_t \sum_{i=1}^n a_{(i)} V_i \sin(\alpha_i + \varphi) \quad (6)$$

Based on kinematic method of limit analysis theorem, the rate of work done by body forces is less than or equal to the rate of energy dissipation in any kinematically admissible failure mechanism (Michalowski 1998). One may solve this unequal and obtain a lower bound value for average strength of the reinforcement ( $k_t$ ) as follows

$$k_t = \frac{n \sum_{i=1}^n w_i V_i \cos(\alpha_i + \varphi)}{H \sum_{i=1}^n a_{(i)} V_i \sin(\alpha_i + \varphi)} \quad (7)$$

This equation provides a lower-bound solution for the reinforcement force necessary to prevent the failure of slope at the three dimensional condition. In order to find the best estimation of  $k_t$ , an optimization procedure needs to be used to maximize  $k_t$  with respect to  $\alpha_i$  ( $i=1$  to  $n$ ),  $P$ ,  $d$  and  $\xi$  (Fig. 1).

In this study the average required reinforcement strength in three dimensional instability conditions are calculated based on above assumptions. For determination of required reinforcement length, in addition to mentioned assumptions, more other assumptions must be made. However, in this study we have focused on the three dimensional conditions influence on the amount of required reinforcement strength to prevent slope failure.

It should be mentioned that the current algorithm is more efficient for steep reinforced slopes (with an angle larger than about 70 degrees) which are common in actual civil engineering practice.

### 3 VERIFICATION OF THE USED METHOD

To verify the current method, no experimental data were available for direct comparison. Also no analytical results were

found for comparison in 3D conditions. Thus the verification of the applied method has been made by comparing the results obtained using the current formulation in 2D conditions with those from existing solutions in the literature. The current method can be compatible to slope analysis in 2D condition in the case of very large width to height ratios (for example  $d/H > 1000$ ). Different slopes are analyzed using current method and the obtained results are compared with those of the procedures presented in the literature such as Ling et al. (1997), Michalowski (1998), Ausilio et al. (2000) and Nouri et al. (2005).

A comparison of the results obtained in this study to those presented by Ling et al. (1997) and Michalowski (1998) is given in Fig. 5 for a slope with angle ( $\beta = 90^\circ$ ). In the calculations performed for verification, the height of slope and the soil density are assumed to be 10m and 20kN/m<sup>3</sup> respectively. Fig. 5 compares the values of  $k_t$  obtained in this study with those proposed by Ling et al. and Michalowski. As can be seen from this figure, the used formulations in this study provide values for  $k_t$  that are in close agreement with those read from the graphs by Ling et al. and Michalowski. The obtained results are closely between those by Ling et al. and Michalowski. The differences between the results obtained by current method and Michalowski's method increase as the value of  $\varphi$  decreases.

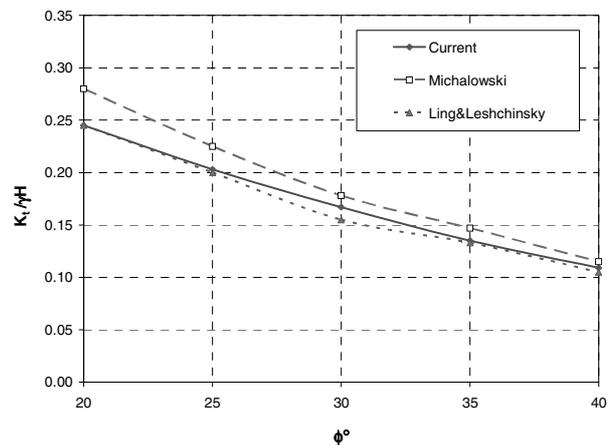
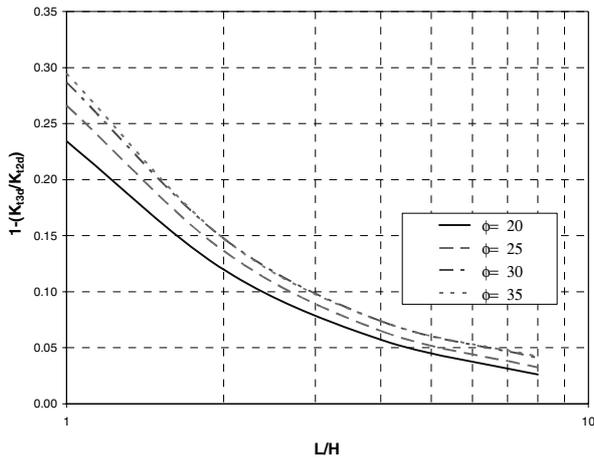
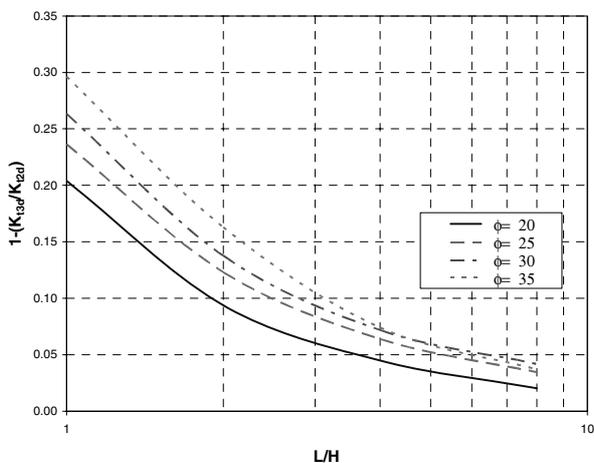


Figure 5 Required reinforcement strength ( $\beta = 90^\circ$ )

### 4 NUMERICAL RESULTS

In this section, a series of calculations is carried out to illustrate the effect of three dimensional conditions on the stability of the reinforced slopes. The soil is assumed to be cohesionless, with angle of internal friction varying from 20° to 35°. Moreover, two slope angles are considered:  $\beta = 80^\circ$  and  $90^\circ$ .

Fig. 6 and 7 present dimensionless parameter  $(1 - k_{t3d}/k_{t2d})$  ( $k_{tid}$  being the required reinforcement for slope stability in i-D analysis) as a function of  $L/H$  for the reinforced slopes. The dimensionless parameter  $(1 - k_{t3d}/k_{t2d})$  is an indication of decreasing required reinforcement strength due to 3-D conditions and  $L$  is the maximum width of failure mechanisms: the width of failure mechanisms is limited to  $L$ .

Figure 6 Required reinforcement strength ( $\beta = 80$ )Figure 7 Required reinforcement strength ( $\beta = 90$ )

## 5 CONCLUSIONS

The most important results of the present analyses can be summarized as follow:

- The required strength of reinforcement to prevent the slope failure in 3D conditions is less than that in 2D conditions.
- In comparison to the 2D conditions, the influence of considering 3D conditions on decreasing the amount of required reinforcement:
  - Decreases with increasing slope angle.
  - Increases with increasing angle of internal friction of the soil.

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