

INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING



This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

<https://www.issmge.org/publications/online-library>

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

An elastoplastic model for unsaturated soils

Un modèle élastoplastique pour les sols non saturés

Daichao Sheng

Centre for Geotechnical and Materials Modelling, The University of Newcastle, NSW 2308, Australia

Delwyn G Fredlund

Golder Associates Ltd., Saskatoon, SK, Canada

ABSTRACT

A number of elastoplastic models have been proposed for unsaturated soils over the past three decades. These models have generally taken the form of simple extensions of elastoplastic models previously proposed for saturated soils. Laboratory testing programs have given rise to several unanswered questions regarding the acceptability of these models for unsaturated soils. There are questions related to: 1.) the variation of the yield stress with soil suction, 2.) the modeling of soils prepared from slurry conditions, and 3.) the existence of a smooth transition between saturated and unsaturated soil conditions. The model proposed in this paper addresses each of these questions by re-formulating the elastoplastic model for unsaturated soils through use of independent stress state variables. The re-formulation provides a smooth transition between the elastoplastic model for saturated and unsaturated soil conditions.

RÉSUMÉ

Durant les trois dernières décennies, plusieurs modèles élastoplastiques pour les sols non saturés ont été présentés. Ces modèles sont habituellement un simple prolongement du modèle élastoplastique précédant pour les sols saturés. Les programmes de test de laboratoire ont soulevé plusieurs questions laissées sans réponses concernant l'acceptabilité de ces modèles pour les sols non saturés. Ces questions sont liées à 1) la variation de la limite d'élasticité par succion du sol, 2) au modèle résultant du sol préparé à partir de la boue, et 3) une transition uniforme créée entre un sol saturé et un sol non saturé. Le modèle présenté dans cet article va aborder chacune de ces questions en reformulant le modèle élastoplastique pour les sols non saturés avec l'aide de variables indépendantes de l'état des contraintes du sol. La reformulation fournit une transition uniforme entre le modèle élastoplastique pour les sols saturés et non saturés.

Keywords : Unsaturated soils, elastoplastic modelling, yield stress, shear strength

1 INTRODUCTION

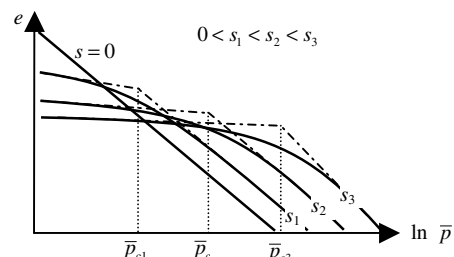
Since the pioneering work of Alonso et al. (1990), a number of elastoplastic constitutive models have been developed for unsaturated soils. Early models only deal with stress-suction-strain relationships of unsaturated soils. These models are based on the same basic assumptions and largely fall in the same framework of Alonso et al. (1990), though different constitutive equations and different stress variables are used. The model by Alonso et al (1990), subsequently referred to as the Barcelona Basic Model, remains as one of the fundamental models for unsaturated soils. More recent models have incorporated suction-saturation relationships with hysteresis into stress-strain relationships (Vaunat et al. 2000; Sheng et al. 2004).

Elastoplastic models for unsaturated soils usually use a loading-collapse yield surface that defines the variation of the yield stress along the soil suction axis. The yield stress is usually assumed to increase with increasing suction. Under such a framework, these models are able to predict the wetting-induced volume collapse. However, some fundamental questions have not yet been fully answered.

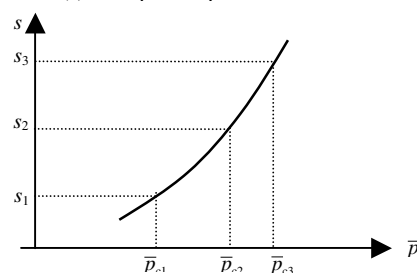
One such question is indeed about the variation of the yield stress with soil suction. Under isotropic stress states, the yield stress is also called the preconsolidation pressure. For unsaturated soils, this yield net mean stress, denoted here by \bar{p}_c , is usually determined from isotropic compression curves obtained under constant suctions. The initial portion such a curve is usually flatter than the ending portion in the space of void ratio versus logarithmic net mean stress, if the suction is larger than zero. Each compressive curve is then approximated by two straight lines, one representing the elastic unloading-reloading line and the other the elastoplastic normal

compression line. The meeting point of the two lines gives the preconsolidation pressure or yield stress (Fig. 1a). The yield stress is then found to increase with increasing suction, irrespective of samples air-dried from slurry or compacted soils, leading to the so-called loading-collapse yield surface (Fig. 1b).

The procedure outlined above for determining the yield stress for unsaturated soils suffers a significant shortcoming.



(a) Isotropic compression curves



(b) Loading-collapse yield surface

Figure 1. Variation of preconsolidation pressure with suction.

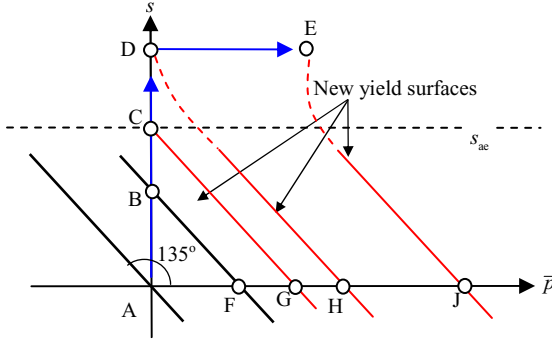


Figure 2. Evolution of the yield stress during drying and compression of a slurry soil (s_{ac} : air entry value, s : suction, \bar{p} : net mean stress).

To demonstrate this shortcoming, we should first realize that the isotropic compression curves shown in Fig. 1a are typical to unsaturated soils prepared from slurry (e.g. Jennings and Burland, 1962) as well as to compacted soils (e.g. Wheeler and Sivakumar, 2000). Because it is relatively easy to understand the preconsolidation stress for a slurry soil than for a compacted soil, we use a slurry soil as an example here. Let us assume that the slurry soil was isotropically consolidated to Point F in Fig. 2 and it has an air entry indicated by the suction at Point C. For saturated soils, the effective stress principle states that the yield stress remains constant as long as the effective stress does not change. The effective stress remains constant along the line inclined to horizontal by 135° . Therefore, the initial elastic zone is then bounded by the two thick lines that go through points A and F and are inclined to horizontal by 135° . Drying the slurry soil under zero mean stress to Point C will cause plastic yielding, because C is outside the initial elastic zone. Further drying will cause desaturation of the soil and plastic yielding as well. Let us dry the soil to Point D. The new yield surface will then pass through points D and H in Fig. 2. Let us now isotropically compress the soil under the constant suction to point E. According to the data by Jennings and Burland (1962) and Cunningham et al. (2003), the isotropic compression line in the space of void ratio against logarithmic mean stress will be curved, in a pattern as those shown in Fig. 1b for $s > 0$. However, the isotropic compression path (DE) is clearly outside the initial elastic zone. Therefore, the isotropic compression path is elastoplastic and does not involve a purely elastic portion as Fig. 1b indicates, suggesting that the method for determining the yield stress in Fig. 1b be incorrect.

Very recently Sheng et al. (2008) proposed a new modelling approach for unsaturated soils. In this approach, yield stress and shear strength of unsaturated soils were derived from the volumetric model that defines the volume change caused by suction and mean stress changes. A specific model was proposed and it was called the SFG model. An essential difference between the SFG model and the other elastoplastic models in the literature is that the former provides a consistent explanation of yield stress, shear strength and volume change behaviour of unsaturated soils as functions of suctions, both for soils prepared from slurry and from compacted specimens. It was shown that all these functions are actually based on one simple equation that defines the volume change caused by suction and stress changes. This equation is written in an incremental form and provides a continuous and smooth treatment of suction or pore water pressure for both saturated and unsaturated states. This paper presents the key elements of the SFG model and some experimental validation of the model.

2 SFG MODEL AND VALIDATION

2.1 Volume change behaviour

In the SFG model by Sheng et al. (2008), the change of the soil volume can be caused by a change in stress or a change in soil suction. For normally consolidated soils under isotropic stress states, we have:

$$dv = -\lambda_{vp} \frac{d\bar{p}}{\bar{p} + s} - \lambda_{vs}(s) \frac{ds}{\bar{p} + s} \quad (1)$$

where v is the specific volume, \bar{p} is the mean net stress and $\bar{p} = p - u_a$, p is the mean stress, u_a is the pore air pressure, s is the soil suction and $s = u_a - u_w$, u_w is the pore water pressure, λ_{vp} is a parameter related to the soil compressibility in terms of stress changes, and λ_{vs} is a parameter related to the soil shrinkability in terms of suction changes. The soil suction used in this paper refers to the matric suction which consists of a capillary and an adsorptive component. When the pore water exists as capillary water at relatively high degrees of saturation, the capillary potential is dominant in the matric suction $s \approx u_a - u_w$. When the pore water exists as adsorbed water films in the soil, the adsorptive potential (ψ_a) becomes dominant in the matric suction. In this case the true water pressure is not well defined since it is not unique at one material point and is dependent on the proximity to the particle surface. An apparent water pressure can be introduced to quantify the adsorptive potential: $u_w = u_a - \psi_a$, i.e. the apparent water pressure represents the negative adsorptive potential measured in excess of air pressure. When the air pressure is atmospheric (zero), the apparent water pressure is then the negative adsorptive potential, and the net mean stress becomes the total mean stress. Such an apparent water pressure is then unique at one material point. With such a definition of u_w , the matric suction can be expressed as $s = u_a - u_w$ and can be used continuously for a relatively large range of saturation.

The parameter λ_{vp} can be determined from normal compression lines (NCL) for $s=0$. It is similar to the slope (λ) of NCL in $e-\ln p$ plots used for saturated soils. In its simplest form this parameter can be treated as a constant for one soil, but more realistically it should be a function of suction. The parameter λ_{vs} is a function of suction. Its value is identical to λ_{vp} for suctions below the saturation suction, but approaches zero as suction increases to infinite. The following simple equation was used in Sheng et al. (2008):

$$\lambda_{vs} = \begin{cases} \lambda_{vp} & s < s_{sa} \\ \lambda_{vp} \frac{s_{sa} + 1}{s + 1} & s \geq s_{sa} \end{cases} \quad (2)$$

where s_{sa} is the saturation suction and its definition is slightly different from the air entry value (see Sheng et al. 2008).

The volumetric model defined by eqs. (1) and (2) is the foundation of the SFG model. The yield stress and shear strength criteria in the SFG model are all based on this volumetric model. It is very simple and the only additional

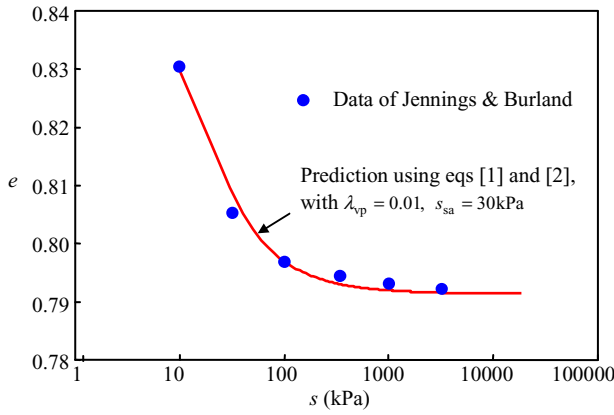


Figure 3. Comparison with data by Jennings and Burland (1962).

parameter needed for unsaturated states is the saturation suction. Its validity was tested against experimental data for different types of unsaturated soils by Zhou and Sheng (2008). In the original model by Sheng et al. (2008), the plastic volumetric strain rate ($d\varepsilon_v^p$) was used in place of the negative volume rate ($-dv$) in eq. [1], which leads to a linear relationship in the double logarithmic space of $\ln v$ - $\ln p$. However, experimental data and associated parameters (such as λ) in the literature are usually presented in the semi-logarithmic space of v - $\ln p$. Therefore, the semi-logarithmic relation is used in this paper, to simplify the comparison. The two alternatives have no essential difference, because the variation of the specific volume is usually less than one order of magnitude.

Data on unsaturated soils air-dried from slurry are not common in the literature. They often represent the missing block in the puzzle of unsaturated soil behaviour. Two such sets of data are reported by Jennings and Burland (1962) and by Cunningham et al. (2003).

In Fig. 3, the predicted void ratio versus suction curve is compared with data by Jennings and Burland (1962). In this figure the parameters λ_{vp} and s_{sa} are estimated for the silty soil tested. The soil was assumed to be normally consolidated. It is shown that the prediction compares quite well with the data.

The predicted isotropic compression curves are compared with the experimental data of Cunningham et al. (2003) in Fig. 4. The slurry soil was isotropically preconsolidated to 130 kPa. Other parameters needed for predictions are given in Cunningham et al. (2003): $\lambda_{vp} = 0.043$, $\kappa_{vp} = 0.007$ and $s_{sa} = 250$ kPa. The parameter κ_{vp} is the slope of the unloading-reloading line in e - $\ln p$ plots. It is observed that the experimental isotropic compression curves for suctions larger than zero don't have a clear point of slope changes (unlike the curve for zero suction). Because the suctions (400, 650 and 1000 kPa) are larger than the air entry value of the soil, the corresponding isotropic compression curves are actually all normal compression lines. They are curved in the e - $\ln p$ space because of the combined effects of the suction and mean stress. Using the method outlined in Fig. 1 to determine the yield stress would clearly lead to a wrong conclusion. The predicted curves agree very well with the experimental data.

As shown by Sheng et al. (2008), the shape of the yield surface in the s - p space for a slurry soil is indeed similar to those shown in Fig. 2. However, compaction or isotropic compression at suctions higher than the air entry value can change the shape of the yield stress loci, e.g. from \bar{p}_c to \bar{p}_{cn} as demonstrated in Fig. 5. The reason for this change is that the stress increments required to generate the same amount of plastic volumetric strain will depend on the suction level.

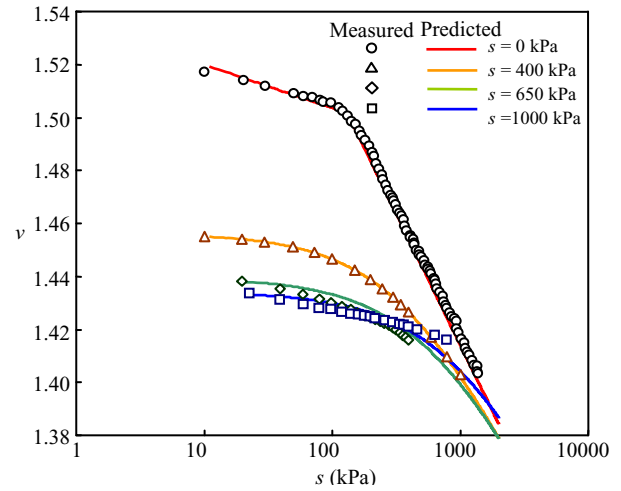


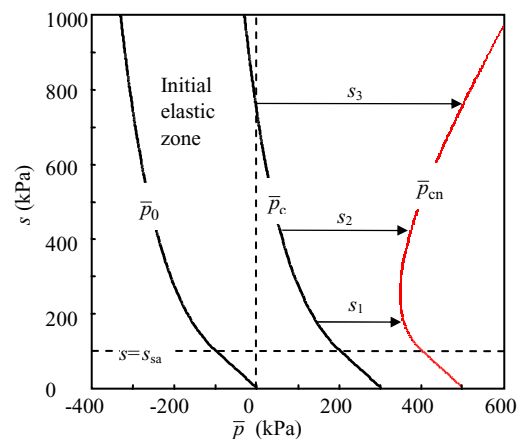
Figure 4. Predicted compression curves versus data by Cunningham et al. (2003).

Starting from the current yield surface defined by \bar{p}_c in Fig. 5, a new yield surface \bar{p}_{cn} represents a contour of plastic volumetric strain for an isotropically hardening material where the plastic volumetric strain is the hardening parameter. The stress increments required to generate the same amount of plastic volumetric strain at the three suction levels (s_1 , s_2 and s_3) are different, leading to a new yield surface of the shape shown by \bar{p}_{cn} in Fig. 5.

Therefore, the yield stress for compacted soils can not be determined as easily as that for a slurry soil. The SFG model is used to predict the isotropic compression curves of compacted kaolin specimens reported by Thu et al. (2007). The preconsolidation stresses are best fit from the test results. Figure 6 shows that the isotropic compression curves under different suctions are very well predicted by the SFG model.

2.2 Shear strength behaviour

The shear strength of an unsaturated soil is usually considered to be a function of suction. Sheng et al. (2008) showed that the shear strength criterion can be derived from the yield stress function. In the SFG model, the yield stress for a slurry soil is defined by:

Figure 5. Yield surface evolution due to compaction or isotropic compression ($s_{sa}=100$ kPa).

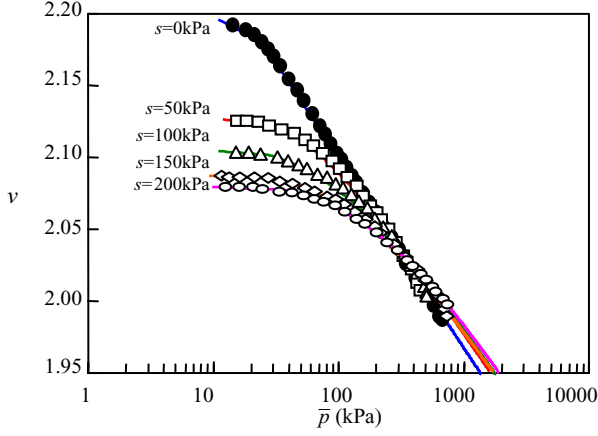


Figure 6. Predictions of isotropic compression curves for compacted kaolin (data by Thu et al., 2007).

$$\bar{p}_0 = \begin{cases} -s & s < s_{sa} \\ -s_{sa} - (s_{sa} + 1) \ln \frac{s+1}{s_{sa}+1} & s \geq s_{sa} \end{cases} \quad (3)$$

The yield stress defined by \bar{p}_0 is a fundamental property of the soil and it is often missing in other models for unsaturated soils. The shape of \bar{p}_0 is shown in Fig. 6. Drying a slurry soil under constant stresses does not change the shape of the yield loci, but only expands the elastic zone (see Fig. 2).

Equation (3) also defines the apparent tensile strength of the soil as a function of suction:

$$\bar{c} = \begin{cases} c' + s \tan \phi' & s < s_{sa} \\ c' + \tan \phi' \left(s_{sa} + (s_{sa} + 1) \ln \frac{s+1}{s_{sa}+1} \right) & s \geq s_{sa} \end{cases} \quad (4)$$

In the equation above, c' is the effective-stress cohesion for saturated soils and is usually zero unless the soil is cemented. It is usually assumed that the slope of the failure lines in the space of deviator stress versus mean stress is constant (e.g. Toll, 1990). The shear strength criterion then becomes:

$$\tau = [c' + s \tan \phi^b] + [(\sigma_n - u_a) \tan \phi'] = \bar{c} + \bar{\sigma}_n \tan \phi' \quad (5)$$

where τ is the shear strength, σ_n is the normal stress on the failure plane, ϕ' is the effective friction angle of the soil, and ϕ^b is the frictional angle due to suction (Fredlund et al. 1978).

Combining equations (4) and (5) leads to

$$\tan \phi^b = \begin{cases} \tan \phi' & s < s_{sa} \\ \tan \phi' \left(\frac{s_{sa}}{s} + \left(\frac{s_{sa}+1}{s} \right) \ln \frac{s+1}{s_{sa}+1} \right) & s \geq s_{sa} \end{cases} \quad (6)$$

In this case, the friction angle ϕ^b is a function of suction as well as the saturation suction. Equation (6) can be used to predict the change of the shear strength against suction.

Vanapalli et al. (1996) reported a series of direct shear tests on compacted glacial till obtained from Indian Head, Saskatchewan. Initial water content and dry density of test sample are 13% and 1.73 Mg/m³, respectively. The test data and the predictions by equation (6) are compared in Fig. 7. The

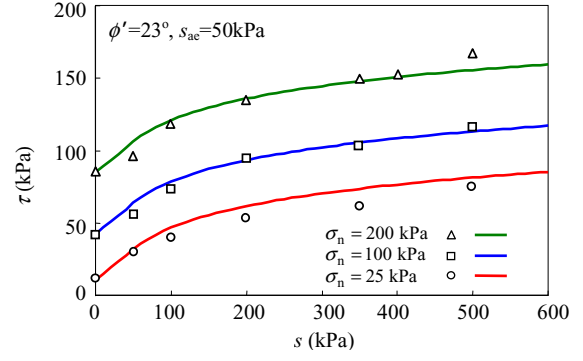


Figure 7. Shear strength versus suction during direct shear tests (data by Vanapalli et al., 1996).

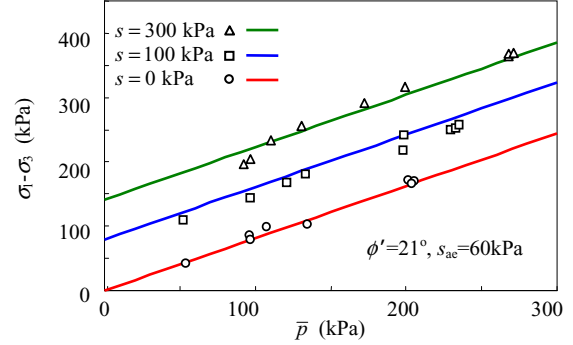


Figure 8. Shear strength versus suction during triaxial compression tests (data by Wheeler and Sivakumar, 2000).

prediction is very good, except that some overestimation is observed for the results with the smallest normal stress.

The triaxial compression test results for compacted kaolin under constant suctions by Wheeler and Sivakumar (2000) are compared with the predictions by the SFG model in Fig. 8. The predictions once more match the data very well.

3 CONCLUDING REMARKS

It should be noted that all soils can be unsaturated with respect to water. In this regard, unsaturated soils are nothing special. The emergence of unsaturated soil mechanics in the last 30 years or so is mainly due to the distinct volume, strength and flow characteristics observed when certain soils become unsaturated with water. Therefore, the key issue in constitutive modelling of unsaturated soils is how the volume change and shear strength behaviour can be considered in a consistent framework both for saturated and unsaturated states. The modelling approach recently proposed by Sheng et al. (2008) seems to be able to provide such a consistent framework.

It was shown that the common procedure for determining the yield stress or preconsolidation stress for unsaturated soils can lead to incorrect conclusion on the shape of the s yield surface. The SFG model recently presented by Sheng et al. (2008) provides a consistent explanation of yield stress, shear strength and volume change behaviour of unsaturated soils. All these functions are based on one single equation that defines the volume change with suction and stress changes. This equation is continuous and smooth over both positive and negative pore pressures. Compared to the volumetric model used for saturated clays, this equation has one additional soil parameter. This paper provides some validation of the equation and the derived shear strength criterion against experimental data. It was shown that the volume change and shear strength behaviour of unsaturated soils can well be predicted by the SFG model.

REFERENCES

- Alonso, E.E., Gens, A. and Josa, A., 1990. A constitutive model for partially saturated soils. *Geotechnique*, **40**, 405–430.
- Cunningham, M. R., Ridley, A. M., Dineen, K. and Burland, J. B. 2003. The mechanical behaviour of a reconstituted unsaturated silty clay. *Géotechnique*, 53(2), 183–194.
- Fredlund, D.G., Morgenstern, N.R., and Widger, A. 1978. Shear strength of unsaturated soils. *Canadian Geotechnical Journal*, 15: 313–321.
- Jennings, J. E. B. and Burland, J. B. 1962. Limitations to the use of effective stresses in partly saturated soils. *Géotechnique*, 12(1), 125–144.
- Sheng, D. C., Fredlund, D. G. and Gens, A. 2008. A new modelling approach for unsaturated soils using independent stress variables. *Canadian Geotechnical Journal*, 45, 511–534.
- Sheng, D., Sloan, S.W. and Gens, A. 2004. A constitutive model for unsaturated soils: thermomechanical and computational aspects. *Computational Mechanics*, 33, 453–465.
- Thu, T. M., Rahardjo, H. and Leong, E. C. 2007. Soil water characteristic curve and consolidation behaviour of a compacted silt. *Canadian Geotechnical Journal*, 44, 266–275.
- Toll, D. G. 1990. A framework for unsaturated soil behaviour. *Géotechnique*, 40(1), 31–44.
- Vanapalli, S. K., Fredlund, D. G., Pufahl, D. E. and Clifton, A. W. 1996. Model for the prediction of shear strength with respect to soil suction. *Canadian Geotechnical Journal*, 33, 379–392.
- Vaunat, J., Romero, E. and Jommi, C. 2000. An elastoplastic hydromechanical model for unsaturated soils. *Experimental Evidence and Theoretical Approaches in Unsaturated Soils* (Tarantino A and Mancuso C, eds), Balkema, Rotterdam, 121–138.
- Wheeler, S. J. and Sivakumar, V. 2000. Influence of compaction procedure on the mechanical behaviour of an unsaturated compacted clay, Part 2: Shearing and constitutive modeling. *Géotechnique*, 50(4), 369–376.
- Zhou, A. and Sheng, D. 2008. Yield stress, volume change and shear strength behaviour of unsaturated soils: validation of the SFG model, *Canadian Geotechnical Journal* (submitted 31 May 2008, manuscript no. 08-142).