INTERNATIONAL SOCIETY FOR
SOIL MECHANICS AND
GEOTECHNICAL ENGINEERING

This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

https://www.issmge.org/publications/online-library

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

Prediction of settlements and structural forces in linings due to tunnelling

S.C. Möller & P.A. Vermeer
Institute of Geotechnical Engineering, University of Stuttgart, Germany

ABSTRACT: The design of tunnels requires a proper estimate of surface settlements and structural forces in linings. As a rule three-dimensional finite element calculations are not feasible, as they are still engineering time consuming. Instead relatively simple two-dimensional analyses are required. Here effects of tunnel installation are accounted for by the so-called Contraction Method or the Load Reduction Factor, as considered in this paper. Unfortunately there is very little data on the appropriate value of the Load Reduction Factor. In order to improve this situation comparison is made between simple 2D and full 3D analyses. In this paper results for NATM-tunnels are shown. Effects of ground and tunnel lining stiffness, strength parameters, tunnel diameter and cutting length are quantified. Moreover, field data from a tunnel are considered in order to arrive at guidelines for the selection of the Load Reduction Factor.

1 INTRODUCTION

A frequently discussed topic is the question, whether structural forces in tunnel linings as well as settlements should be computed by a fully three-dimensional analysis or whether more simple two-dimensional models are sufficient. No doubt, tunnel installation involves a three-dimensional stress-strain-situation and three-dimensional Finite-Element (FE) analyses have been adopted in engineering practice, but such analyses are still time consuming. For large tunnel projects with several kilometres of excavation and various cross sections three-dimensional analysis cannot be used as a design tool and two-dimensional analyses are required.

At present two-dimensional FE-analyses are common in tunnelling practice; complex geometries and ground behaviour can be accounted for. For the calibration of such analyses assumptions have to be made in order to incorporate effects of 3D tunnel installation. In NATM tunnelling this is mostly done by means of the so-called load reduction factor, and assumptions have to be made about its magnitude. In shield tunnelling the contraction method is more popular and assumptions have to be made about the amount of volume loss.

The present paper shows, that solutions of an analytical continuum model agree well with results of the FE-Method. Hereafter three-dimensional analyses are systematically compared to two-dimensional FE-analyses in order to arrive at more information on proper load reduction factors. Analyses are carried out for the design of tunnel linings, i.e. bending moments and normal forces, as well as for settlements due to tunnelling. Finally results of structural forces in tunnel linings and surface settlements are shown for a underground railroad tunnel in Stuttgart, Germany.

2 ANALYTICAL SOLUTION

In present engineering practice numerical approaches are becoming more and more important, as in contrast to limited analytical models they can account for complex tunnel geometries and ground profiles. At the same time analytical models play a role as they provide insight and can be taken to validate complex numerical approaches.

Analytical solutions have been obtained for a circular cavity in linear elastic ground. An elastic-perfectly plastic continuum has been considered by Clough & Schmidt (1981) a.o., assuming purely radial loading.

Figure 1 presents an analytical continuum model as used by Ahrens et al. (1982). Due to non-isotropic geostatic stresses, radial and tangential displacements of the tunnel lining are predicted, i.e. the lining deforms into an oval shape. The analytical solution is based on the following simplifications and assumptions: The tunnel lining is circular with radius R. The tunnel longitudinal axis is parallel to the ground surface, inducing plane strain conditions. The thickness of the lining is constant. The lining is deforming without any expansion, i.e. Poisson's ratio is equal to zero. Soil weight is neglected in the sense, that there is a uniform initial stress field, with $\sigma_h = K_0\sigma_v$, where K_0 is the coefficient of lateral earth pressure and $\sigma_v = \gamma H$. Here γ is the unit soil weight and H is the tunnel depth as indicated in Figure 1. The lining is installed before

Copyright © 2006 Taylor & Francis Group plc, London, UK

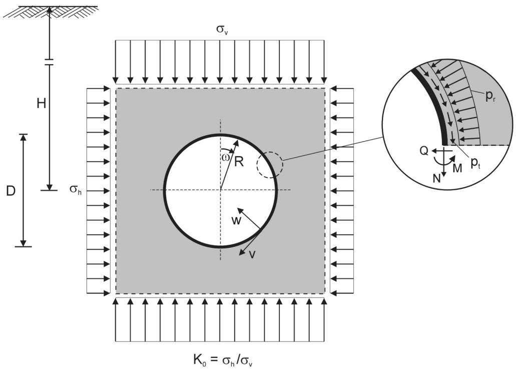

Figure 1. Analytical continuum model with initial stresses.

tunnel excavation. The lining is either rough (full bonding) or smooth with no bonding at all. Both lining and ground behave linear elastic. Second order theory can be neglected.

Erdmann (1983) supplemented the findings of Ahrens et al. to obtain Equations 1–4 for a lining with full bonding. He expressed the maximum normal force as $N = N_1 + N_2$ with

$$N_1 = \gamma H \frac{1+K_0}{2} \frac{R}{1+\frac{1}{1+\nu}\beta+\frac{\beta}{\alpha}} \tag{1}$$

$$N_2 = \frac{\gamma H \frac{1-K_0}{2} R\left(1+\frac{1}{12(1+\nu)}\alpha+\frac{1}{4(1+\nu)}\beta\right)}{1+\frac{3-2\nu}{12(3-4\nu)(1+\nu)}\alpha+\frac{5-6\nu}{4(3-4\nu)(1+\nu)}\beta+\frac{1}{12(3-4\nu)(1+\nu)^2}\alpha\beta} \tag{2}$$

where ν is Poisson's ratio of the ground and

$$\alpha = \frac{ER^3}{(EI)_l} \quad \text{and} \quad \beta = \frac{ER}{(EA)_l} \tag{3}$$

EA and EI are the normal stiffness and the flexural rigidity of the lining respectively. E is the elasticity modulus of the ground. For the maximum bending moment Erdmann found

$$M = \frac{\gamma H \frac{1-K_0}{2} R^2\left(1+\frac{1}{2(1+\nu)}\beta\right)}{2+\frac{3-2\nu}{6(3-4\nu)(1+\nu)}\alpha+\frac{5-6\nu}{4(3-4\nu)(1+\nu)}\beta+\frac{1}{6(3-4\nu)(1+\nu)^2}\alpha\beta} \tag{4}$$

3 ANALYTICAL VERSUS FE-ANALYSIS

In order to compare results of the analytical solution after Erdmann to the FE-Method, numerical analyses have been carried out for linear elastic ground and lining. The initial stresses have been taken constant over depth with $\sigma_v = 500$ kPa, $\sigma_h = K_0 \cdot \sigma_v$ and $K_0 = 0.5$. For the lining stiffness we have $EA = 3$ GN/m and

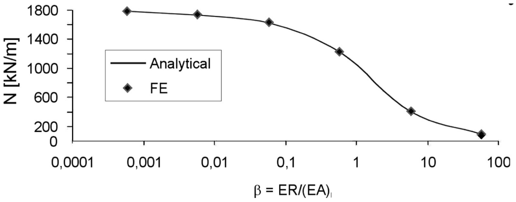

Figure 2. Analytical and numerical results for normal forces.

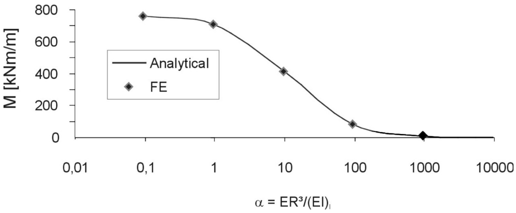

Figure 3. Analytical and numerical results for bending moments

$EI = 0,225$ GNm^2/m and a Poisson's ratio of $\nu = 0$ has been used. The radius of the tunnel has been taken as $R = 3.5$ m. The continuum has been modelled with a Poisson's ratio of $\nu = 1/3$ and the Young's modulus E has been varied such, that $0.1 < \alpha < 10{,}000$ and $0.001 < \beta < 100$.

Figure 2 and Figure 3 show results of analytical and numerical methods for bending moments and normal forces respectively. It can be seen, that FE-analyses perfectly match analytical findings.

4 2D FEM INSTALLATION PROCEDURES

Both the analytical solution and the above FE-procedure is primitive in the sense that the lining is "wished in place" without any consideration of installation procedures. On incorporating installation effects, we have to distinguish between shield and NATM tunnelling.

It would seem that shield tunnelling is best simulated by the so-called contraction procedure after Brinkgreve et al. (2002). First the lining is "wished in place" and then contracted to simulate a prescribed amount of volume loss. The overall position and the shape of the lining are left free. In contrast to analytical approaches, the ground surface, soil layers and initial stresses are properly accounted for. Moreover, advanced constitutive models can be applied.

In order to approximate sequential NATM tunnelling by two-dimensional FE-analyses, it is popular to use the load reduction method. This method involves an artificial support pressure $\beta \cdot \sigma_0$, being applied

Copyright © 2006 Taylor & Francis Group plc, London, UK

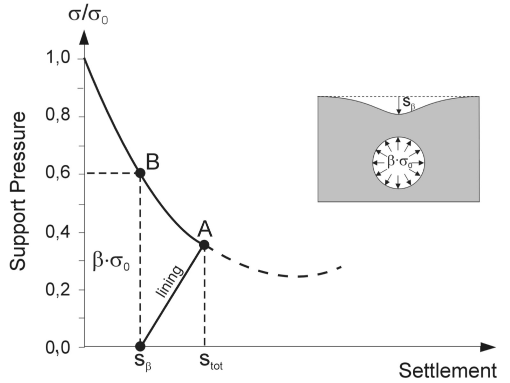

Figure 4. Display of load reduction method adopting ground response curve.

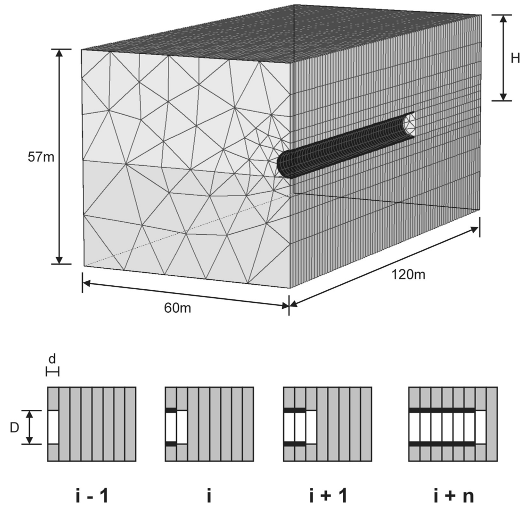

Figure 5. Three-dimensional FE-model, D = 7 m and d = 1.5 m.

before installation of the lining, as indicated in the insert of Figure 4. Here σ_0 is the initial radial stress before construction of the tunnel and β is the load reduction factor. In this paper we will simply refer to β as the unloading factor. For $\beta < 1$ a volume loss and a related settlement trough is introduced. After unloading from σ_0 down to $\beta \cdot \sigma_0$, the lining is "wished in place" and the remaining support pressure $\beta \cdot \sigma_0$ is taken away in a subsequent phase of calculation.

Figure 4 shows the two calculation phases of the load reduction method, being related to the so-called ground-response curve, i.e. the Fenner-Pacher curve. In the first phase of calculation the Fenner-Pacher curve is computed down to point B in Figure 4 with settlement S_β. At this point the lining is activated. Hereafter the second stage of computation will yield point A. In this second phase the support pressure is completely removed, the lining is loaded and the final settlement S_{tot} is reached. Structural forces in the lining occur only during the second phase. In fact, the analytical solution of Section 2 applies to this second phase.

The magnitude of the support pressure factor β, influences both settlements and structural forces. A large factor β leads to large structural forces and small settlements. A small factor leads to low structural lining forces and large settlements.

5 THREE-DIMENSIONAL FE-ANALYSIS

Until now β-values have been based on engineering judgement and values between 0.3 and 0.7 have been suggested. However, for a true calibration of the procedure, β-values may be obtained by comparing results of 2D-analyses with data from 3D-analyses. Before making such calibrations we will consider the 3D-analysis of NATM tunnels.

Figure 5 shows a three-dimensional FE-mesh in a half-symmetric condition. NATM-tunnels with a circular cross section have been analyzed. The mesh of Figure 5 comprises a tunnel diameter of $D = 7$ m and a cutting length of $d = 1.5$ m. It has a total height of 57 m, a length of 120 m and a width of 60 m. Different meshes with diameters of 9 m as well as 11 m, and cutting lengths of $d = 0.5$ m as well as 1 m have also been analysed. All tunnels had a depth of $H = 21\text{ m} + 0.5 \cdot D$ and a mesh width of 60 m. The minimum mesh length was taken eighty times the cutting length d. All meshes were large enough to exclude influences of mesh boundaries.

To simulate the excavation process of NATM-tunnels the so-called 'step-by-step'-method has been used. Starting from initial geostatic stresses, the excavation sequence is as indicated in Figure 5. Ground elements inside the tunnel are removed to simulate an unsupported excavation with a particular cutting length. Each computational phase i consists thus of an excavation, in which one slice of soil elements is switched off. Within the same phase a ring of lining elements is switched on to support the previous excavation $i - 1$. These calculation phases are repeated in steps $i + 1$ to $i + n$ until a representative steady-state solution is obtained.

Figure 6 shows computed normal forces after an excavation of $50 \cdot d$. At first sight the displayed zigzagging pattern of normal forces seams somewhat peculiar. But in fact it is logical, as the unsupported tunnel head is arching on the front and not on the back of a tunnel segment, resulting in high normal forces at the front and low normal forces at the back of a ring of lining. These normal force distributions approach a

Copyright © 2006 Taylor & Francis Group plc, London, UK

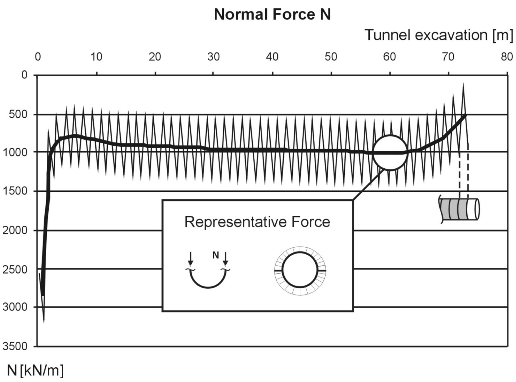

Figure 6. Evaluation of three-dimensional normal forces.

steady-state solution after a total excavation length of about $10 \cdot d$.

The significant increase of normal forces at the left mesh boundary is non-physical but related to effects of the FE-mesh boundaries. In order to compute β-values for the two-dimensional analyses the average value of these zigzag distributions (thick line in Figure 6) has been considered. The representative structural forces from the three-dimensional analyses were taken in the middle of the tunnel lining as indicated in the insert of Figure 6.

6 ON GROUND AND LINING STIFFNESSES

In tunnelling a large volume of ground is excavated so that the ground underneath the tunnel is unloaded. Moreover, stress decrements decrease with depth so that the ground underneath the tunnel experiences on average only very small stress decrements. In such cases the ground will behave extremely stiff. Rather than using complex material models which account for stress and strain-level depending stiffnesses and which distinguish between loading and unloading, computations will be carried out for relatively simple material models with a constant stiffness modulus E both for loading and unloading. In order to account to some extend for the unloading and the small strain in the deep ground below the tunnel, a two-layer mesh is used for all analyses and the deep ground layer is taken much stiffer than the upper layers.

The lining shell elements were simulated by using a linear elastic model. The increase of the shotcrete lining stiffness E_l with time has been accounted for by using a stepwise increase with excavation phases. Starting from an initial stiffness of $E_l = 7500\,\text{MN/m}^2$ in step i, an increase to $E_l = 15000\,\text{MN/m}^2$ in step $i + 1$ has been adopted.

7 2D- VERSUS 3D-FE-ANALYSES

For a particular tunnel geometry and a particular set of material properties, the finding of the corresponding β-value sometimes requires an iterative procedure. This is best explained by considering the ground response curve as shown in Figure 4. Here a complete ground response curve is obtained from a two-dimensional FE-analysis and the settlement S_{tot} is obtained from a three-dimensional FE-analysis. For tunnels in truly elastic ground, the line from point A to point S_β in Figure 4 is linear. In such a linear case it is straightforward to compute the inclination of line A–S_β and to find point B with the corresponding β-value. This simple procedure even applies to some non-linear grounds. Indeed, for deep tunnels with $K_0 = 1$, there is a more or less axis symmetric state of stress around the tunnel giving a cylindrical compression of the lining. As long as this lining behaves elastically line A–S_β in Figure 4 will be linear. For shallow tunnels in non-linear ground, however, circular tunnel linings deform into ellipses with complex stress distributions and line A–S_β is not longer linear. In such cases point B with its corresponding β-value has to be computed iteratively.

The above β-finding procedure applies to settlements. Instead of considering settlements, one may also compute ground response curves for bending moments and/or normal forces in linings. The settlement in Figure 4 might for instance be replaced by the maximum bending moment or the maximum normal force, as both of them increase when reducing the support pressure $\beta \cdot \sigma_0$. No doubt, all three curves will be somewhat different and one will find three different factors: β-settlement, β-bending moment and β-normal force. In the following results of β-values of bending moments and normal forces will not be plotted as discrete curves but are shown in a band width for structural forces in general.

8 LINEAR ELASTIC ANALYSES

First of all a linear elastic constitutive model for the ground behaviour has been used to investigate the influences of ground stiffness E, tunnel diameter D and cutting length d on β. All analyses were carried out for situations without ground water. A unit weight of $\gamma = 20\,\text{kN/m}^3$ and a Poisson's ratio of $\nu = 0.3$ was taken for all analyses. Moreover, results have been obtained from a tunnel with a depth of $H = 24.5\,\text{m}$ and $K_0 = 0,67$. Full bonding between lining and ground has been assumed. The results shown here were obtained from a tunnel with $D = 7\,\text{m}$, but are also valid for $D = 9\,\text{m}$ and $D = 11\,\text{m}$.

Figure 7 shows the influence of ground stiffness and cutting length on the unloading factor. Both for

Copyright © 2006 Taylor & Francis Group plc, London, UK

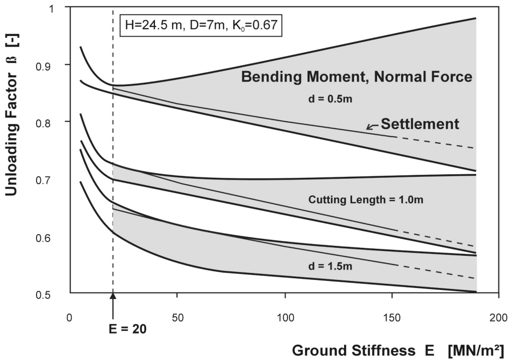

Figure 7. Influence of ground stiffness and cutting length for linear elastic ground.

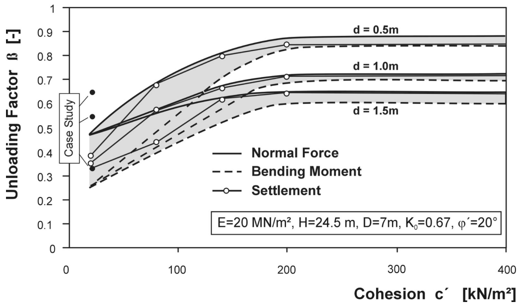

Figure 8. Influence of friction angle, cohesion and cutting length for elasto-plastic ground.

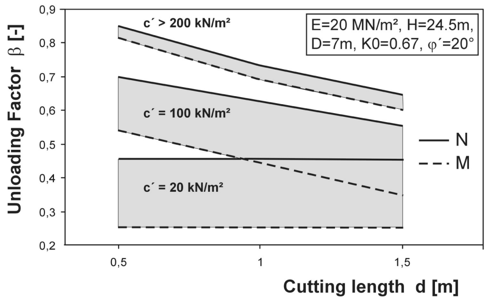

Figure 9. Influences of cutting length and cohesion.

structural forces and settlements the unloading factor is decreasing with increasing cutting length. This seams logical, as a tunnel without a lining, i.e. $d=\infty$, has an unloading factor of $\beta=0$ (no forces in the lining). Because of different unloading factors for bending moments and normal forces a band width for structural forces is plotted. With increasing ground stiffness a decrease of unloading factors is observed. The β-values for settlements always remain within the bandwidth for structural forces.

The elasticity calculations show the considerable influence of the cutting length on β. On considering for example a ground stiffness of $E=20$ MPa, we find $\beta\approx 0.85$ for a very short cutting length of $d=0.5$ m, but a much lower value of $\beta\approx 0.65$ for a long cutting length of $d=1.5$ m.

Elasticity computations give qualitative information on the β–d relationship and possibly quantitative information for tunnels in rock. Indeed, rock has a considerable shear strength and tunnelling will be dominated by the quasi elastic rock properties. On the other hand, soil has a relatively low shear strength and NATM tunnelling will be dominated by plastic deformation rather than quasi elastic deformation. Therefore plastic soil behaviour is considered in the following section.

9 ELASTO-PLASTIC ANALYSES

Following the linear elastic evaluations the elasto-plastic Mohr-Coulomb model has been used to determine the influences of the shear strength parameters by considering $\varphi'=20°, 30°$, and $40°$. For this realistic range of friction angles, their influence appeared to be relatively small. In all analyses dilation was neglected by using a dilatancy angle equal to zero.

Figure 8 shows computed unloading factors as a function of the effective cohesion for different cutting lengths. With increasing cohesion the computed β-values approach a horizontal plateau that corresponds to the linear elastic solution. Up to $c'=200\,\text{kN/m}^2$ Figure 8 shows a significant influence of the cohesion, i.e. for tunnels in soft to hard soils. Here, the c–β relationship is nearly linear. Beyond an effective cohesion of $200\,\text{kN/m}^2$, i.e. for tunnels in rock, the cohesion plays no role at all. The information of Figure 8 is also plotted in Figure 9, but this time β is plotted as a function of the cutting length. Again one observes that the unloading factor decreases both as a function of cutting length and cohesion.

10 CASESTUDY OF A SUBWAY TUNNEL

Figure 10 shows a three-dimensional FE-mesh of the “Steinhaldenfeld” tunnel in Stuttgart. This NATM tunnel was recently constructed and constitutes an extension of the Stuttgart subway system. In addition to a full 3D FE-analysis, 2D analyses have been carried out in order to arrive at unloading factors for this particular tunnel in layered ground.

The ground is dominated by two layers of over consolidated marl. The drained ground behaviour was described by the Mohr-Coulomb model with parameters given in Table 1. The ground around the tunnel was improved by nails and this was numerically taken

Copyright © 2006 Taylor & Francis Group plc, London, UK

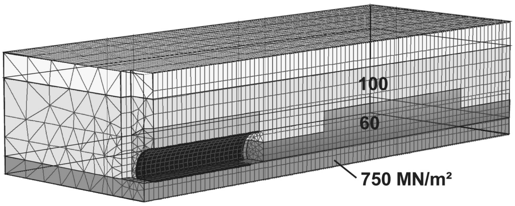

Figure 10. Three-dimensional FE-model of the subway tunnel.

Table 1. Ground properties of Steinhaldenfeld tunnel.

	Cover layer	Upper Keuper Marl	Lower Keuper Marl	Limestone
γ [KN/m^3]	20	24	23	23
E [MN/m^2]	15	100	60	750
ν [−]	0.375	0.2	0.35	0.2
φ' [°]	25	25	25	35
c' [KN/m^2]	10	25	25	200

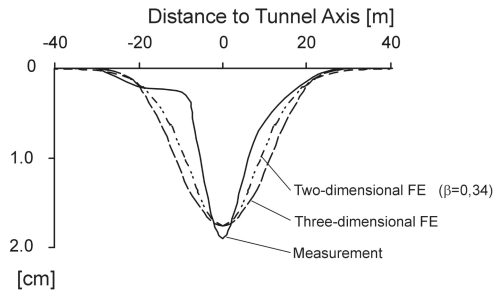

Figure 11. Calculated and measured settlement trough.

into consideration by an increase of the cohesion in a zone around the tunnel.

Figure 10 shows a relative thin ground layer underneath the tunnel. The deeper rock is simply omitted as the stiffness of the deeper layers is approximately ten times larger than the stiffness of the layers on top. For the three-dimensional FE-analysis of the top heading hoarse shoe profile an excavation of 57 cutting lengths with $d = 1.2$ m has been simulated.

Figure 11 shows the results of computed and measured settlement troughs. The best fit for a 2D settlement analysis was found for $\beta = 0.34$. Structural forces in Figure 12 were best matched with $\beta = 0.54$ for the bending moment and $\beta = 0.64$ for the normal force. Again it becomes obvious, that different unloading factors for settlements as well as for bending moments and normal forces have to be used. The β-values for the settlement of Steinhaldenfeld tunnel perfectly match the present study of Figure 8 but

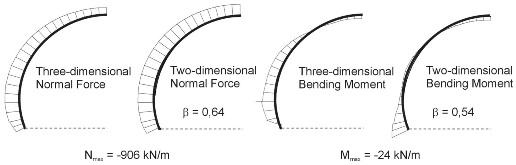

Figure 12. Comparison of 3D- and 2D- structural forces.

unloading factors for structural forces are larger than the results obtained. However, this is believed to be an effect related to the hoarse shoe profile of this particular tunnel, as structural forces are strongly influenced by the shape of the lining cross section.

Further details on the settlement trough of Steinhaldenfeld-tunnel can be found in Möller et al. (2004).

11 CONCLUSIONS

It has been shown that analytical solutions are well matched by the FEM, which proves the accuracy of 2D FE-analyses in tunnelling. In order to incorporate effects of 3D tunnel installation, 2D FE-analyses are applied in combination with an empirical Load Reduction Factor. This factor represents roughly the portion of the initial soil pressure that is carried by the lining. For assessing this factor, we have performed series of 3D FE-analyses.

The load reduction factor was found to decrease as a function of ground stiffness. This is logically as the load and the lining will decrease as the ground stiffness increases. Moreover, it was found that the reduction factor decreases as a function of the cohesive strength and the cutting length in NATM tunnelling. Again it is logical that the load portion of the linings is reduced for increasing cohesion and cutting length.

It was somewhat surprising to find that the soil friction angle has relatively little influence on the load reduction factor, but this is an issue of further research as relatively few variations were carried out.

In the presented studies we modelled $K_0 = 1 - sin\ \varphi'$. It was observed that larger K_0-values lead to a slight increase of bending moments. Effects will be more significant for over consolidated ground, but this has not been considered in these studies.

In this paper effects of depth H and tunnel diameter D have not been discussed in detail. However, variations that were carried out on the relative tunnel depth H/D showed that unloading factors are slightly decreasing with increasing H/D. The influence of the relative tunnel depth will be considered in detail in further research.

Considering load reduction factors it has been shown, that it is not suitable to use one single β-value for bending moments, normal forces and settlements. Instead one needs to use three different values in

Copyright © 2006 Taylor & Francis Group plc, London, UK

order to compute appropriate structural forces and settlements.

The load reduction factor for settlements of particular tunnel with a non-circular cross section in layered ground perfectly matches present findings, whereas load reduction factors of structural forces were found to be somewhat larger. This makes it clear, that present β-values for structural forces only hold true, when considering circular tunnels. Moreover, influences of securing means like anchors etc. or complex ground layering as adopted for this case study still have to be evaluated. For assessing structural forces of such complex tunnels it is more appropriate to perform a 3D FE-analysis.

For tunnels in soil the displacements are well matched when using β-values between 0.3 and 0.4. For bending moments we find approximately the same values as long as the soil cohesion is below 50 kN/m^2 and circular tunnels are considered. For a particular non-circular tunnel however, the bending moment was matched with $\beta = 0.54$ and it would seem that low β-values are not appropriate for bending moments. A conservative approach for structural forces would be to use β-values of at least 0.5–0.7 in soil.

REFERENCES

Ahrens, H., Lindner, E., Lux, K.-H. 1982. *Zur Dimensionierung von Tunnelausbauten nach den "Empfehlungen zur Berechnung von Tunneln im Lockergestein (1980)"*. Die Bautechnik 8: 260–273.

Brinkgreve, R., Vermeer, P.A. 2002. *Plaxis Version 8*. A.A. Balkema. Rotterdam.

Clough, G.W., Schmidt, B. 1981. *Design and performance of excavations and tunnels in soft clay.* In: Soft Clay Engineering. Elsevier: 569–634.

Erdmann, J. 1983. *Vergleich ebener und Entwicklung räumlicher Berechnungsverfahren für Tunnel.* Bericht Nr. 83–40. Institut für Statik, TU Braunschweig.

Möller, S., Lehmann, T., Rogowski, E. 2004. *Dreidimensionale Finite-Element-Berechnung der Setzungsmulde am Beispiel des Steinhaldenfeldtunnels in Stuttgart*. Kolloquium Bauen in Boden und Fels. Tagungsband 4. Ostfildern: TAE: 275–282.

Copyright © 2006 Taylor & Francis Group plc, London, UK