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Probabilistic back-analysis of K_0 and deformation properties of fine soils found in Line 2 of Santiago Metro, Chile

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ABSTRACT: Field measurements were taken during construction of a double track tunnel for the northern extension of Santiago Metro Line 2 in Chile, between 2004 and 2005. The tunnel was driven through a thick deposit of a fine fluvial-lacustrine sediment, consisted mainly by silts and clays with low deformability, with consistency moderate to high and with presence of isolated thin lenses of silty fine sand, gravel and volcanic ash. The monitoring data made possible Bayesian updating of the coefficient of earth pressure at rest (K_0) and of the distribution of soil deformability with depth, represented by a hyperbolic model that requires knowing the initial tangent modulus (E_i). The back-analysed values for K_0 and modulus distribution with depth were different from what was used for design of Line 2 and were incorporated into the design of the new Line 3 under construction for Santiago Metro.

1 INTRODUCTION

The construction of the new Line 3 of Santiago Metro will add about 22 km to the current underground network of Chile capital city, delivering 18 new stations and enabling connections with the existing Lines 1, 2, 4 and 5 and with the future Line 6, also under construction. Figure 1 presents the Santiago Metro network on a simplified geological map.

Two of the most important types of ground found in Santiagoare a well graded gravel with a fine matrix, called Santiago Gravel (see for instance Queiroz *et al.*, 2005) and a fine fluvial-lacustrine sediment, referred to as North-Western Fine Soils of Santiago (see for instance Poblete, 2004). The former, laying in the central portion of the city, has been thoroughly investigated, whereas the latter demands further studies.

Both soils present reasonably complex geological formation, involving alluvial, gravitational and glacial mass transport. To this is added a complex and active tectonic scenario of the bedrock, conditioned by the colliding Nazca – South America plates in a subduction fault below the Pacific Ocean along the western continent rim, causing compressive straining accounting about 10 cm of convergence per year.

Thus, the assessment of the coefficient of earth pressure at rest (K_0) for Santiago soils is not a simple task.

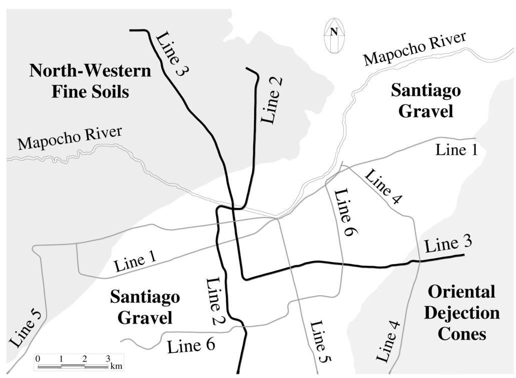

Figure 1. Location of the Metro network on a simplified geological map of Santiago city, modified from Valenzuela (1978).

Though recognised as a parameter that strongly conditions tunnel design, it is rarely investigated in routine practice that prefers estimates based on accumulated geotechnical knowledge.

In order to compensate the lack of accumulated knowledge on the Santiago fine soils encountered along the Metro Line 3, a probabilistic back-analysis of K_0 and of deformation properties was undertaken on the basis of field monitoring data gathered during construction of Metro Line 2 also driven through fine soils.

Estimation of soil parameters based on probabilistic back-analysis of tunnel monitoring data is a known technique. Queiroz *et al.* (2004, 2005) have used Bayesian updating to estimate K_0 of Santiago gravels for tunnels of Metro Line 5 and found values from 0.79 to 1.04, higher than the 0.30 estimate based on Jacky (1944) expression used in the design of Line 5 tunnels.

2 THE BAYESIAN UPDATING TECHNIQUE

The Bayesian probabilistic back-analysis consists on the application of the Bayes theorem, by means of conditional probabilities. A conditional probability relates the chance of certain event to occur, having the information that another one has already occurred.

For instance, having established the probability distribution of a given geotechnical parameter (mean value and standard-deviation), one can determine its probability distribution conditioned to the realisation of some event, such as a collapse, a change of the predicted behaviour or any other type of performance measurement. This new probability relates to a specific case and, therefore, is associated to a minor variability than the former one (smaller standard-deviation).

The updating begins with the preceding acknowledge of the first and second order moments of the variables to be updated (state variables), i.e. the vector of mean values {m'} and the matrix of covariance Cov[m'], respectively.

The monitoring data from Line 2 of Santiago Metro were used as performance parameters. They act as the conditional event of the Bayes theorem. Consistently to the state variables, the performance parameters are also represented by a vector of mean values {P} and a matrix of covariance Cov[P].

Moreover, the monitoring may also present a vector of errors $\{\nu\}$, related to a possible measuring error trend of a certain instrument (systematic errors). A positive value represents the data overestimated by the monitoring, whereas a negative value corresponds to the underestimated data.

The Bayesian updating is based on the comparison of the performance parameters {P} and the predicted values for the performance parameters {mp}.

The probabilistic back-analysis methodology presented herein is based on Sage & Melsa (1971) and Hachich (1981), which considers a linear relationship between the initial state variables (m') and the predicted values of the performance parameters (mp):

$$\{mp\} = [S].\{m'\} + \{A\} + \{\nu\} \qquad (1)$$

where [S] is the linear coefficients matrix and [A] is the independent terms vector for the hyperplan adjusted to the performance prediction values (least squares method).

Hyperplan is a generalisation of a more than three dimensions plan. For this study, the linear relationship is represented by a three-dimensional plan, in which the estimate depends on K_0 and the deformability modulus.

The updating of {m′} is accomplished by:

$$\{m''\} = \{m'\} + Cov[m'].[S]^T.([S].Cov[m'].[S]^T + Cov[P])^{-1}.(\{P\} - \{mp\}) \qquad (2)$$

while the updating of Cov{m′} is done by:

$$Cov[m''] = Cov[m'] - Cov[m'].[S]^T.([S].Cov[m'].[S]^T + Cov[P])^{-1}.[S].Cov[m'] \qquad (3)$$

where {m''} is the vector of updated parameters K_0 and deformability modulus, accordingly to the monitoring data.

One of the advantages of the Bayesian back-analysis in comparison to the deterministic back-analysis is that the latter requires that the number of performance parameters (observed data) should be equal to the number of parameters to be updated. Conversely, the probabilistic back-analysis allows any number of observed data to be confronted with the parameters to be updated.

Furthermore, the Bayesian back-analysis takes into account the variability of each performance parameter (standard-deviation of each monitoring data), giving more importance (or more weight in the calculations) for the data that presents minor dispersion (more certainty). This reduces the chances in obtaining unrealistic parameters, what might be the case in some deterministic back analysis.

3 TUNNEL PERFORMANCE PREDICTION

In order to determine the predicted values of the performance parameters (mp), the method developed by Negro (1988) was used. It is a numerically derived model based on 2D and 3D finite elements analysis. This method enables the performance prediction of shallow circular tunnels, driven in soil under plane and horizontal ground surface.

Negro's method considers a non-linear stress-strain relationship for the soil, by means of a hyperbolic constitutive model. The method accounts for the three-dimensional ground stress release occurring during tunnel excavation, for a given distance of lining activation behind the tunnel face.

The increasing ground stiffness with depth is also considered within this model. A linear elastic behaviour is assumed for the lining.

Although derived for circular tunnels, Negro's method also yields reasonable predictions for tunnels with quasi-circular cross-sections, such as the majority of the conventionally built tunnels (commonly known as NATM tunnels). This method has been used in numerous tunnel designs with a wide variety of soil conditions and types, resulting in performance predictions that represent well the actual performance measured in the field, whenever good ground control conditions are fulfilled, with absence of ground loosening. Applications of this method to conventional tunnelling are discussed by Negro & Eisenstein (1997) and Negro *et al*. (1998).

The prediction of tunnels performance furnished by Negro's method is expressed in terms of surface and sub-surface ground displacements, transversal and longitudinal distortions at the surface and over the tunnel crown, radial lining displacements, radial and tangential stresses acting on the soil/lining interface and lining acting loads. The results for the lining and for the soil/lining interface are obtained for points around the tunnel (crown, springline and floor).

4 THE BACK-ANALYSIS OF SANTIAGO FINE SOILS

4.1 *Parameters taken as constant*

The double track tunnel from the northern sector of Line 2 of Santiago Metro, considered in this analysis, was built according to the geometry shown in Figure 2.

This tunnel was built by conventional tunnelling method using sprayed concrete as initial support, in a heading and bench excavation sequence, with 1 m

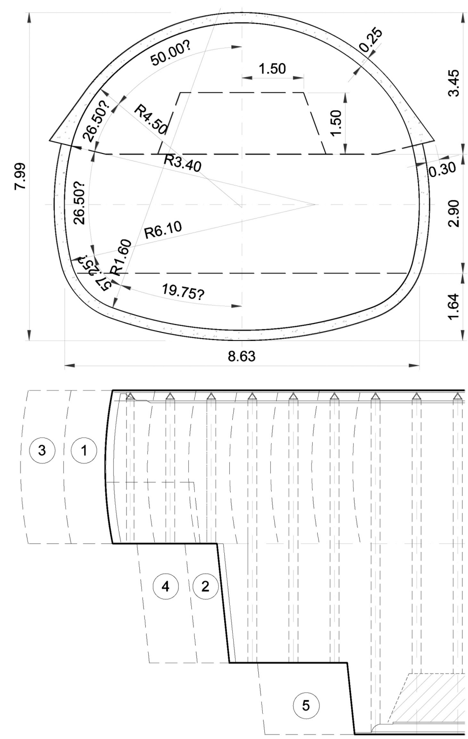

Figure 2. Geometry of the double track tunnel in the northern sector of Line 2: a) cross-section and b) construction sequence.

advancing steps and with inverted arch closure varying from 4 to 6 m behind the tunnel leading face. The support system includes a sprayed concrete primary lining reinforced with steel meshes and lattice girders at 1 m interval, as well as cast in place secondary lining.

For the present study, the following dimensions were taken as constant values along the tunnel lengths under analysis:

- excavation area, $A = 60.42\,m^2$
- excavation height, $H = 7.99\,m$
- excavation width, $W = 9.13\,m$
- soil cover, $C = 11.50\,m$
- lining thickness, $t = 0.25\,m$
- distance of lining activation, $d = 6.00\,m$

For the sprayed concrete lining, a deformability modulus of 10 GPa and a Poisson ratio of 0.20 were considered. The low deformability value is consistent with previous experiences for early age shotcrete, which accounts for hardening and creep under first loading.

The soil profile was very uniform and constant along the sections of analysis. The tunnels were driven through the Santiago's North-Western Fine Soils, under a cover of the same soil, part of a thick deposit of fine fluvial-lacustrine sediments consisted mainly by silts and clays with low deformability, with consistency moderate to high and with isolated presence of thin lenses of silty fine sand, gravel and volcanic ash.

Besides the back-analysed K_0 and deformability modulus, the other geotechnical parameters considered for the North-Western Fine Soils were those also used for the Line 3 design:

- specific weight, $\gamma = 17.5\,kN/m^3$
- effective cohesion, $c' = 30\,kPa$
- effective friction angle, $\phi' = 30°$
- Poisson ratio, $\nu = 0.30$

The soil deformability was represented by a hyperbolic stress-strain relationship, based on the model proposed by Duncan y Chang (1970) and expressed by:

$$\varepsilon_1 = \frac{1}{E_i} \cdot \frac{q}{1 - \dfrac{q}{q_a}}, \qquad \text{for } q < q_f, \tag{4}$$

where the major principal strain (ε_1) depends on the initial tangent deformability modulus (E_i), on the deviatoric stress (q) and on the deviatoric stress of the hyperbol asymptote (q_a). The latter relates to the failure deviatoric stress (q_f) by the resistance ratio (R_f), as:

$$R_f = \frac{q_f}{q_a} \tag{5}$$

The initial tangent modulus (E_i) relates to the deformability modulus at 50% of failure (E_{50}) by:

$$E_i = \frac{2 \cdot E_{50}}{2 - R_f} \tag{6}$$

For the design of Line 3 tunnels, and therefore also for this Line 2 back-analysis, it was considered an exponential variation of the *in-situ* modulus E_{50} with the effective confinement stress σ'_c (and consequently with depth), according to the equation:

$$E_{50} = E_{50}^{ref} \cdot \left(\frac{\sigma'_c \cdot sen(\varphi') + c' \cdot \cos(\varphi')}{p^{ref} \cdot sen(\varphi') + c' \cdot \cos(\varphi')} \right)^m \quad (7)$$

where the reference modulus (E_{50}^{ref}) refers to the reference stress (p^{ref}), while the exponent (m) defines the format of the curve that represents the modulus variation.

These complementary parameters (R_f, p^{ref} and m) considered for the North-Western Fine Soils were also those used for the Line 3 design:

- resistance ratio, $R_f = 0.90$
- reference stress, $p^{ref} = 100\,kPa$
- curve exponent, $m = 1.00$

It should be noted that, by taking the curve exponent equal to one, a linear relationship between E_{50} and σ'_c results.

4.2 *Initial probability distributions of the parameters to be back-analysed*

The Bayesian updating starts by establishing the mean values and the variability of the parameters to be updated, the state variables K_0 and E_{50}^{ref}:

$$\{m'\} = \begin{Bmatrix} \mu_{K_0} \\ \mu_{E_{50}^{ref}} \end{Bmatrix} \quad (8)$$

$$Cov[m'] = \begin{bmatrix} Var\,[K_0] & Cov\,[K_0; E_{50}^{ref}] \\ Cov\,[E_{50}^{ref}; K_0] & Var\,[E_{50}^{ref}] \end{bmatrix} \quad (9)$$

The covariance between K_0 and E_{50}^{ref} was assumed equal to zero, thus a given value of a parameter does not influence the other. A normal (Gaussian) probability distribution was assumed for the state variables, whose mean values and standard-deviations are defined in what follows.

The equation by Mayne & Kulhawy (1982) for over-consolidated sedimentary soils was considered valid for application to the soils under analysis:

$$K_0 = (1 - sen\,\varphi').OCR^{\,sen\,\varphi'} \quad (10)$$

Poblete (2004) determined the over-consolidation ratios (OCR) for a soil deposit called Oriental Dejection Cones (see Figure 1), with values ranging from 4 to 10. Considering liberally OCR values varying from 1 to 10 for the North-Western Fine Soils, equation 10 yields $K_0 = 0.50$ and $K_0 = 1.58$, respectively. Conveniently, the lower and upper limits 0.40 and 1.60 were adopted. Considering a confidence level of 5% for the lower limit and of 95% for the upper limit, the first and second order moments of the K_0 probability distribution were determined as:

- mean value, $\mu_{Ko} = 1.00$
- standard-deviation, $\sigma_{Ko} = 0.365$

During the design of Line 2 (North-Western Fine Soils) and the design of Line 4 (Oriental Dejection Cones), samples were collected from open pits in depths of up to 25 m for laboratory testing which included physical characterization, consolidation tests, unconfined compression and triaxial tests. Moreover, plate horizontal loading tests were performed on pit walls and Ménard pressuremeter tests were run in probing boreholes. The values of E_{50} obtained from these tests are presented in the Figure 3(a). For each value of E_{50}, a corresponding value of reference deformability modulus (E_{50}^{ref}) was calculated through equation 7. The results found are presented in the Figure 3(b).

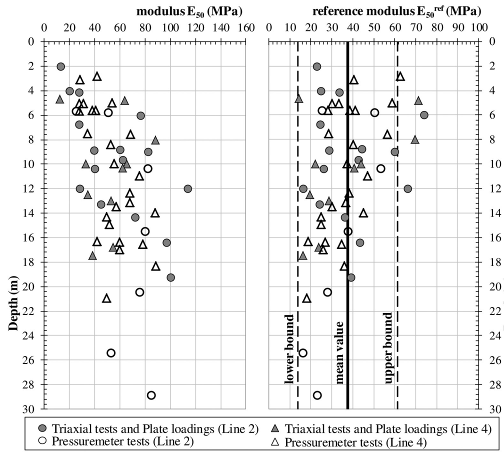

Figure 3. Variation of the deformability modulus with depth: a) E_{50} tests results and b) calculated values of E_{50}^{ref}.

From all calculated values of E_{50}^{ref}, the first and second order moments of its probability distribution were determined:

- mean value, $\mu_{E50ref} = 37.64\,MPa$
- standard-deviation, $\sigma_{E50ref} = 18.49\,MPa$

4.3 *Probability distributions of the performance parameters*

The field monitoring installed during construction of Line 2 included measurements of surface settlements, tunnel lining displacements measured by total station and pressures onto the lining by embedded contact pressure cells, as shown in the Figure 4.

A total of 38 monitoring sections were selected for analysis, taking into account the largest possible number and types of instruments in each section.

The performance parameters used as conditional event of the Bayes theorem were the surface settlement

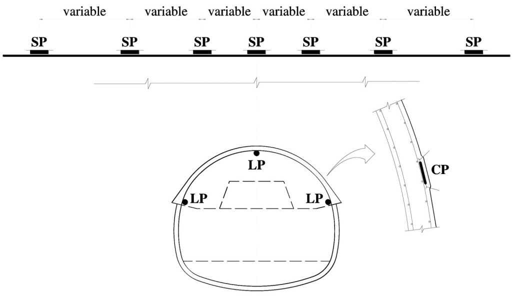

Figure 4. Field monitoring: surface settlement points (SP), lining displacements points (LP) and contact pressure cell (PC).

at the tunnel symmetry axis (ρ_{sup}), the calculated values of the maximum transversal angular distortion at surface (β), the lining displacements in the vertical direction at the tunnel crown (δ_v) and in the horizontal direction at the tunnel springline (δ_h), and the radial ground stresses acting onto the lining at the crown (σ_v) and at the springline (σ_h).

The values of maximum angular distortion at surface were calculated from the Gauss curves adjusted to the transversal distribution of surface settlements, fitted to measured data by the least squares method.

There is a strong non-linearity between the parameters to be back-analysed and the predicted performance parameters ρ_{sup} and β, as discussed in the next section. Accordingly, their logarithms were used instead, $Ln(\rho_{sup})$ and $Ln(\beta)$, to render linearization possible.

Thus, the other monitoring data score 7 performance parameters. The vector of mean values {P} and the matrix of covariance Cov[P] were determined as:

$$\{P\} = \begin{Bmatrix} \mu_{Ln(\rho_{sup})} \\ \mu_{Ln(\beta)} \\ \mu_{\delta_{h\,left}} \\ \mu_{\delta_v} \\ \mu_{\delta_{h\,right}} \\ \mu_{\sigma_v} \\ \mu_{\sigma_h} \end{Bmatrix} = \begin{Bmatrix} 1.62 \\ 7.76 \\ 0.38 \\ 2.50 \\ 0.43 \\ 0.04 \\ 0.06 \end{Bmatrix}$$

$$Cov[P] = \begin{bmatrix} 0.25 & -0.20 & -0.07 & 0.08 & 0.36 & 0 & 0 \\ -0.20 & 0.29 & 0.50 & 0.81 & -0.63 & 0 & 0 \\ -0.07 & 0.50 & 4.44 & 0.17 & -0.38 & -0.07 & 0.04 \\ 0.08 & 0.81 & 0.17 & 6.91 & -2.02 & -0.03 & 0.01 \\ 0.36 & -0.63 & -0.38 & -2.02 & 2.81 & -0.04 & 0.02 \\ 0 & 0 & -0.07 & -0.03 & -0.04 & 0 & 0 \\ 0 & 0 & 0.04 & 0.01 & 0.02 & 0 & 0 \end{bmatrix}$$

A systematic error of −0.046 MPa was accounted for the pressure cells, as suggested by Queiroz, *et al.* (2005). The negative sign is due to the fact that embedded pressure cells tend to underestimate ground stresses acting on tunnel linings.

4.4 *Predicted performance parameters*

According to equation (1), the predicted performance parameters shall present a linear relationship with the state variables.

A strong non-linearity was noted between the predicted values of surface settlements at the tunnel axis (ρ_{sup}) and the state variables K_0 and E_{50}^{ref}, as well as between the predicted values of maximum angular distortion (β) at surface transversally to the tunnel and the state variables. Conversely, a linearity could be noted between the $Ln(\rho_{sup})$ and $Ln(\beta)$ and the state variables, justifying the use of the logarithms instead of the performance parameters proper.

A singularity was observed for the prediction of the lining displacements at the tunnel springline (δ_h) and at the tunnel crown (δ_v). Values higher than $K_0 = 1.00$ resulted in wall convergence and in crown heave, while lower values resulted in wall divergence and in crown settlement. Though these performance parameters presented a reasonably linear relationship with the state variables, the three-dimensional plan fitted to the predicted values did not depict properly the intercepting curves resulted from the referred singularity.

The predicted values of radial ground stresses acting onto the lining at the tunnel crown (σ_v) and at the springline (σ_h) revealed an also reasonably linear relationship with the state variables. The influence of E_{50}^{ref} was noted to be more pronounced the higher is K_0.

The resulting three-dimensional plan that delivers the linear relationship between the state variables and the predicted performance parameters is thus represented by the linear coefficients matrix and the independent terms vector:

$$[S] = \begin{bmatrix} -4.00x10^0 & -2.86x10^{-2} \\ 5.15x10^0 & 2.86x10^{-2} \\ 7.72x10^0 & 3.84x10^{-3} \\ -5.76x10^0 & -8.79x10^{-3} \\ 7.72x10^0 & 3.84x10^{-3} \\ 1.41x10^{-1} & 1.42x10^{-3} \\ 7.03x10^{-2} & -3.73x10^{-4} \end{bmatrix} \{A\} = \begin{Bmatrix} 6.59x10^0 \\ 2.06x10^0 \\ -7.26x10^0 \\ 7.14x10^0 \\ -7.26x10^0 \\ -8.51x10^{-2} \\ 6.49x10^{-2} \end{Bmatrix}$$

4.5 *Results of the probabilistic back-analysis*

The initial probability distributions of the parameters to be back-analysed (state variables) were presented in section 4.2. These parameters were updated based in the monitoring data registered during construction of the Line 2 tunnels (performance parameters), whose probability distributions were presented in section 4.3. The linear relationship between the state variables and the predicted performance parameters is presented in section 4.4.

The back-analysed parameters were updated according to equations (2) and (3), furnishing:

$$\{m\} = \begin{Bmatrix} 0.84 \\ 40.13 \end{Bmatrix} and Cov[m] = \begin{bmatrix} 0.0036 & -0.93 \\ -0.93 & 245.82 \end{bmatrix}$$

From this, the mean values and standard-deviations of the back-analysed parameters are:

for K_0, $\mu = 0.84$ and $\sigma = 0.06$
for E_{50}^{ref}, $\mu = 40.1$ MPa and $\sigma = 15.7$ MPa

It was noted that the variance of K_0 is less than that of E_{50}^{ref}, since a small variation of K_0 produces significant changes in all performance parameters,

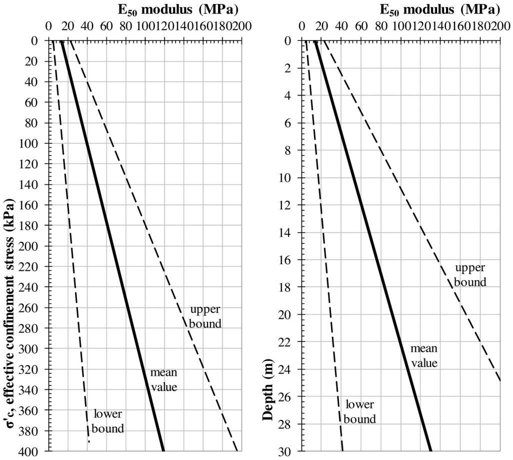

Figure 5. Variation of E_{50} modulus a) with the confinement stress and b) with depth, calculated with the updated values of E_{50}^{ref} and K_0.

whereas a large variation of E_{50}^{ref} does not change much them. Thus, as the effect of K_0 is more pronounced, the Bayesian updating attributes more certainty to its updated value, reducing its variance.

It is difficult to fully interpret the meaning of E_{50}^{ref}, since the variation of the E_{50} modulus with the confinement stress also depends on the specific weight (γ), the effective cohesion (c′), the effective friction angle (ϕ'), the reference stress (p^{ref}) and on the exponent for the curve format (m), as expressed by the equation (7). For this reason, Figure 5 presents the variation of E_{50} with depth, calculated with those parameters as well as with the updated values of E_{50}^{ref}. In order to present the variation with depth, the K_0 had also to be considered. The lower and upper limits presented in Figure 5 were determined considering a normal (Gaussian) probability distribution and respectively 5% and 95% of confidence level, for both parameters E_{50}^{ref} and K_0.

5 PERFORMANCE PREDICTIONS USING THE BACK-ANALYSED PARAMETERS

The performance parameters were calculated using the updated values of K_0 and E_{50}^{ref}, presented in the section 4.5, considering Negro (1988) method.

The probability distribution of each predicted performance parameter was determined by a first order approximation of the Taylor Series, according to Duncan (2000). Having as an example the surface settlement at the tunnel axis (ρ_{sup}), its mean value and standard-deviation can be calculated respectively as:

$$\mu_{\rho_{sup}} = R[\mu_{K_0};\mu_{E_{50}^{ref}}] \tag{11}$$

$$\sigma_{\rho_{sup}} = \sqrt{\left(\frac{\Delta R_{K_0}}{2}\right)^2 + \left(\frac{\Delta R_{E_{50}^{ref}}}{2}\right)^2} \tag{12}$$

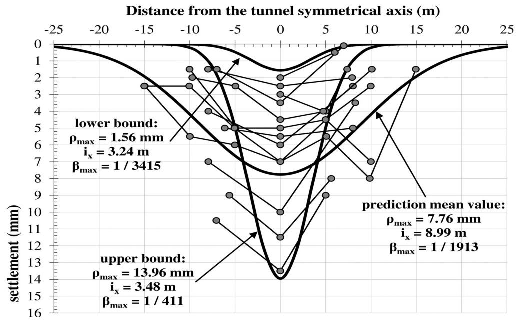

Figure 6. Transversal profiles of surface settlements calculated with back-analysed parameters compared to field data.

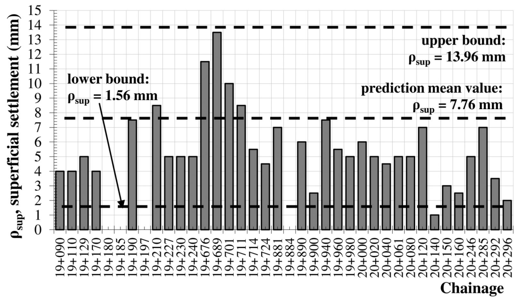

Figure 7. Surface settlements at the tunnel axis calculated with back-analysed parameters compared to field data.

where $R[\mu_{Ko};\mu_{E50ref}]$ is the predicted value calculated using the mean values of K_0 and E_{50}^{ref}, and ΔR is the difference of the predicted with addition or subtraction of one standard-deviation from the mean value of the parameter, expressed as:

$$\Delta R_{Ko} = R[(\mu_{Ko} - \sigma_{Ko});\mu_{E50ref}] - R[(\mu_{Ko} + \sigma_{Ko});\mu_{E50ref}] \tag{13}$$

$$\Delta R_{E50ref} = R[\mu_{Ko};(\mu_{E50ref} - \sigma_{E50ref})] - R[\mu_{Ko};(\mu_{E50ref} + \sigma_{E50ref})] \tag{14}$$

This procedure is repeated to other performance parameters. A comparison between predicted performance parameters and monitoring data is shown in the next set of figures. The mean value of the each performance parameter is presented, as well as their lower and upper limits, relative to 5% and 95% of confidence level, respectively.

A fairly consistent and favourable comparison between calculated and measured data is noted throughout the set of figures, indicating that the approach used is robust despite its shortcomings and limitations.

6 DISCUSSIONS AND CONCLUSIONS

The back-analysed parameters found for the North-Western Fine Soils of Santiago city are $K_0 = 0.84$ and

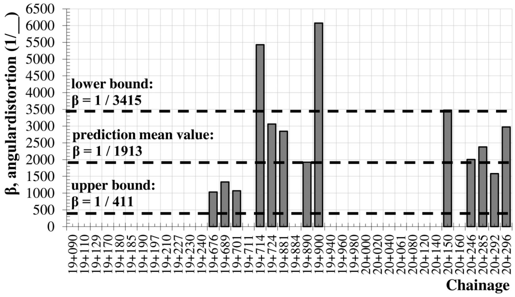

Figure 8. Maximum angular distortion calculated with back-analysed parameters compared with field data.

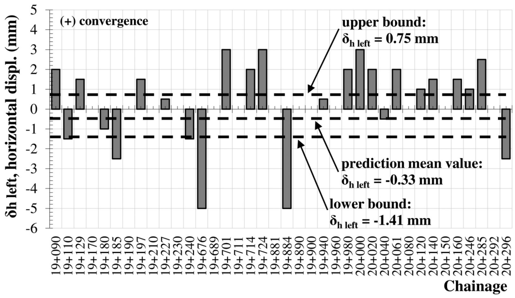

Figure 9. Horizontal displacements at the left springline calculated with back-analysed parameters compared to field data.

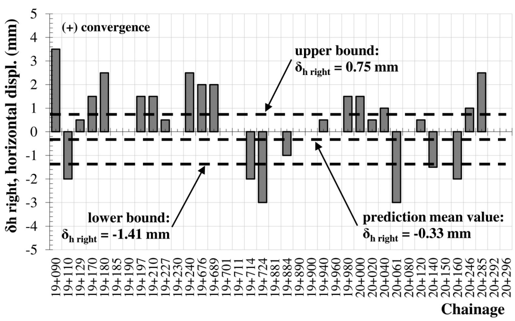

Figure 10. Horizontal displacements at the right springline calculated with back-analysed parameters compared to field data.

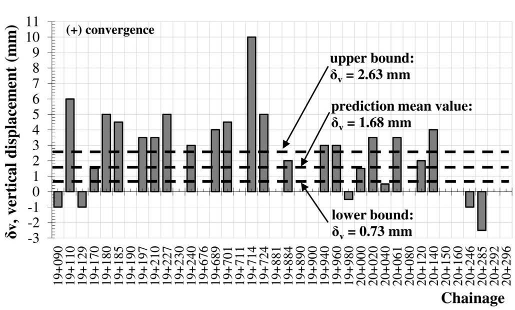

Figure 11. Vertical displacements at tunnel crown calculated with back-analysed parameters compared with field data.

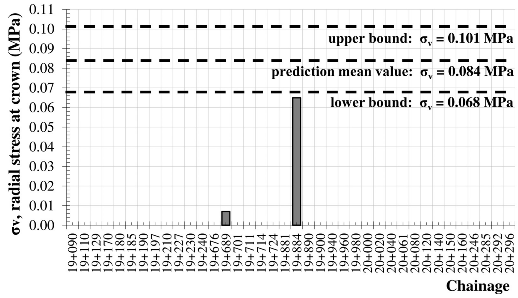

Figure 12. Radial ground stress on lining crown calculated with back-analysed parameters compared with field data.

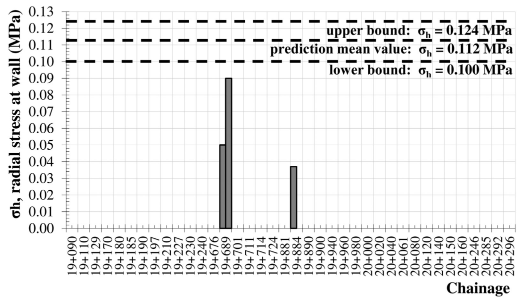

Figure 13. Radial ground stress on lining springline calculated with back-analysed parameters compared with field data.

$E_{50}^{ref} = 40.1$ MPa, with standard-deviations of 0.06 and 15.7 MPa, respectively.

Considering its updated value and a normal (Gaussian) probability distribution, K_0 is bounded by the range 0.74 and 0.94 for 5% and 95% of confidence level, respectively. This K_0 range is intriguingly close to the range proposed by Queiroz *et al.* (2005) for the Santiago gravels.

The back-analysed values obtained herein departed from what was used for Line 2 design. A higher K_0 was found, making possible lining optimisation. For vertical retaining walls, however, this newly assessed K_0 value represents a more critical condition. The variation of the E_{50} modulus with depth renders higher stiffness than used in the design of Line 2. The design considered E_{50} values near to the lower bound presented in Figure 5. Higher stiffness can be considered favourable regarding ground settlements, reducing potential damages to nearby structures.

It should be noted that the variance of the performance parameters lies within limits of the field measurement errors. In fact, the magnitudes of displacements and the accuracy of their measurements are low, resulting in larger relative errors than what is normally accepted. More accurate measuring techniques could provide lower variance for the back-analysed parameters.

The Bayesian updating resulted in a covariance of −0.93 between both state variables. This yields the coefficient of correlation −0.98, this meaning an almost perfect inversely correlation between K_0 and E_{50}^{ref}: an increase in one produces a proportional decrease of the other.

A thorough discussion on the causes of the higher value of K_0 obtained herein is beyond the scope of this study, but it is clear the need of further geological and geotechnical studies. Possible causes for this finding are over-consolidation from erosion of superficial soil layers, tectonic conditioning of the area and increase of effective stress due to matrix suction during periods of dry climate, among others.

Some limitations of the analyses performed were pointed out. The non-linear response of some predicted values to the state parameters was bypassed by a convenient introduction of the performance parameter logarithm instead of the performance parameter proper. However, the singularity approached for K_0 tending to unity, changing the sign of the lining displacements, could not and would not be bypassed by a hyper plan approximation. These values, however, do not affect much the result of the back-analysis, the maximum surface settlement being the controlling value. The overall approach used seems sound and robust.

The parameters resulted from the probabilistic back-analysis of Line 2 field data were incorporated into the design of the new Line 3, currently under construction for Santiago Metro. More specifically, the design considered the mean value of K_0 and the lower limit for the variation of the E_{50} modulus with depth.

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