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Numerical analysis of one-dimensional consolidation of unsaturated soil with non-Darcian fluid flow

Analyse numérique de la consolidation unidimensionnelle d'un sol non saturé avec un écoulement de fluide non darcien

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ABSTRACT: The aim of this article is to present numerical solutions of the one-dimensional consolidation in partially saturated soils on the basis of non-Darcian flow caused by non-Newtonian liquid. Two partial differential equations corresponding to water and air phases are used to model the transient process of the consolidating layer having a finite thickness and subjected to vertical loading. The flow of the water phase is assumed to be governed by four parameter-dependent velocity-gradient nonlinear model Except for the Darcian relationship, all other assumptions considered in Terzaghi's classical one-dimensional consolidation theory for saturated clays are considered herein. Diffusion of air through water and the movement of water vapor is not considered in the analysis. The numerical analyses are performed through the Crank-Nicolson implicit scheme. The results are plotted in terms of dimensionless isochrones and consolidation curves (average degree of consolidation with respect to time) corresponding to both water- and air- phases and different drainage boundary conditions. It is well observed from the numerical simulations that the non-Darcian flow delays the process of consolidation.

Keywords: non-Darcian; unsaturated; consolidation; Crank-Nicolson; clays

1 INTRODUCTION

Consolidation of soil is the primary concern of the geotechnical engineers. When it comes to address unsaturated soil, the analysis become more complicated because of having three phase system. Analysis of unsaturated soil relates the field problem more accurately. Application of external load over the partially saturated consolidating layer generates initial excess pore water pressure (u_{w0}) as well as initial excess pore air pressure (u_{a0}) both were developed. Both the excess pore pressures dissipate over time, which eventually leads to consolidation settlement of the entire stratum. To described the behavior of consolidation in unsaturated soil various researchers (Biot 1941, Blight 1961, Scott 1963, Barden (1965,1974) Fredlund and Hasan 1979) proposed their consolidation equations. Fredlund and Hasan's (1979) equations are most popular equation of analysis of unsaturated soil. Assuming both phases continuous, Fredlund and Hasan (1979) proposed the consolidation of unsaturated soil with the help of two separate partial differential equation (PDE). Fredlund and Dakshanamurthy 1980 and Dakshanamurthy *et al.* 1984 extended the Fredlund and Hasan 1979 theory for 2D and 3D cases respectively. Consolidation equation proposed by Fredlund and Hasan 1979 are nonlinear in nature. To simplify the equations Fredlund and Rahardjo (1993) presented 1D consolidation equations in the form of linear equations assuming all the soil parameter are unchanged during the consolidation. Several researchers (Qin et al (2008, 2010) Shan et al (2012, 2013), Ho et al 2014, Ho and Fatahi 2016, Zhou et al 2014) provided the analytical solution for the differential equation proposed by Fredlund and Rahardjo (1993).

All the aforementioned works were based on the linear relationship between hydraulic gradient and fluid flow velocity (Darcy's law). Several researchers (King, 1899; Hansbo, 1960; Swartzendruber, 1962; Kutilek, 1967; Elnagger et al.1973) had earlier shown through experimental processes that although Darcy's law represents the water flow for coarse-grained soil quite realistically, however, Darcian fluid flow law is not at all suited for modelling the fluid flow behaviour for low permeable soil subjected to low hydraulic gradient. A few works (Dubin and Moulin, 1985; Xie et al., 2007; Liu et al., 2009; Li et al., 2010,2012; Li and Xie, 2013) were earlier conducted for estimating the consolidation of saturated soil by incorporating the non Darcian flow law. Li and Wang (2014) provided an analytical solution for consolidation of unsaturated soil following non-Darcian flow. However, as per the authors findings, never before any rigorous research was carried out for predicting the consolidation behavior of variably saturated soil by duly incorporating the non-linear velocity-hydraulic gradient relationship. This is prime reason for executing the present work.

In this article, a detailed 1-D consolidation analysis of the unsaturated clayey soil is carried out by considering the non-Darcian flow rule proposed by Elnaggar *et al.*(1973) incorporate with pervious top pervious bottom (PTPB) and pervious top impervious bottom (PTIB) boundary condition. The analyses are performed by using the finite difference method, and the results are presented by varying the model parameter and permeability ratio (k_w/k_a); where, k_w and k_a are the permeabilities of water and air, respectively.

2 PROBLEM STATEMENT AND NUMERICAL ANALYSIS:

A constant uniform load of magnitude q is applied over a horizontal surface of a clayey stratum of height H, as shown in Figure 1. Initial excess pore water pressure (u_{w0}) and pore air pressure (u_{a0}) both generated instantaneously upon the application of external load and a transient flow is taken place for the entire dissipation. The consolidation of soil strata is assumed to be governed by the Fredlund and Rahardjo's classical consolidation equation for partially saturated soil (1993) except for the fact that the flowing of water is dictated by the following non-Darcian flow.

$$v_w = k_w\left(i-(1-a)i_1\left[1-\exp\left(\frac{-i\theta}{i_1}\right)\right]\right) \qquad (1)$$

Here, a, θ, (empirical parameters) and i_1 (threshold gradient) are the model parameters and k_w is the ultimate permeability of water as a function of matric suction and v_w=velocity of water. If, a=1 and/or θ= 0 the flow rule turns into Darcian law. On the basis of Equation (1), the water and air diffusion through compressible soils are given by the following expressions:

$$\frac{\partial u_w}{\partial t}+C_w\frac{\partial u_a}{\partial t}=Mc_v^w\frac{\partial^2 u_w}{\partial z^2} \qquad (2)$$

$$\frac{\partial u_a}{\partial t}+C_a\frac{\partial u_w}{\partial t}=c_v^a\frac{\partial^2 u_a}{\partial z^2} \qquad (3)$$

Where $M=\left(1-(1-a)\theta\left[1-\exp\left(\frac{-\theta}{i_1\gamma_w}\frac{\partial u}{\partial z}\right)\right]\right)$;

$$C_w=\frac{m_1^w}{m_2^w}-1;\ C_a=\frac{\frac{m_2^a}{m_1^a}}{1-\frac{m_2^a}{m_1^a}-\frac{(1-S)n}{\overline{u}_a m_1^a}};\ c_v^w=\frac{k_w}{\gamma_w m_2^w}$$

and $c_v^a=\frac{RT_k k_a}{\omega_a}\frac{1}{\overline{u}_a m_1^a\left(1-m_2^a/m_1^a\right)-(1-S)n}$

Here, (i) c_v^w and c_v^a coefficient of consolidation with respect to the water and air phases, respectively.

(ii) m_1^w and m_2^w are the coefficient of water volume change with respect to the change in net normal stress and matric suction, respectively. For saturated case m_1^w=m_2^w =m_v coefficient of volume change.

(iii) m_1^a and m_2^a are the coefficient of air volume change with respect to the change in net normal stress and matric suction, respectively.

(iv) S=degree of saturation, n=porosity, and k_a = coefficient of permeability of the air phase.

(v) R, ω_a, $\overline{u}_a$, and T_k represent the universal gas constant, molecular mass of the air, absolute air pressure, and absolute temperature, respectively. The assumptions taken for deriving these equations and the constraints imposed in the formulations are listed in the following section.

2.1 *Assumption:*

1. Soil solid and water phase are incompressible.
2. Water and air phase are assumed to be continuous.
3. Vapor pressure and dissolution of air into water are neglected.
4. Water and air flow independently (Fredland and Hasan 1979) in unsaturated soil. Both the phases follow continuity condition.
5. Flow of water and air are governed by non-Darcian and Fick's law respectively.
6. The loading deformation takes place only in vertical direction, net normal and matric suction both are responsible for the volume change of unsaturated soil.
7. The coefficient of permeability of air and water phase remain to be constant throughout the process of consolidation for unsaturated soil.

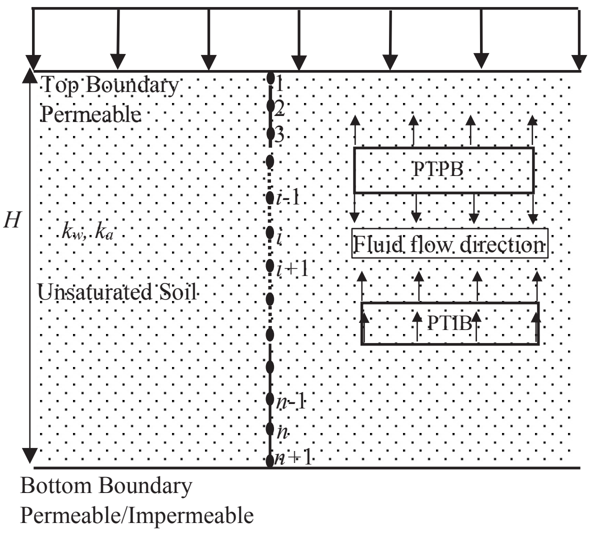

Figure 1. Soil spectrum subjected to strip loading.

2.2 *Initial condition and Boundary condition*

It is assumed that upon the application of external load, initial excess pore water (u_{w0}) and air pressure (u_{a0}) are generated in an instant within the consolidating stratum and that can be calculated by employing Equations 28 and 30 proposed by Fredlund and Hasan (1979. The initial and the boundary conditions are as follows:

Initial Condition: $u_{w,a}\big|_{z,0}=u_{w0,a0}$

$$u_{w,a}\Big|_{0,t} = u_{w,a}\Big|_{H,t} = 0 \text{ (PTPB)};$$

Boundary Conditions: $$\left.\frac{\partial u_{w,a}}{\partial z}\right|_{H,t} = 0 \qquad \text{(PTIB)}$$

3 RESULTS AND DISCUSSIONS

For analyzing one-dimensional consolidation, the soil stratum is discretized in n segments that results in (n+1) number of nodes as shown in Figure 1. The field variable (u_w and u_a) are computed at each node with the advancement in dissipation process. The parameters are taken as n=0.5, $k_w=10^{-10}$ m/sec, S=0.80, m_1^s = -2.5×10^{-4} kPa , m_2^s/m_1^s=0.4, m_1^w/m_1^s=0.2, and m_1^w/m_2^w=0.25. If applied load q=100 kPa and if it is considered that gauge pressure=0 kPa, then initial excess pore water pressure u_{w0}=40 kPa and initial excess pore water pressure u_{a0}=20 kPa (approximately).

The equations are discretized through Crank Nicolson Implicit Scheme of Finite Difference Method and a suitable code is written for the purpose of analysis.Corresponding to different model parameters (a,θ and i_l) and boundary conditions the results are represented in the form of normalized isochrones and consolidation curve. The difference between Darcian flow rule (DLA) and non - Darcian flow rule (NDL) are also shown in both isochrones and consolidation curve.

3.1 *Isochrones*

After application of load the excess pore pressures (water and air) for entire depth can be represented by isochrones (Powrie 1977) at any time. Thus, the whole consolidation process can be represented by a series of isochrones. Figure 2 and 3 shows the isochrones for different combination of model parameters corresponding to two different time values (2×10^8 and 5.38×10^8 second) for PTPB and PTIB boundary condition respectively. All the isochrones are drawn at k_w/k_a=10. Isochrones drawn by DLA method are represented by dotted line, isochrones drawn by NDL method represented by dashed line.

Figure 2(a-c) represents the isochrones for model parameter θ=0.5, i_l=10 and the value of a is chosen as 0.1, 0.5, and 0.9, As the value of a approaches unity, the difference between DLA and NDL minimizes, and behaves like a Darcian flow at a=1. The value of θ lies between 0 and 1 and the effect can be observed in Figure 2(d-f). The isochrones are drawn at a=0.5, i_l=90 for three different values of θ, namely, 0.2, 0.5 and 1. From Figure it can be observed that as the value of θ increases, the difference between DLA and NDL increases. The effect of i_l can be observed from Figure 2b and 2e. There is hardly any difference being observed in the isochrones while i_l is being varied.

For PTIB case isochrones cut the bottom orthogonally because of no flow condition. Effect of model parameters are

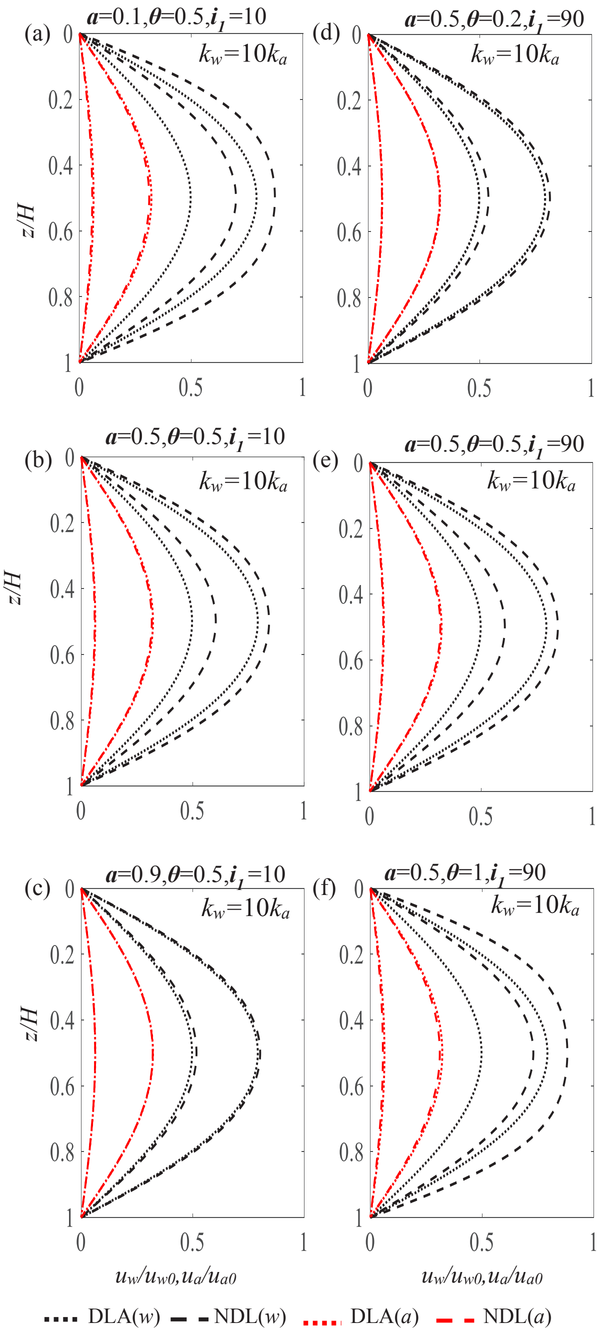

Figure 2. Isochrones corresponding to different model parameters subjected to PTPB condition.

the same as PTPB condition. At the same time dissipation of pore water pressure is less in PTIB case than PTPB case. At the same time, the dissipation of both phases is less in PTIB condition than PTPB condition.

One important thing can be observed from the figures as the time increases the difference between DLA and NDL is also increases. Air phase dissipate more rapidly than water phase. Dissipation of excess pore water pressure calculated by NDL is always less than or equal to the dissipation calculated by

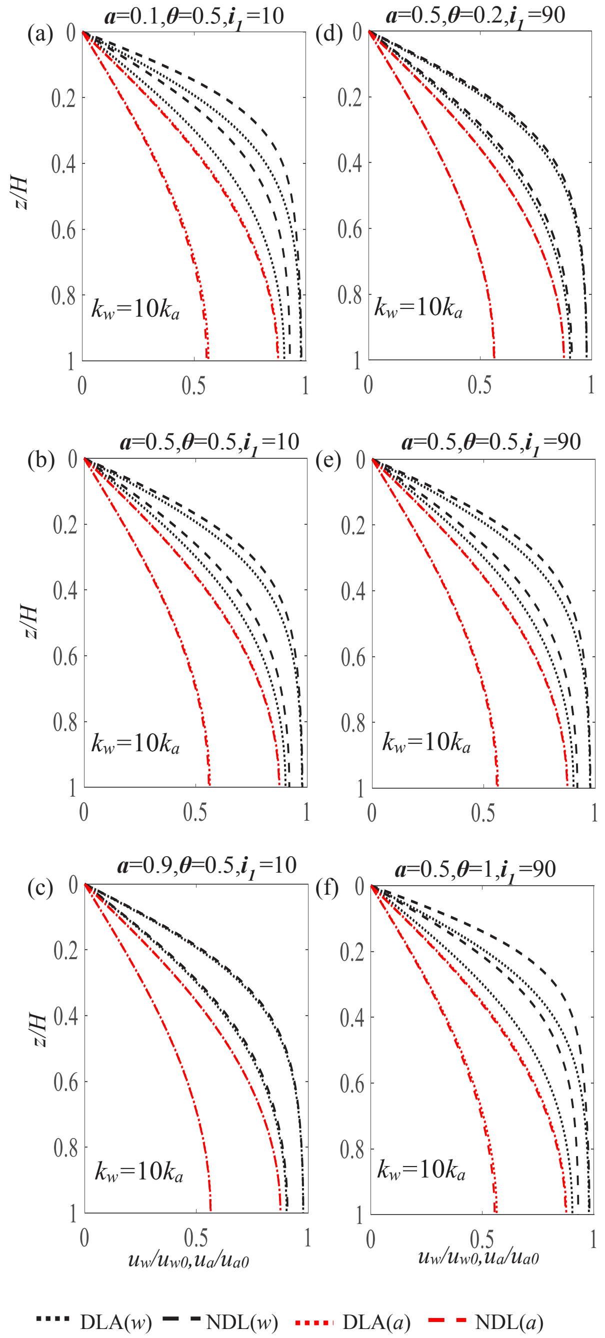

Figure 3. Isochrones corresponding to different model parameters subjected to PTIB condition.

DLA method. The difference in DLA and NDL are only visible at water phase but for air phase both are overlapped. This means that the non-Darcian flow rule of water phase has no appreciable effect on the dissipation of air phase.Isochrones provide the picture of consolidation phase at a specific time. To view the process of consolidation over time scale, U_{avg} vs T graphs are plotted. In a sense, these graphs provide the picture of consolidation in totality. As c_{vw} and c_{va} are different, therefore, it is not possible to draw the degree of consolidation with respect to time factor. So the graph are plotted with respect to time instead of time factor

3.2 U_{avg} vs T

Figures 3 and 4 depicts the consolidation curve which is the graph between average degree of consolidation (U_{avg}) and the time (T). It is well known that U_{avg} is the representative of degree of settlement. For unsaturated case, U_{avg} for water phase ($U_{avg,w}$) and air phase ($U_{avg,a}$) are represented as:

$$U_{avg,w} = 1 - \frac{\int_0^H u_w dy}{\int_0^H u_{w0} dy} ; U_{avg,a} = 1 - \frac{\int_0^H u_a dy}{\int_0^H u_{a0} dy} \qquad (4)$$

Figure 3 represents the U_{avg} vs T for PTPB case by varying all the considered model parameters. It can be observed that for water phase, the DLA and NDL curves overlapped over each other at the beginning of the consolidation and as time progresses, the difference between the curves increases. After a certain time, it remains to be constant till the end of consolidation. This can be interpreted as non- Dracian flow rule does not affect much the initial phase of consolidation but as the time increases the dissipation calculated by NDL is less that the DLA. The effect of model parameters on the consolidating curves are as the same as that on the isochrones. However, for air phase the graphs obtain from NDL and DLA coincide with each other.

Influence of the ratio of water and air permeability (k_w/k_a) are also rigorously observed. Three values of k_w/k_a namely 0.1,1 and 10 are selected for the analysis. From figures it can be observed that as the ratio of k_w/k_a increases the consolidation curves of water phase shifted to right side at the initial phase of consolidation and as the time increases the curve are overlapped with each other. This can be inferred as no matter whatever be the value of k_w/k_a ratio, the process of consolidation completes at the same time but at initial phase this ratio affects the dissipation of water phase. However, the effect of k_w/k_a ratio seems to be significantly high in air phase in comparison to water phase. With the variation in k_w/k_a ratio, the consolidating curves for the air phase markedly shift from each other. Figure 5 represents the U_{avg} vs T curves for PTIB case. The effect of model parameters and k_w/k_a ratio are similar as discussed for the PTPB case. The basic difference between the PTPB and PTIB lies in the consumption of time for the consolidation process to complete.

4 VERIFICATION

Figure 6 shows the comparison of the present numerical solutions with the analytical solutions reported by Fredlund and Hasan (1979) for PTIB drainage conditions for Darcian flow. A small amount of deviation (variation ranges between 0.01% - 5%.) greater height. The close proximity between these solutions provides the necessary confidence on the methodology and the written code.

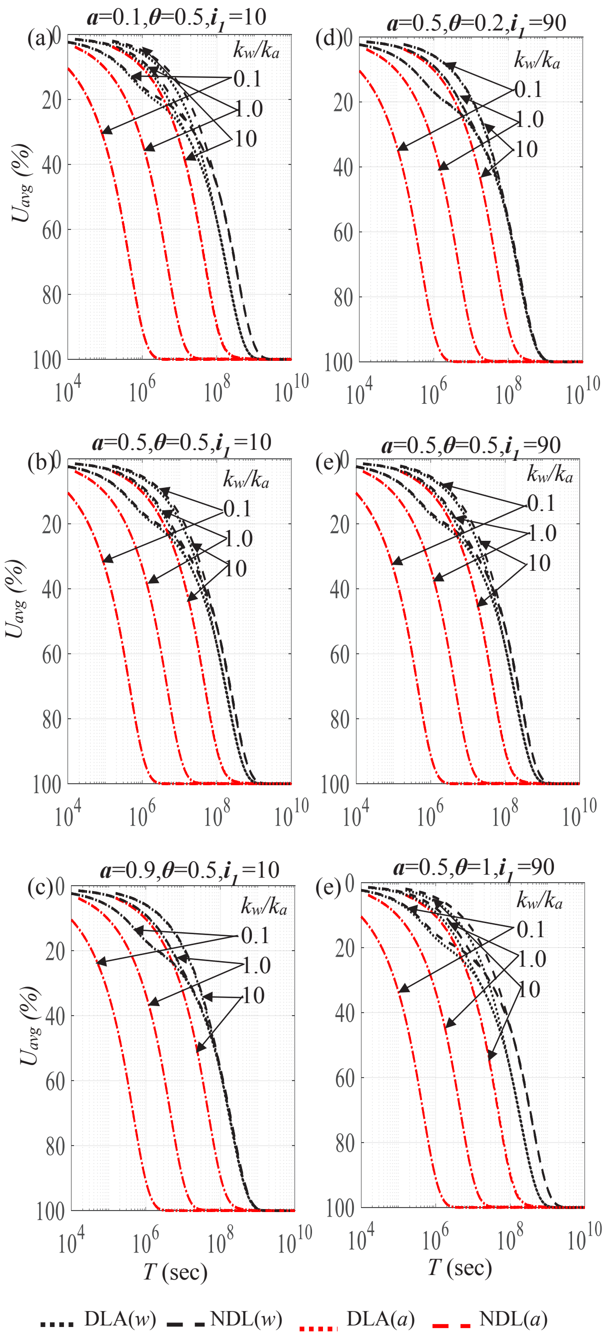

Figure 4. U_{avg} vs T graph corresponding to different model parameters for PTPB condition

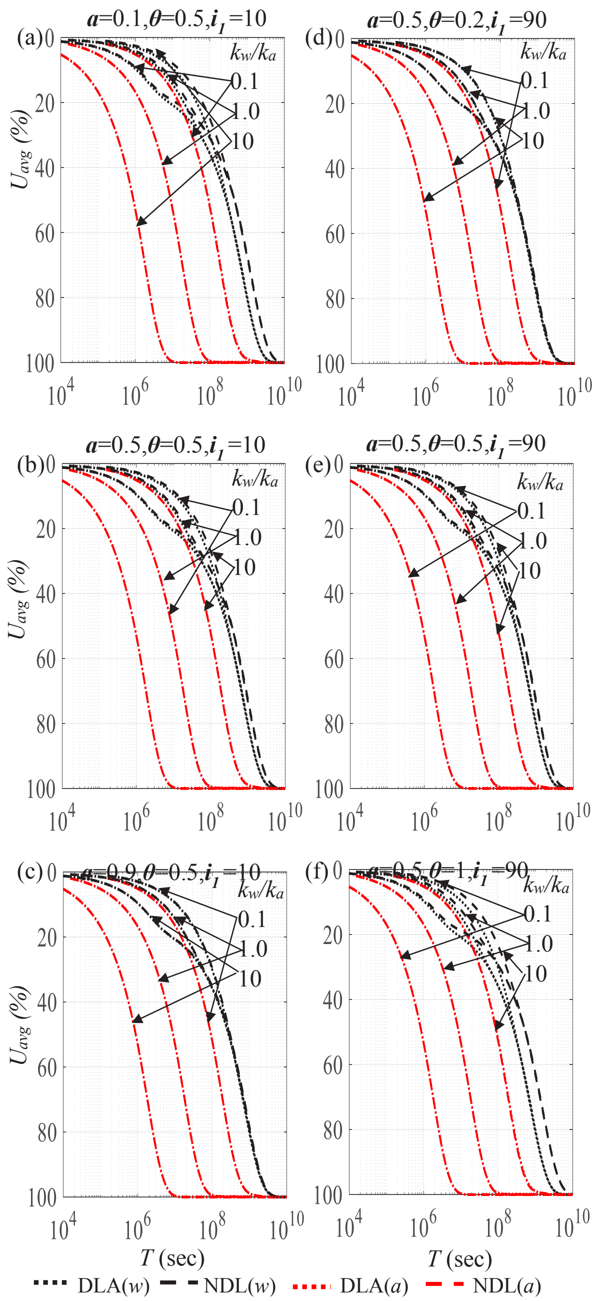

Figure 5. U_{avg} vs T graph corresponding to different model parameters for PTIB condition

5 CONCLUSION

The prime focus of this study is to understand the effect of non Darcian flow rule on consolidation of unsaturated soil for one-way as well as two-way drainage. Crank-Nicolson implicit scheme is employed for the analysis. The isochrones and the settlement curves are plotted and viewed intuitively for understanding the combined effect of flow laws, material and model parameters, and the drainage conditions. The effect of model parameters and k_w/k_a are thoroughly observed

The analysis clearly revels that non-Darcian flow rule affect only the water phase but not the air phase. The rate of dissipation estimated by NDL generally remains on the higher side than its DLA counterpart. At early phase of consolidation, the differences between NDL and DLA is less and as the time increases the difference increases. The ratio of k_w/k_a has a pronounced effect on the air phase than the water phase. After a certain progress of consolidation irrespective of the k_w/k_a ratio, the consolidation curves merge with each other.

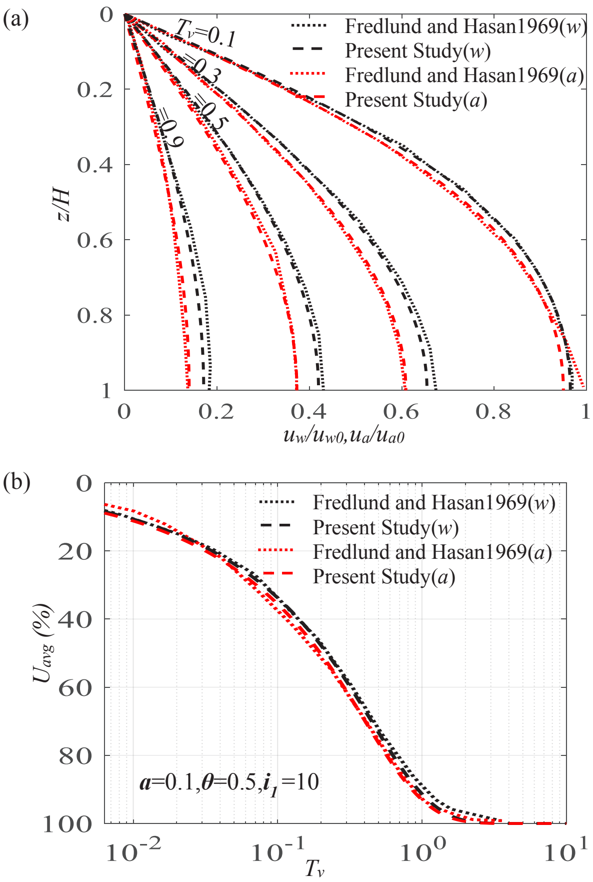

Figure 6. Comparison of the present numerical solutions with the available analytical solutions of Fredlund and Hasan (1969)

6 ACKNOWLEDGEMENTS

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