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Finite difference analysis on settlements of piled mat foundation considering the effects of soil springs and pile-to-pile interactions

Analyse des différences finies sur les tassements des fondations de tapis empilés en tenant compte des effets des ressorts du sol et des interactions pieu-pieu

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ABSTRACT: A finite-difference analysis for piled raft foundation at the ground surface under vertical loads is introduced in this paper. Formulas from the Thin Plate Theory were adopted to monitor the loaded foundation in clays. Modified Lysmer's analog soil spring and the equivalent pile springs were implemented to monitor the foundation settlements. The effects of pile-to-pile interactions on foundation settlements were approximated. As a result, the foundation displacements were found agreeable with three-dimensional finite element analysis. It was found that the piled raft foundation with flexible and rigid raft will behave differently on pile load distributions and pile stiffness. Application of the FD analysis requires further attention because the pile stiffness and soil resistance will interactively change with the pile orientations and soil stiffness

RÉSUMÉ : Une analyse par différences finies pour les fondations de radeaux sur pilotis à la surface du sol sous des charges verticales est présentée dans cet article. Des formules de la théorie des plaques minces ont été adoptées pour surveiller la fondation chargée en argiles. Le ressort de sol analogique de Lysmer modifié et les ressorts de pieux équivalents ont été mis en œuvre pour surveiller les tassements de fondation. Les effets des interactions pieu-pieu sur les tassements des fondations ont été approximés. En conséquence, les déplacements des fondations ont été jugés acceptables avec une analyse par éléments finis tridimensionnelle. Il a été constaté que les fondations du radeau sur pilotis avec radeau flexible et rigide se comporteront différemment en ce qui concerne la répartition des charges et la rigidité des pieux. L'application de l'analyse FD nécessite une attention supplémentaire car la rigidité du pieu et la résistance du sol changeront de manière interactive avec les orientations du pieu et la rigidité du sol.

KEYWORDS: finite difference analysis, piled raft foundation, thin-plate theory, soil springs, pile-to-pile interactions

1 INTRODUCTION.

The three-dimensional finite element (FE) analysis is known as the best tool in analyzing piled raft foundation. The modelling and material model in use is the key to the success of such analysis. For simplicity, many engineers prefer to use the approximate methods in which the analysis is easier to conduct with ordinary material parameters that can be obtained easily. Poulos (2001) has pointed out the importance of approximate computer based analysis in designing the piled raft foundation.

The rigidity of the raft can be estimated by the suggestions following ACI (1988) and Egyptian code (El Gendy, 1998). Rigid raft can provide more uniform settlements whereas the flexible raft will results in large differential settlements. The contact pressures between the foundation and the soils are also complicated and they can be influenced by a number of criterions (Chang et al., 2021b). The study intends to introduce a newly proposed finite difference (FD) analysis for the piled raft foundation, in which the variation of the soil stiffness and the effects of pile-to-pile interactions were taken into account throughout the analysis. Figure 1 depicts the layout of the proposed FD analysis.

2 METHODOGIES.

2.1 Finite difference analysis for raft on soil springs

The finite difference analysis based on Thin-Plate theory for the settlements of raft foundation and piled raft foundation under vertical loads can be found in Chang et al. (2018). A two-dimensional soil spring model was then suggested (Chang et al., 2020 and 2021a). To model the soil springs underneath the raft, the authors (Chang et al., 2021a) have suggested a modified version of the Lysmer's analog spring model.

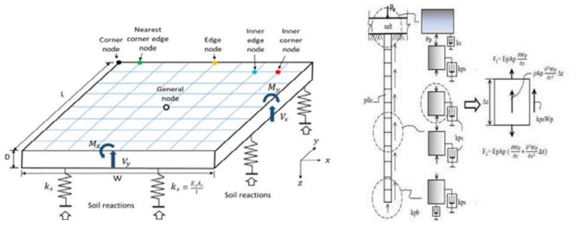

Figure 1 Layout of the discrete piled raft foundation

The soil springs, k_s underneath the raft were modeled as $k_s=k_o/m \times F(x,y) \times$□, where k_o is the Lysmer's spring constant, m is the number of nodes of the raft, F(x,y) is a two-dimensional function that was modeled by F(x,y)=f(x)×g(y), where f(x) is the normalized displacement function in the x-direction, and □ is the calibration factor for flexible foundation. Function g(y) is the same as f(x) which can be defined in the y-direction. Table 1 depicts f(x) with the dependence of Poisson's ratio (□) for soils with shear wave velocity (V_s) at 150m/s (Chang et al., 2021a).

Table 1 Normalized function for modified Lysmer's analog soil spring (Chang et al., 2021a)

V_s	□	f(x), 0≤ ≤1	r^2
150m/s	0.3	$f(x)= 2.6612x^5 -2.8259x^4 +0.2343x^3 + 0.5606x^2-0.08024x+1.00156$	0.991
	0.4	$f(x)= 8.6613x^5- 16.9555x^4 +11.2677x^3 -2.6108x^2 - 0.1867x+0.997909$	0.976
	0.5	$f(x)= 8.1555x^5- 16.3070x^4+11.2631x^3-2.9018x^2- 0.2897x+0.996683$	0.991

Note: x needs to be normalized with B/2 where B is the width of raft

2.2 Equivalent pile stiffness

Using the correlations between V_s and the undrained shear strength (S_u) for clays (Dickenson, 1994; Ashford et al., 1997), the undrained shear strength of the soils for V_s at 120, 150 and 180m/s were obtained as 32, 52 and 76kPa. The Mohr-Coulomb model was used to simulate mobilized frictions and end bearing for soils along the single piles. Assuming that the soils can be fully mobilized under the loads, the equivalent pile stiffness can be obtained by performing a wave equation analysis of a single pile and dividing the pile load by the displacement of pile head. The equivalent pile stiffness obtained from such modeling was implemented into the finite difference analysis for piled raft foundation.

It should be noted that the equivalent pile stiffness will be significantly affected by the soil model used along the pile. The authors had examined the soil model along the pile for the best approximation with the pile stiffness obtained from the three-dimensional finite element analysis. For better results of the finite difference analysis of piled raft foundation, the authors selected the pile stiffness model that was able to compare with the FE ones for the approximation. Table 2 reveals the comparisons of the equivalent pile stiffness for part of the numerical models. More details can be found in Hung (2020).

Table 2 Comparisons of equivalent pile stiffness, k_p where S/d=4

V_s		FE analysis	FD analysis
(m/s)	Location	k_p (kN/m)	
120	Center	32794	35778
	Edge	33836	36085
	Corner	31935	35378
150	Center	49848	55268
	Edge	50145	55695
	Corner	48380	54603
180	Center	64644	78369
	Edge	64271	78951
	Corner	61920	77403

2.3 Pile-to-pile interactions

The approximate pile-to-pile interactions (PPI) formulas suggested by Dobry and Gazetas (1988) was adopted to model the PPI effects in the proposed finite-difference analysis. For example, if a n×n piled raft foundation was subjected to vertical load. The loads taken by the piles can be calculated assuming that the raft was rigid enough to yield equal settlements of the piles. The relevant computations can be found in Chang et al. (2001). Now since the flexible raft is encountered, the FD analysis will provide the unequal pile displacements. With the ratios of the displacements found from the FD analysis, one can substitute the displacements ratios into the above analytical calculations to obtain the new load distributions.

In such way, the loads carried by the piles would be affected by the PPI accordingly. The equivalent pile stiffness can be repeatedly computed using the pile loads and displacements, and will be used in the FD computations. With a number of iterations and the satisfaction of convergence on pile displacements, the analysis was stopped. In such a manner, the PPI effects can be taken into account the proposed FD analysis. More details of the procedures can be found in Hung (2020). The above modeling will result in over-conservative estimations of the PPI effects on the equivalent pile stiffness for piles in the piled raft foundation. This is especially true when the piles are closely oriented.

3 NUMERICAL MODELLING.

A concrete piled raft foundation at the surface of clayey ground was presumed. The length, width and thickness of the raft is assumed as 26m, 26m and 1m. 20m length round piles with 1m diameter (d) were assumed. The ratio of the spacing distance (S) between two adjacent piles and the pile diameter (d), i.e., S/d, was assumed at 4, 6 and 8 to monitor the influences of pile orientations. Uniform vertical load of 100 kPa intensity was applied at top of the raft. The shear wave velocity (V_s) of the soils was varied at 120m/s, 150m/s and 180m/s. Poisson's ratio of the soils were assumed as 0.3, 0.4 and 0.5. Corresponding pile stiffness pre-obtained from above discussions were implemented into the FD analysis.

For validation of the proposed FD analysis, Midas GTS NX program was adopted to provide the solutions for comparisons. The dimensions of the analytical zone in FE analysis was kept as 200m×200m×60m, in which the essential boundary criteria (rollers and hinges) were used. Hexagon and pentagon solid elements were used in the FE mesh, and Stability of the solutions (foundation displacements) was ensured in the FE analysis. Figure 2 presents the FE model of the piled raft foundation.

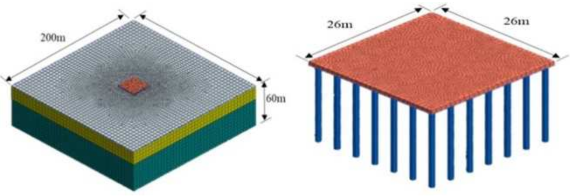

Figure 2 Discrete FE model used for the validations

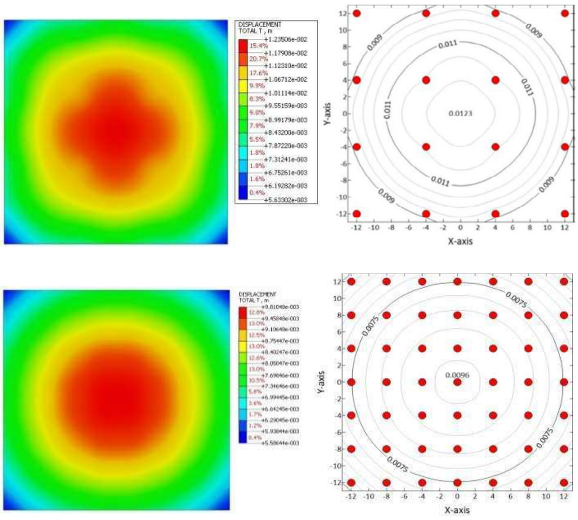

Figure 3 Comparison of the foundation settlements for numerical examples with S/d=8 and S/d=4 (Color plots from FE analysis and contour plots from FD analysis)

For simplicity, compatibility condition was assumed at the interfaces between piles and the soils. Validations of the foundation displacements are shown in Figure 3 for cases of numerical foundations with S/d=8 and 4 where V_s=150m/s and ν=0.4. The comparisons of the displacements at the center, edge and corner of foundation that were affected by shear wave velocity of the soil and S/d are shown in Figure 4.

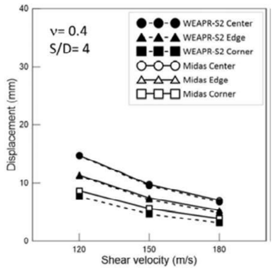

Figure 4 Comparisons of the foundation settlements with the influences of design parameters

4 OBSERVATIONS AND DISCUSSIONS

In general, it was found that the flexible raft will result in large load for the center pile and smaller loads for the corner piles. This is different from the pile group with a rigid cap. It was known that the pile foundation with a rigid cap will have smaller load at the center, and larger loads at the corner. Table 3 reveals the load sharing of the piles in the piled raft foundations of 7×7 piles where the raft is rather flexible.

Table 3 Load sharing of the piles of the piled raft foundation with 7×7 piles at various ground soils

V_s (m/s)	Location	FE analysis	FD analysis
120	Center	2.5%	2.5%
	Edge	2.1%	2.0%
	Corner	2.6%	1.6%
150	Center	2.5%	2.5%
	Edge	2.0%	2.0%
	Corner	1.5%	1.5%
180	Center	2.5%	2.5%
	Edge	1.9%	2.0%
	Corner	1.4%	1.5%

The corresponding pile stiffness at the foundation with 4×4 piles was found larger at the inner piles. The difference between the pile stiffness was found mostly significant in such case. As the number of piles increased e.g., 5×5 and 7×7, the differences between the pile stiffness became smaller. In general, the pile stiffness were reduced when more piles involved. It was found that the pile stiffness in the proposed FD analysis can't be fixed to obtain agreeable solutions with the FE analysis. Figure 5 reveals the contour plots of the original pile stiffness obtained in the FE analysis when V_s of the soil is kept at 150m/s. Figure 6 presents the equivalent pile stiffness k_p calculated from the FE analysis on the numerical model for 26m raft. It can be obviously seen that the pile stiffness was affected by S/d and shear wave velocity (V_s) of the soil. If S/d decreased (more piles) and so does V_s, k_p will decrease. The center pile seems to have the largest pile stiffness compared to the edge pile and the corner pile when V_s=180m/s and 150m/s. However, for V_s=120m/s, corner piles at S/d=8 will have the largest pile stiffness.

The values of k_p for the center piles are in the range of 196~39 MN/m whereas the edge piles are 165~44 MN/m, and the corner piles are 128~58 MN/m. It seems that corner piles will become more resistible when more piles were involved at soft stratum. The pile-to-pile interactions (PPI) are the main reason to cause such phenomena. More detailed discussions of the pile stiffness varied with the dimension of the raft can be found in Lai (2021). Figure 7 illustrates the corresponding reduction rates (k_p*/k_{pmax}) of the pile stiffness for all the piles shown in Fig. 6, where k_p* is the affected pile stiffness, k_{pmax} is the maximum pile stiffness when S/d=8. Note that the reduction rate was computed by dividing the individual pile stiffness to pile stiffness of the center pile at S/d=8 for V_s=180m/s and 150m/s respectively. But for V_s=120m/s, the rates were computed by taking the corner pile at S/d=8 as the reference since the corner pile preserve the largest pile stiffness in such case. It can be found that the effects of PPI are not only dependent of the pile orientations but also the soil stiffness. The reduction rates of the pile stiffness from the FE analysis are approximately between 48%~100% in such cases. For the results shown in FD analysis, it was found that the PPI influences are over conservative since the approximate PPI formula was adopted (Hung, 2020).

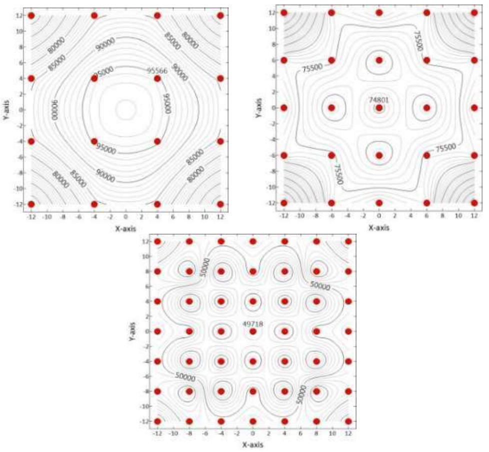

Figure 5 Contour plots of the pile stiffness at various pile orientations (Units: kN/m)

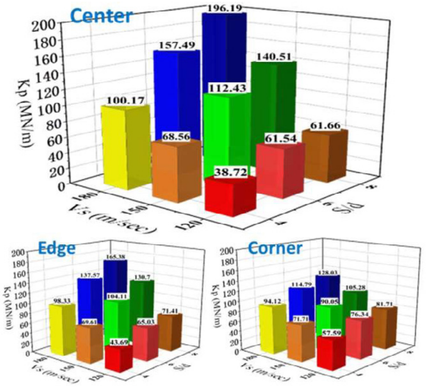

Figure 6 Equivalent pile stiffness affected by S/d and V_s for piled raft foundation with 26m square raft

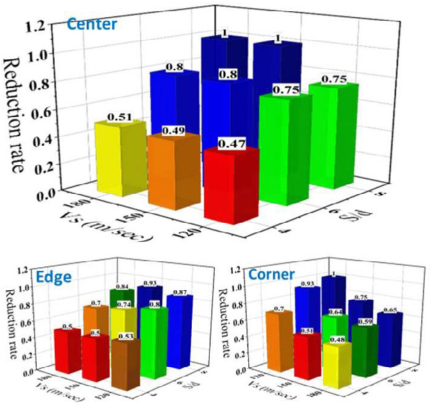

Figure 7 Reduction rates of equivalent pile stiffness affected by S/d and V_s for piled raft foundation with 26m square raft

To show the results of the numerical examples of this study, the settlement ratios (piled raft fdt./raft fdt.) vs load ratios (pile loads/total load, i.e., □) obtained from this study were plotted in the diagram suggested by Katzenbach and Choudhury (2013). Figure 8 depicts the data associated with the example studies. Note that C1, C2 and C3 soils stand for clays with V_s at 120, 150 and 180m/s, respectively. The piles obviously took more loads especially in soft soils when more piles were involved in the piled raft foundation.

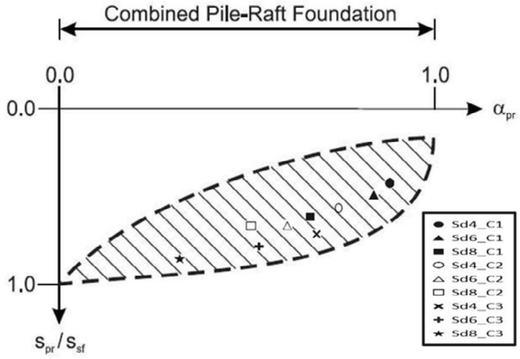

Figure 8 Data allocated in the diagram of settlement ratio (S_{pr}/S_{sf}) vs. load ratio (□) for piled raft foundations with flexible/rigid raft

Additionally, it should be pointed out that the estimations of the pile loads must be very careful since the loads exerted in the raft above the piles and the load calculated from the internal stresses in the pile head will be very different owing to the geometric change between the raft and the piles

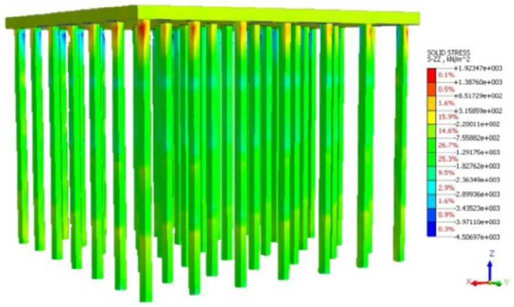

Figure 9 3D view of the piled raft foundation on stress concentrations under vertical loads

Figure 9 depicts such mechanism revealed by FE analysis. In the numerical examples of this study, the stresses found by FE analysis at top of the piles were about three times of those exerted in the raft above the piles. Such dramatic change of the stresses in structural elements is not able to simulated in the FD analysis. Although the proposed FD analysis seems to be adequate in predicting the foundation displacements, its use in estimating the internal stresses of the foundation needs further attentions.

5 CONCLUSIONS

A newly proposed finite difference analysis was introduced in this study for piled raft foundation subjected to vertically uniform load. The displacements resolved from the analysis were found comparable with those obtained from three dimensional finite element method. The foundation settlements under the uniform load were found larger at the center and smaller at the edge. The loads carried by the center piles were found larger rather than the corner piles. As the number of piles increased, the load carried by the piles will become more consistent. The load distribution of the piles in this case seems to be very different from what was known in the pile group with a rigid cap. Meanwhile the pile stiffness and the reductions due to pile-to-pile interactions needs further attentions since they will be affected by the number of piles involved, the locations of piles, and the soil stiffness. To keep the pile stiffness as a constant and to interpret them adequately throughout the FD analysis seems to be infeasible. The authors indeed found that the simplified FD analysis has the disadvantage in predicting the internal stresses of the foundation because the analysis simplifies the pile raft foundation system using the springs for soils and piles.

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7 REFERENCES

ACI Committee 336. 1988. Suggested analysis and design for combined footings and mats. Report ACI 336.2R-88, American Concrete Institute, Farmington Hills, MI, USA.

Ashford, S.A., Jakrapiyanum, W., and Lukkanaprasit, P. 1997. Amplification of earthquake ground motion in Bangkok. Research Rep. Cu\CE\EVR\1997.002, Chulalongkorn Univ., Bangkok, Thailand.

Chang, D.W., Hung, M.H. and Jeong, S.S. 2021a. Modified Lysmer's analog model for two dimensional mat settlements under vertically uniform load. Int. J., Geomechanics and Engineering, KGS, 25(3), 221-231

Chang, D.W, Lien, H.W., and Hung, M.H. 2020. FD analysis on piled raft foundation settlements under verical lodas. Geotechnical Engineering, Journal of SEAGS and AGSSEA, 51(2), 159-165.

Chang, D.W., Lien, H.W. and Wang, T.Y. 2018. Finite difference analysis of vertically loaded raft foundation based on the Plate Theory with boundary concern. Journal of GeoEngineering, 13(3), TGS, 135-147.

Chang, D.W., Tu, Y.J. and Cheng, S.H. 2021b. Settlements, contact pressures and coefficients of subgrade reactions of surface raft foundations subjected to uniform vertical loading. Int. J., Geomechanics and Engineering, KGS (under review)

Chang, D.W. and Wen, C.H. 2001. Direct wave equation analysis on vertically loaded raft-pile. Procds., The 10th International Conference on Computer Methods and Advances in Geomechanics, Tucson, Arizona, USA, 1451-1456.

Dobry, R. and Gazetas, G. 1988. Simple method for dynamic stiffness a damping of floating pile groups. Geotechnique 38, 557-574.

Dickenson, S. E. 1994. Dynamic response of soft and deep cohesive soils during the Loma Prieta earthquake of October 17, 1989, Ph.D. thesis, Univ. of California, Berkeley, CA.

El Gendy, M. 1998. An analysis for determination of foundation rigidity. The 8th Int. Colloquium on Structural and Geotechnical Engineering, Ain Shams U., Cairo Egypt, Dec. 15-17.

Hung, M.H. 2020. Stiffness reduction of piles due to pile-to-pile interactions for piled raft foundation under vertical load, Mater Thesis. Dept. of Civil Engineering, Tamkang University, New Taipei City, Taiwan. (in Chinese)

Katzenbach, R. and Choudhury, D. 2013. ISSMGE Combined Pile-Raft Foundation Guideline. Technische Universität Darmstadt, Darmstadt, Germany.

Lai, Y.Y. 2021. Load sharing and pile stiffness of piled raft foundations. Master Thesis, Dept. of Civil Engineering, Tamkang University, Taiwan. (in Chinese)

Poulos, H.G. 2001. Pile-raft foundation: design and applications. Geotechnique, 51(2), 95-113