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New constitutive equation for viscoplastic material behaviour

Nouvelle loi constitutive pour la simulation de matériaux viscoplastiques

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ABSTRACT: Granular rock salt has a strong time dependent and stress dependent material behaviour. For the numerical simulation of this material behaviour a new viscoplastic constitutive equation was developed and implemented into a Finite-Element-(FE-)Programme. The numerical simulation of this material behaviour is necessary for the analysis of the stability and the serviceability of large tailings heaps and infrastructure construction in the area around these heaps. The tailings heaps occur during the production of potassium fertilizer and consist of granular rock salt. Granular rock salt has a strongly time variant deformation behaviour and a strongly velocity dependent strength behaviour. In addition the material behaviour of granular rock salt is influenced by the stress level, the temperature and the compaction state. The paper explains the mathematical formulation of the new viscoplastic constitutive equation which is called VISCOSALT 2017 and the implementation into a FE-Programme. The salt mechanical material parameters for this constitutive equation are determined by load controlled triaxial creep tests and strain rate controlled triaxial fracture tests. For the verification of the new constitutive equation and the correct implementation into the FE-Programme the back analysis of the triaxial tests is used. The comparison of the measurement data of the triaxial tests and the results of the numerical simulations of the back analysis show a good accordance. Now the new constitutive law is used for the analysis of the stability and the serviceability of large tailings heaps and the infrastructures in the area around these heaps.

RÉSUMÉ : Les sels de roche granulaire ont un comportement fortement dépendant du temps et du niveau de contrainte. Une nouvelle loi de comportement pour la simulation numérique de ce matériau a été développée et implémentée dans un programme aux éléments finis. La simulation numérique est nécessaire pour analyser la stabilité de larges terrils et des infrastructures environnantes. Ces terrils proviennent de la fabrication de fertilisants potassiques et sont constitués de sels de roche dont les minéraux ont été extraits. Les sels de roche granulaire ont un comportement en déformation qui dépend fortement du temps et une résistance qui dépend fortement de la vitesse de déformation. Le comportement du matériau dépend également du niveau des contraintes, de la température et de l'état de compaction. Cet article explique comment cette nouvelle loi constitutive intitulée VISCOSALT 2017 est formulée mathématiquement et comment elle a été implémentée dans un programme aux éléments finis. Les paramètres numériques pour modéliser les sels de roche sont déterminés à partir des résultats d'essais triaxiaux, en particulier des essais de fluage sous compression et des essais de rupture à vitesse de déformation donnée. Les résultats numériques obtenus par une analyse inverse présentent une bonne concordance avec les résultats expérimentaux. Cette loi constitutive est utilisée actuellement pour analyser la mise en service et la stabilité des terrils et des infrastructures environnantes.

KEYWORDS: viscoplasticity; rock salt; tailings heap; Finite-Element-Method; VISCOSALT 2017

1 INTRODUCTION

Pottasium salt is mined in underground mines for the production of fertilizer and special products for the chemical industry. During the production processes granular residual material occurs which is stored on large tailings heaps (Lukas 2002). Figure 1 shows the fill up of a tailings heap in Germany. In the back ground there is another heap (left) and the production facilities (centre).

Figure 1. Tailings heaps of rock salt in Germany

This granular residual material consists mainly of rock salt which is stored on large tailings heaps. The granular rock salt has a strong time dependent stress-deformation behaviour.

The dimensions of the tailings heaps can be enormous. The ground view can be bigger than 1 km^2 and the height can be more than 200 m above the surface (Katzenbach et al. 2013). Because of the dimensions, the complex stress-deformation behaviour and the distinctive interaction between tailings heap, subsoil and infrastructure construction in the area around these heaps have to defined as structures belonging to the Geotechnical Category GC 3 according to the current technical regulation Eurocode EC 7 (Katzenbach & Leppla 2012). The Geotechnical Category GC 3 is the category for constructions with the highest complexity factor.

The time variant stress-deformation behaviour is based on the viscoplastic characteristic of granular rock salt. Due to this behaviour the stress distribution in the tailings heaps and in the subsoil changes continuously. The resulting soil-structure interaction between the heaps, the subsoil and infrastructure construction around these heaps is stress and time dependent (Katzenbach et al. 2009). Using the Finite-Element-Method (FEM) the stability and the serviceability of the tailings heaps and the infrastructure constructions around these heaps are analized. For the subsoil elastoplastic constitutive laws are used. For the granular rock salt a viscoplastic constitutive equation is used, which has been developed at the Institute and Laboratory of Geotechnics of the Technische Universität Darmstadt, Germany (Leppla 2017).

The paper explains the material behaviour of granular rock salt, the developed and verified constitutive law and the implementation in a FE-Programme.

2 BEHAVIOUR OF GRANULAR ROCK SALT

2.1 *Basics*

Viscous material behaviour, which is also defined as creeping, means that the deformation and strain develops continuously under a constant stress level. Viscous material behaviour can exist of three phases: primary creeping, secundary creeping and tertiary creeping. At the primary creeping, which is also defined as transient creeping, the strain rate $\dot{\varepsilon}_{ij}$ decreases with the time until the secundary creep phase is reached. In this secundary creep phase, which is also defined as stationary creeping, the strain rate is constant. The tertiary creeping does not occur at all materials and shows an increasing strain rate.

The creep deformation of granular rock salt starts below the border of elasticity and plasticity (Boley 1999, Günther 2009, Wachter 2009). The strain of granular rock salt consists of an instantaneous elastic part ε_{ij}^{el}, an instantaneous plastic part ε_{ij}^{pl} and of a viscous part ε_{ij}^{vpl}. Granular rock salt has no viscoelastic strain (Wachter 2009, Leppla 2017).

The viscoplastic material behaviour can be discribed by the rheological model according to BINGHAM (Figure 2). This model combines the characteristics of the ST. VENANT friction element, the NEWTON absorber element and the HOOK spring element (Reiner 1969, Worthoff 2013). This model shows the elastic behaviour until the flow stress in the friction element is not reached. After reaching the flow stress plastic behaviour occurs which is influenced time variant by the absorber element.

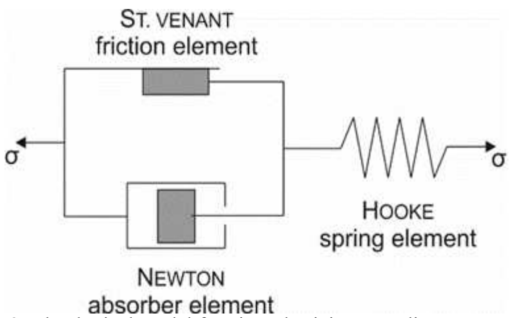

Figure 2. Rheological model for viscoplasticity according to BINGHAM.

For the mathematical description the ARRHENIUS-equation k(T) is used, which is extended by a stress function f and an internal parameter A (Eq. 1).

$$\dot{\varepsilon}_{ij}^{vpl} = \mathrm{t}(\sigma_{ij}) \cdot \mathrm{A} \cdot \mathrm{k}(T) = \mathrm{t}(\sigma_{ij}) \cdot \mathrm{A} \cdot e^{-\frac{Q}{R \cdot T}} \quad (1)$$

The plastic strain rate $\dot{\varepsilon}_{ij}^{vpl}$ depends on the temperature T, the activation energy Q and the common gas constant R (Plewinsky et al. 2012). The letter e is the EULER number.

An overview on constitutive laws of intact rock salt and granular rock salt is given in Leppla (2017).

Granular rock salt has a strong time variant deformation behaviour and a strong velocity dependent strength behaviour. In addition the material behaviour depends on the stress level, the temperature and the compaction state. The viscoplastic material behaviour basis on the crystalline structure and the along going crystal defects (Leppla 2017).

The material behaviour of the granular rock salt is strongly dependent to the deformation rate. High, overcritical deformation rates lead to high strength and stiffness of the rock salt and to brittle fracture. Small, undercritical deformation rates lead to minor strength and stiffness and to a plastic creep without fracture (figure 3).

Under constant deviatoric stress granular rock salt presents a constant creep behaviour despite a huge strength. The material behaviour of this material is strongly dependent on the state of stress and of the deformation rate. Figure 3 shows the results of two strain controlled triaxial tests on specimen of granular rock salt stored on heaps, obtained from core drillings from the top of a heap. For both tests a cell pressure of σ_3 = 0.5 MN/m² was specified. Both deformation rates vary by the factor 1,000.

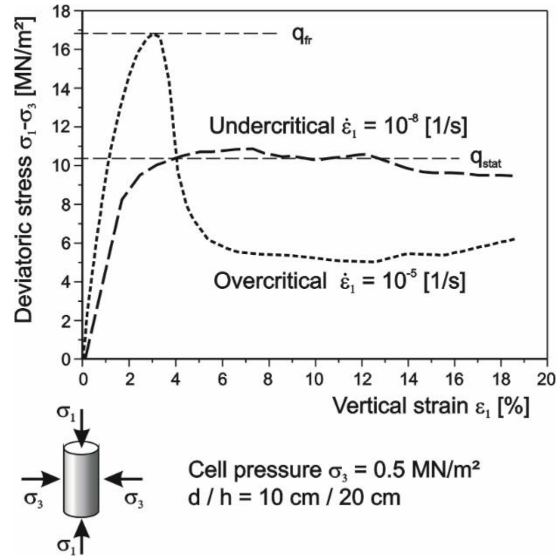

Figure 3. Typical results of strain controlled triaxial tests on stockpiled material samples.

The sample with the higher, overcritical deformation rate of dε/dt = 10^{-5} 1/s presents the usual material behaviour on high deformation rates. After the peak stress q_{fr} follows a deep drop of the deviatoric stress (brittle fracture). The sample with the lower (undercritical) deformation rate of dε/dt = 10^{-8} 1/s does not collapse. The sample creeps under a constant deviatoric stress q_{stat}.

For the identification of the viscoplastic material behaviour several laboratory tests have been carried out. Main part of these laboratory tests are 199 triaxial tests, which can be divided into load-controlled creep tests and strain-controlled fracture tests.

During a load-controlled creep test the time dependent displacement of the specimen is investigated in relation to the state of stress. The necessary test time may last up to years.

Strain-controlled fracture tests are characterized by the fact that the combination of cell pressure σ_3 and given strain rate $\dot{\varepsilon}_1$ leads to a failure or shearing of the rock salt specimen with a subsequent stress decrease.

2.2 *Time variant deformation behaviour*

In principle granular rock salt behaves like intact rock salt. In addition the time and stress dependent compaction leads to a change of the shape and the volume. Filled fresh granular rock salt has a density of about ρ = 1.4 t/m³, which is much smaller than the grain density of intact rock salt of about ρ = 2.15 t/m³. As a result of the compaction the pore volume of the granular rock salt decreases.

Fresh granular rock salt has a non-cohesive texture which fades in a cemented cohesive texture with high strength after short time (Ankes 1972). This process depends on the moisture and the stresses of the overburden. Due to the comparatively big pore volume the material is still named granular rock salt. The macroscopic changing is shown in figure 4. The two pictures are made of a drilling core of a 120 m high tailings heap in Germany. The left picture shows a texture with single grains. In the right picture single grains cannot be detected.

The compaction rate increases with an increasing hydrostatic stress. The deviatoric stress has no significant influence on the compaction rate (Zeuch & Holcomb 1990, Brodsky 1994).

Figure 4. Macroscopic texture of granular rock salt.

As an example for the time variant material behaviour of granular rock salt the results of two load-controlled creep tests are shown in figure 5. The two tests differ in the stress level and in the load reduction.

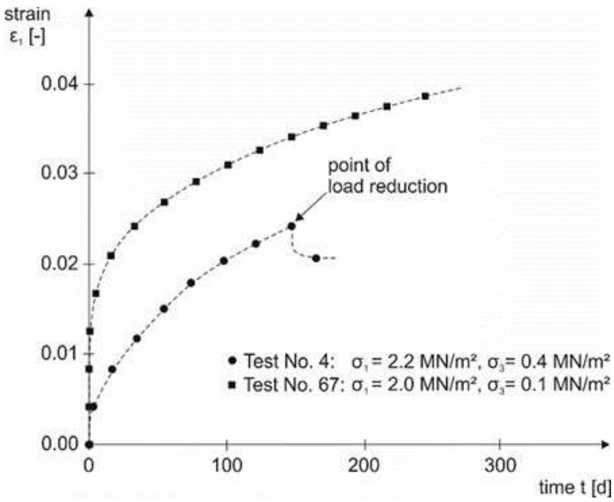

Figure 5. Influence of the stress level.

The strain consists of an elastic and a plastic part at the beginning and of a viscous part caused by creeping. The load reduction of test no. 4 shows, that the viscous strain is non-reversible. This means that the strain is viscoplastic and no viscoelastic part occurs.

The viscoplastic strain consists of a transient part and of a stationary part (Eq. 2).

$$\varepsilon_{ij}^{vpl} = \varepsilon_{ij}^{vpl,tr} + \varepsilon_{ij}^{vpl,st} \tag{2}$$

The transient part goes towards a limes value. The stationary part depends on the deviatoric stress and the temperature. Figure 6 shows the development of the viscoplastic strain of test no. 67 over the test time.

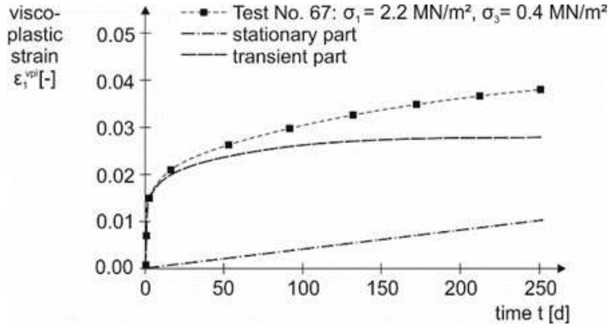

Figure 6. Transient and stationary parts of viscoplastic strain.

The temperature has a significant influence on the viscoplastic material behaviour. As an example figure 7 shows the results of a load-controlled creep test with changing in temperature. The reduction of the temperature after 75 days leads to a slowdown of the strain rate. The increased temperature after 170 days and 230 days leads to a speedup of the strain rate.

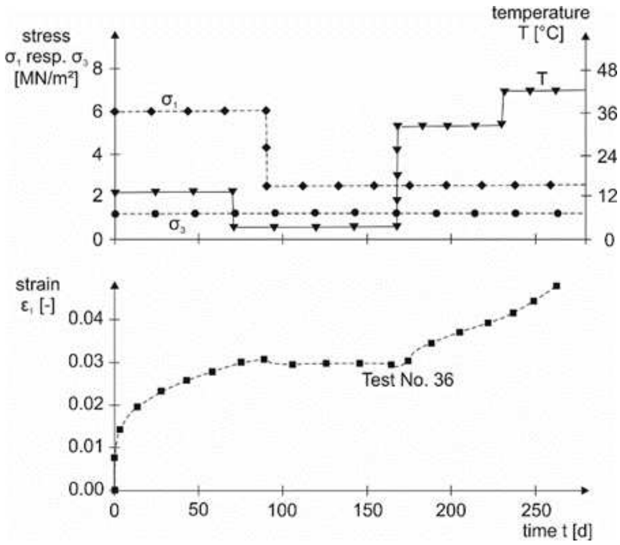

Figure 7. Influence of the temperature.

The time and stress dependent compaction leads to a change of the shape with the so called deviatoric strain e and a change of the volume with the so called volumetric strain ε_{kk}. The deformation behaviour of granular rock salt depends on the compaction level which included the current density. Further information is given in Leppla (2017).

For the determination of the long term behaviour of granular rock salt triaxial tests have been carried out over several years. As examples the results of three long term tests are shown in figure 8.

The tests no. 131 and no. 132 are hydrostatic triaxial tests which lasted between 4 years and 7.5 years. The comparatively small hydrostatic stresses lead to a strain between 4 % and 6 %. At about 500 days the system reaches a specific equilibrium for this stress level for this density. The strain rate is zero.

The test no. 182 is a triaxial test with a significant deviatoric stress. The test time is 4 years. After about 100 days the transient part of the strain rate is zero and only stationary creeping occurs.

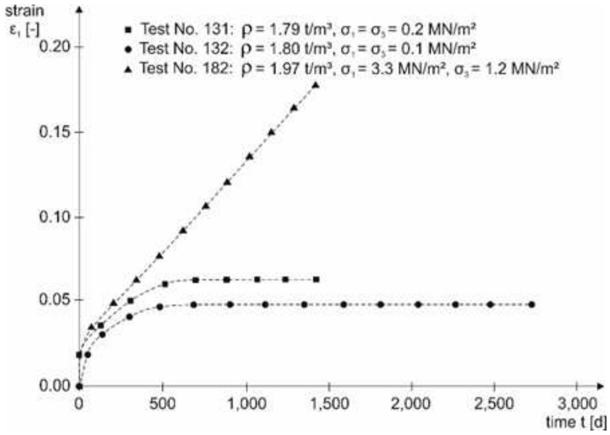

Figure 8. Load-controlled triaxial long term creep tests.

2.3 *Velocity variant fracture behaviour*

The fracture behaviour of granular rock salt depends strongly on the velocity of the loading. As an example the test results of a strain-controlled fracture test is shown in figure 9.

Test no. 45 has a high strain rate $\dot{\varepsilon}_1$ and a clear maximum of the possible deviatoric stress s. Test no. 43 has a smaller strain rate and a smaller maximum, but a bigger strain ε_1. The strain rate at test no. 130 was significantly smaller. In this case the material has a fractureless creeping.

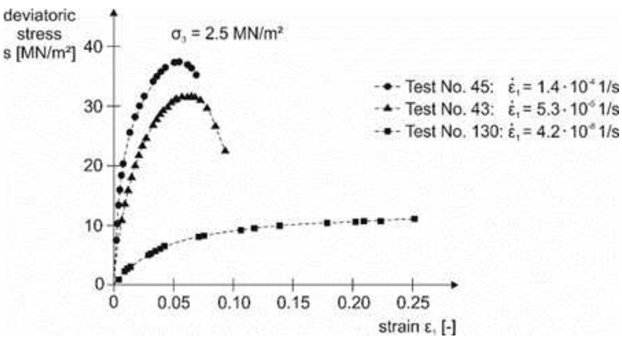

Figure 9. Influence of the strain rate.

Figure 10 shows the specimen of two different strain-controlled fracture tests. The specimen with the high, overcritical strain rate is broken (left) and the specimen with the smaller, undercritical strain rate has creeped (right).

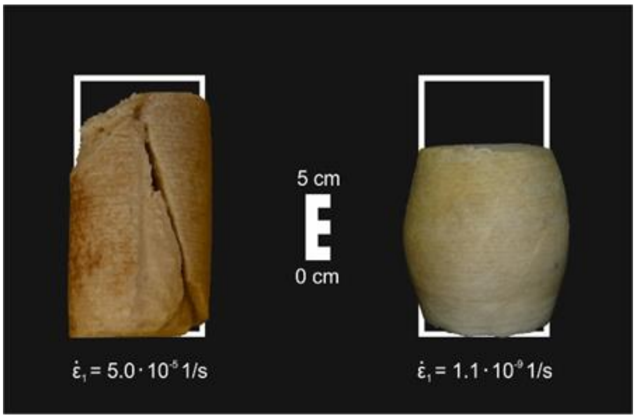

Figure 10. Strain-controlled fracture test (left) and load-controlled creep test (right).

3 NUMERICAL SIMULATION

For the numerical simulation of the complex soil-structure interaction of tailings heaps consisting of granular rock salt a phenomenological, empirical constitutive equation was developed (Leppla 2017). It is written in a subroutine which is implemented into a FE-Programme. The subroutine is called VISCOSALT 2017.

3.1 *Constitutive equation*

The constitutive law for granular rock salt is based on the principle of the superposition which considers the dependency of stress, time, temperature and density including the effect of the time and stress dependent compaction. The single strain parts are calculated seperately and added together. The total strain consists of the instantaneous (ε_{ij}^{el}), the instantaneous plastic (ε_{ij}^{pl}) and the viscoplastic (ε_{ij}^{vpl}) parts as shown in equation 3.

$$\varepsilon_{ij} = \varepsilon_{ij}^{el} + \varepsilon_{ij}^{pl} + \varepsilon_{ij}^{vpl} \tag{3}$$

The instantaneous elastic and instantaneous plastic strain is time independent. The viscoplastic strain is time dependent and desribes the creep behaviour. For the calculation of the single strain parts material parameters are used that are gained by mathematical functions of regression analysis.

The HOOK law is used to calculate the instantaneous elastic strain ε_{ij}^{el}. For the calculation of the instantaneous plastic strain ε_{ij}^{pl} a cap model is used (Fig. 11).

Using the approach of Wallner (1983) the velocity dependent fracture behaviour is transferred into a velocity independent fracture behaviour. The flow volume of the cap model is defined by the conus flow surface F_e and the cap flow surface F_c. Both flow surfaces have a associated flow rule. They have not a consistent connection but fulfill the requirement of convexity.

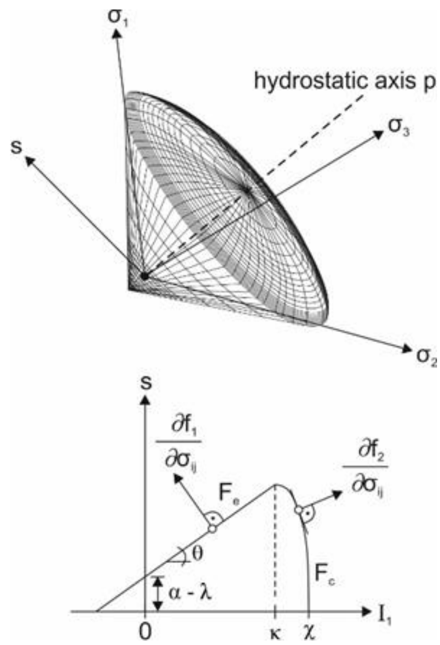

Figure 11. Cap model for plastic behaviour.

The time variant viscoplastic strain ε_{ij}^{vpl} consist of two parts. One part is the shape changing e (also deviatoric strain) which is effected by the deviatoric stress. The other part is the volumetric strain ε_{kk} which is effected by the hydrostatic stress level. The deviatoric strain consists of the transient part $\varepsilon_{ij}^{vpl,tr}$ and the stationary part $\varepsilon_{ij}^{vpl,st}$ (Eq. 2). The strain rate decreases during the transient creeping and leads to the stationary creeing with a constant strain rate.

The viscoplastic transient deviatoric strain rate $\dot{e}^{vpl,tr}$ is described by the maximum pore volume n_{max} and the time t (Eq. 4). The constant C_1 and D_1 are determined by mathematical functions of regression analysis of load-controlled triaxial creep tests.

$$\dot{e}^{vpl,tr} = \frac{D_1 \cdot n_{max}}{C_1} \cdot e^{-\frac{t}{C_1}} \tag{4}$$

The viscoplastic stationary strain rate $\dot{e}^{vpl,st}$ is described by the devatoric stress s and the temperature t (Eq. 5). The constants B_1 to B_4 are determined by mathematical functions of regression analysis of load-controlled triaxial creep tests. Using the approach of Betten (2001), the temperature T is considered by the ARRHENIUS-equation with activation energy Q and the common gas constant R.

$$\dot{e}^{vpl,st} = e^{-\frac{Q}{R\cdot T}} \cdot [B_1 \cdot s + (B_2 \cdot s^{B_4} - B_1 \cdot s) \cdot \tanh(B_3 \cdot s)] \tag{5}$$

The viscoplastic volumetric strain rate $\dot{\varepsilon}_{kk}$ is described by the hydrostatic stress level using the first invariant I_1, the temperature T and the compaction state (Eq. 6). According to Zhan et al. (1993) the ARRHENIUS-equation and the NORTON–BAILEY approach are used. The maximum volumentric strain $\varepsilon_{kk,max}$ is the initial compaction state and consists of the instantaneous elastic part, the instantaneous plastic part and the viscoplastic part. The constants E_5 to E_8 are determined by mathematical functions of regression analysis of load-controlled triaxial creep tests. The compaction state is described by a logarithmic saturation function which considers the transition of the granular rock salt to intact rock salt.

$$\dot{\varepsilon}_{kk}^{vpl} = e^{-\frac{Q}{R\cdot T}} \cdot E_7 \cdot e^{E_8 \cdot I_1} \cdot \left[ln\left(\frac{\varepsilon_{kk,max}}{\varepsilon_{kk,max}-\varepsilon_{kk}}\right)\right]^{-(E_5+E_6\cdot\varepsilon_{kk,max})} \tag{6}$$

3.2 *Implementation*

The whole subroutine VISCOSALT 2017 consists of 21 calculation modules which are used in 10 different calculation steps (Leppla 2017). The subroutine is implemented into the FE-Programme ABAQUS.

During the calculation the stress state and the strain state, the material stiffness as well as further solution dependent state variables are calculated and used as components of the system stiffness. Therefore, the material behaviour is updated during every integration in every integration point of each element.

In the calculation step 1 the three-dimensional modelling is checked. In calculation step 2 the elastic material stiffness is determined. In calculation step 3 the results of the converged increment before are taken. Using the Return-Mapping-Algorithm (Hofstetter et al. 1993, Wilkins 1964) the conus flow conditions and the cap flow conditions are calculated in the calculation steps 4 and 5 for determination of the inelastic strains. In the calculation step 6 the necessary material modus is selected. The elasticity modus, the conus modus, the cap modus and the corner modus are time invariant. The viscous modus is time variant. The strain is calculated in the calculation steps 7 and 8. With these strains the stresses and other material parameters are updated in calculation step 9. The numerical simulation is finalized in calculation step 10 by determining the whole material stiffness of the system and the setup of the tangent operator.

3.3 *Verification*

For verification of the subroutine VISCOSALT 2017 several triaxial test have been analised by back-calculation (Leppla 2017). These numerical simulations show the complex material behaviour of granular rock salt in a very good accordance to the test results. For example the results of the load-controlled triaxial creep test no. 182 are shown in Figure 12.

The whole test time is about 4 years. The initial density was $\rho_0 = 1.97$ t/m³ with a pore volume of $n_0 = 0.10$. The load $\sigma_1 = 3.2$ MN/m² and the cell pressure $\sigma_3 = 1.2$ MN/m² are constant. The big initial density and the small pore volume refer to the depth below the top of the tailings heap. The granular rock salt is compacted significantly. The comparison of the test results and the numerical simulations show a very good accordance.

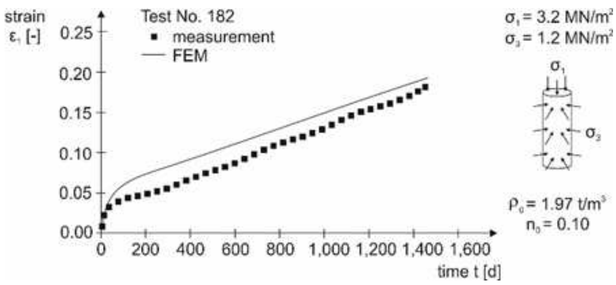

Figure 12. Time-strain behaviour of a long term creep test.

4 CONCLUSIONS

For the numerical simulation of the time variant stress-deformation behaviour of granular rock salt the subroutine VISCOSALT 2017 was developed. This subroutine was verified by back-calculation of triaxial tests. The results of these numerical simulations show a very good accordance to the test results. This is the proof that the developed subroutine can be used for the investigation of the stability and the serviceability of large tailings heaps consisting of granular rock salt and infrastructure constructions in the area around these heaps. Currently the subroutine is used at several projects of the engineering practice (Leppla 2017).

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