Proceedings of the 20th International Conference on Soil Mechanics and Geotechnical Engineering– Rahman and Jaksa (Eds)

Comparison of small and full scale shear box tests on large particle size aggregate

Comparaison d'essais de boîte de cisaillement à petite et à grande échelle sur des granulats de grande taille

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ABSTRACT: Aggregates with a large particle size are widely used in geotechnical engineering for a range of applications such as load distribution platforms, filling, drainage and railway ballast. It can consist of fresh aggregate or recycled components, and its engineering characteristics can vary widely. Independent of the application, it is important to characterise accurately its material properties in order to guarantee safe, economical solutions. The angle of friction is often the main factor required for geotechnical design and a common laboratory method to measure this property is the direct shear test. Nonetheless, when using standard laboratory equipment there is an upper limit to the size of particle that can be tested. However, significant differences can occur in the results compared with full-scale testing. The paper describes the assessment of a large direct shear box apparatus, with a shear plane area 2.25 m^2, that was fabricated to test aggregates with large particle sizes (≤100 mm). The results of initial full-scale tests on crushed limestone with a maximum particle size of 63 mm are presented and compared with results from standard shear box tests conducted on reduced scale samples of the same material.

RÉSUMÉ : Les agrégats de larges particules sont largement utilisés en génie géotechnique pour de nombreuses applications telles que les plates-formes de distribution de charge, le remplissage, le drainage et le ballast ferroviaire. Ils peuvent être constitués d'agrégats frais ou de composants recyclés, et leur caractéristiques techniques peuvent varier considérablement. Indépendamment de l'application, il est important de caractériser précisément leurs propriétés matérielles afin de garantir des solutions sûres et économiques. L'angle de frottement est souvent le principal facteur requis pour la conception géotechnique. L'essai de cisaillement direct est une méthode de laboratoire courante pour mesurer cette propriété. Néanmoins, lorsque l'on utile des équipements de laboratoire standard, les particules pouvant être testées ne peuvent dépasser une taille maximale. Pour passer outre cette limitation des appareils standard, une approche courante est de tester un échantillon mis à l'échelle. Cependant il peut y avoir des différences significatives entre ces résultats et ceux obtenus lors d'un test à échelle réelle. L'article décrit l'évaluation d'un grand appareil à boîte de cisaillement direct, comprenant une surface plane de cisaillement de 2,25 m^2, qui a été fabriqué pour tester des agrégats de particules de grandes taille (≤ 100 mm). Les résultats des premiers essais grandeur nature sur du calcaire concassé d'une granulométrie maximale de 63 mm sont présentés et comparés aux résultats d'essais de bôite de cisaillement standard menés sur des échantillons à petite échelle du même matériau.

KEYWORDS: large scale tests, shear strength, scale effects.

1 INTRODUCTION

The use of large particle size aggregate material is extensive in geotechnical engineering for different type of applications. The material can comprise fresh aggregate (e.g. crushed rock) or recycled components, such as construction demolition waste (which most commonly consists of brick and concrete). The characteristics of the material can largely influence the design of the engineering structures in which it is utilised and the parameter having most impact is represented by the angle of friction. For example, in the case of working platforms for piling, a change of the angle of friction used in the design calculations from 40° to 45° would cause a decrease of platform thickness equal to 150 mm (BRE 2004). This decrease in platform thickness implies a decrease in the overall factor of safety of the structure but across a platform of 50 m square would mean a difference in volume of material required equal to 375 m^3, the equivalent of almost 40 truck loads. It is evident that even a small overestimation of this angle could lead to an unsafe design but, alternatively, an underestimation would cause a conservative and, therefore, uneconomical design. It is therefore crucial to determine this property for the material to be utilised as accurately and reliably as possible.

One of the problems associated with the correct estimation of the angle of friction is the large variability of the material components, angularity and particle size distribution, especially when the material used is made of recycled constituents. This difficulty can be overcome by testing the material in order to identify the shear properties with a certain level of accuracy. The second problem however, related to testing, is the difficulty of conducting tests on large particle size materials in standard apparatus because the number of particles in the sample is too small to be representative or the particles are simply too big to fit into the apparatus.

A commonly used laboratory method for determining the angle of friction of soils is the direct shear test that, compared with other test methods (such as the triaxial test), is a relatively quick and inexpensive tool for determining the shear properties of the material under drained conditions. Standard shear box apparatus typically have a width (or diameter) of 64-73 mm (Bareither *et al.* 2008) so the maximum particle size which could normally be accommodated whilst respecting the guidance in the ASTM Standards (D3080, 2011) is approximately 5 mm and, depending on the testing device, rarely exceeds 10 mm with standard apparatus (Simoni and Houlsby 2006). Testing material in an apparatus that is not large enough (based on the ratio of particle to apparatus size) is widely reported to generate results for the angle of friction that are overestimated (Taylor and Leps 1938, Bishop 1948, Cerato and Lutenegger 2006, Sobol *et al.* 2015). An alternative approach for testing aggregates with large particle sizes consists of preparing a sample corresponding to a smaller scale representation of the original material and testing it using a standard apparatus (Moulay Smaîne *et al.* 2014, Kim and Ha 2014). The problem related to this procedure is the presence of scale effects affecting the test results and this method (independent of the scaling type used) often results in the reported angle of friction being underestimated.

Presented in this paper is a possible solution to this problem. This involves testing the material at full scale using a large shear box apparatus that has been designed and manufactured especially for this purpose at the laboratory of City, University of London. A number of large shear boxes have been constructed in the past, often for specific testing applications e.g. mine spoil (Bradfield *et al.* 2015), municipal waste (Dehdari *et al.* 2021) or soil reinforcement such as nails (Davies and Le Masurier 1997) or grids (Palmeira 1987). These examples (amongst other), coupled with small scale tests were used as the basis of the current design. A full description of the design process for the large shear box utilised here is presented in Tanghetti *et al.* (2019) and this paper mainly focuses on the description and assessment of its functionality and test procedures. The initial tests are conducted using a crushed limestone material having a particle size distribution corresponding to an average grading of Highways Agency material specification 6F2 (Highways Agency 2004).

The study was carried out in order to compare the results of the large shear box tests with the ones obtained from testing a downscaled sample of the same material using a standard shear box apparatus having an internal (sample) size 100 mm by 100 mm in plan and 44 mm high. In this way, the effect of testing the material at different scales could be evaluated.

The first results obtained from testing limestone material with a maximum particle size of 63 mm in the large shear box are presented in this paper and compared with small scale test results. A method for analysing the results is presented and finally, an evaluation of the performance of the large apparatus is considered.

2 TESTING PLAN

The first full scale test in the large shear box apparatus was conducted under a vertical stress corresponding to the sum of the self-weight of the material above the shear plane and weight of the top lid which has a mass of 991 kg (i.e. no further vertical load was applied by the vertical hydraulic jack). The resulting vertical stress at the shear plane during the test was therefore equal to 11 kPa and the sample was sheared at constant rate of displacement.

The small-scale tests were conducted on the same material (smaller particles being sieved out and then recombined to create the appropriate particle size distribution) were carried out under vertical stresses equal to 100, 200 and 500 kPa. The three different values of normal stress were decided in order to identify with more accuracy the critical state line defining the angle of friction of the sample. Specimens prepared at a low void ratio (dense samples) were sheared under a low level of vertical stress (100 and 200 kPa) while loose samples were tested under a normal stress of 500 kPa.

From the results of small scale tests it was possible to derive the shearing behaviour of the material which was compared with the results obtained from the full scale test.

3 APPARATUS DESCRIPTION

The full-scale direct shear test was conducted in a large shear box apparatus with a sample plan area of 2.25 m^2 and a maximum height of 1 m. An overview of the design of the large shear box is provided in Figure 1.

The maximum height allowed for the sample was slightly less than 1 m to allow the top lid (platen) to sit within the box when the sample was set up. This prevents the possibility of the lid moving laterally although, in practice, the design of the apparatus means this is unlikely. The maximum particle size of the sample which can be tested in this shear box whilst minimising any scale effects is equal to 100 mm. This specific size was chosen for the design of the large apparatus as representing the average maximum particle size of the material class 6F2 being considered. Therefore, the design of the large apparatus was produced starting from the geometry of the box obtained by considering the minimum width/height of the box to satisfy the maximum particle size ratio requirements to avoid scale effects indicated by the study of Fu *et al.* (2015).

Vertical load can be applied on top of the sample by a 5 MN hydraulic jack. A hydraulic control system can maintain a constant vertical force accommodating any movements of the lid due to dilation or contraction of the sample during the test. The

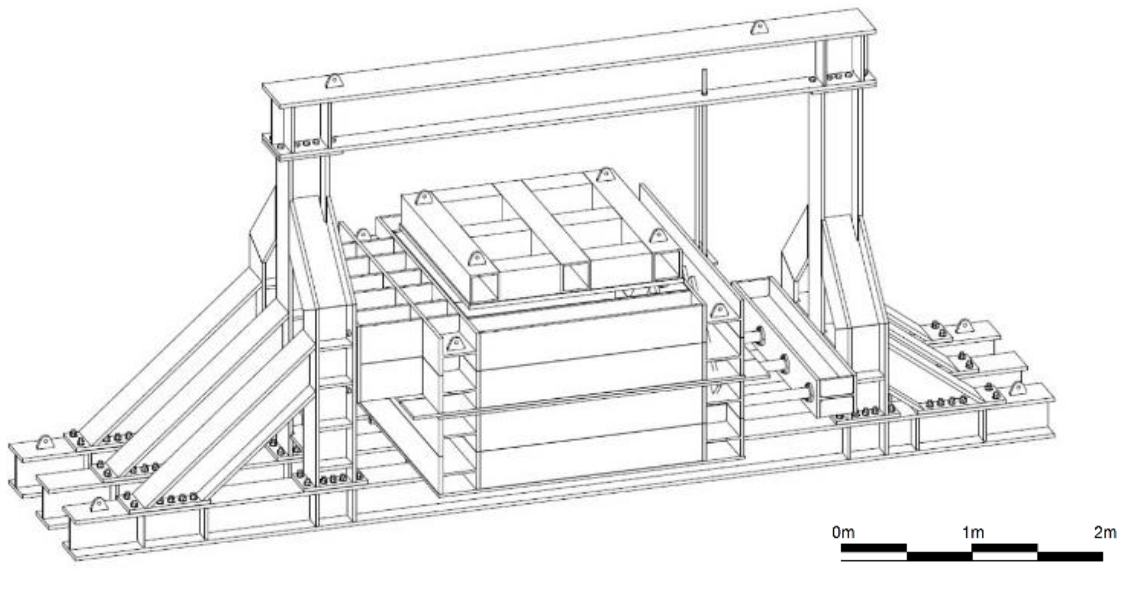


Figure 1. Overview of the design of the large shear box apparatus (Tanghetti *et al.* 2019)

horizontal load moving the bottom half of the shear box is applied by four 500 kN hydraulic jacks which push directly against the lower half of the box while the top half is restrained by the reaction frame. Two load cells are located between the top half of the box and the reaction frame on the opposite side to the horizontal jacks. These measure the horizontal force applied to the sample along the shear plane (after accounting for friction in the system).

In order to minimize the frictional losses acting between the two halves and at the bottom of the box, the top surface of the lower half of the box and the three beams supporting the box were covered with Acetal sheets. These were glued to the surfaces and covered with grease in order to further reduce the friction. The maximum horizontal displacement allowed (corresponding to the maximum stroke of the jacks) is equal to 337 mm which was considered sufficiently large to reach the critical state of the material (based on the results conducted at small scale which allowed an estimate of the level of shear strain required). The rate of displacement can be altered by a needle valve controlling the flow of the oil to the jacks so that the speed can be increased or reduced based on the requirements of the test. In order to maintain the similarity between small and large scale tests, the rate of displacement in the large apparatus was made as close as possible to 1 mm/min giving the same shear strain rate employed during the small scale tests. For clarity, in this paper, shear strain is defined as dh/H where dh is the horizontal movement of the sample and H is the initial height. A full description of the design of the large shear box apparatus is presented in Tanghetti *et al.* (2019).

4 SAMPLE PREPARATION AND TEST PROCEDURE

The material which was used for the large-scale test is a crushed Devonian limestone sourced from a quarry in Ashburton, UK. The same soil was used for the small-scale tests conducted in the standard shear box so no differences in the mineralogy or angularity of the material would affect the results and only the effect of scaling the size of the particles would be examined. The grading curve of the sample used for the full-scale tests together with those representing the samples tested at smaller scale are shown in Figure 2.

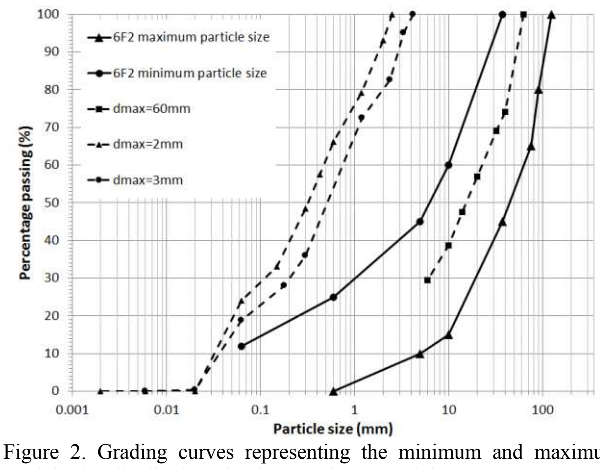


Figure 2. Grading curves representing the minimum and maximum particle size distributions for the 6F2 class material (solid curves) and the ones representing the limestone samples used for small and full scale tests (dashed curves).

As can be observed from the graph, the material selected for the full scale sample has a grading that was approximately equivalent to an average of the grading curves representing the minimum and maximum particle size distribution of 6F2 class material (Highways Agency 2004). All particles of the sample used for large scale tests passed the 63 mm sieve opening size. The grading curves for the small scale samples were determined by translating towards the left the curve representing the full scale sample such that their maximum particle size was respectively corresponding to 3 and 2 mm. Samples used for the small scale tests were prepared by sieving and recombining material from the full scale sample to represent these grading curves.

The large particle size material was delivered to the laboratory of City, University of London in bulk bags, each one containing about 1000 kg of material. The bags were provided with loops which allowed them to be lifted by the use of a crane. Therefore, in order to prepare the sample for testing, the bags were weighed and then lifted into the shear box (the reaction beam and attached vertical hydraulic jack having been removed to facilitate the filling operation). The bottom of the bags were cut open using a knife so that the material was allowed to pour into the box. This operation was repeated using a bag at the time until almost reaching the top of the box (a small height of about 100 mm was left empty in order to accommodate the top lid so that it was safely kept in the same position during the test and also so that no loss of material would occur during testing). The material was not compacted in any way and the surface was carefully levelled.

Once the sample was in the box and its surface levelled, the lid was placed on top of it. After this, the beam holding the vertical hydraulic jack was bolted to the vertical reaction frame so that the jack was aligned with centre of the top lid. For this first test the vertical jack was not used to apply pressure on top of the sample, nonetheless it was necessary to secure the horizontal beam to the vertical components of the reaction frame in order to guarantee the correct functionality of the structure.

The initial height of the sample was measured from the distance between the top of the lid and the top of the shear box which was subtracted to the sum of the internal height of the box and the thickness of the lid. The preparation process therefore resulted in a sample having a density of 17.25 kN/m^3.

Changes in height of the sample during the test were measured by four displacement transducers which were placed vertically at the four corners of the top lid. These transducers are held in position by a frame made by assembled slotted channels which were used to create an independent solid structure resting on the floor of the laboratory. Similar independent structures were built in order to support the two horizontal displacement transducers which were used to measure the horizontal displacement of the bottom half of the shear box. Figure 3 shows the completed and assembled test apparatus prior to testing.

Figure 3. Completed and assembled test apparatus prior to testing.

At this point the test was ready and shear force was applied by pushing the bottom half of the box using the four horizontal jacks to shear the sample at a constant rate of displacement along the horizontal plane between the two halves of the box. The shear force was calculated as the difference between the sum of the forces measured by the two load cells reacting against the top side of the box less the frictional force acting at the contact surface of the two halves of the box. This was calculated from the coefficient of friction between steel and Acetal (validated in a proof test conducted with the box empty).

Once the test was complete and sufficient horizontal displacement was achieved, the bottom half was pushed backwards to its original position by the use of two small hydraulic jacks which were placed between the reaction frame and the side of the bottom half of the box (in the opposite direction to the four jacks previously used to move the bottom half of the box and shear the sample). At this point the horizontal beam and top lid were removed and the box was emptied.

5 TEST RESULTS

The results obtained for the large-scale test are presented in Figures 4 and 5, which show the variation of shear stress and volumetric strain during the test.

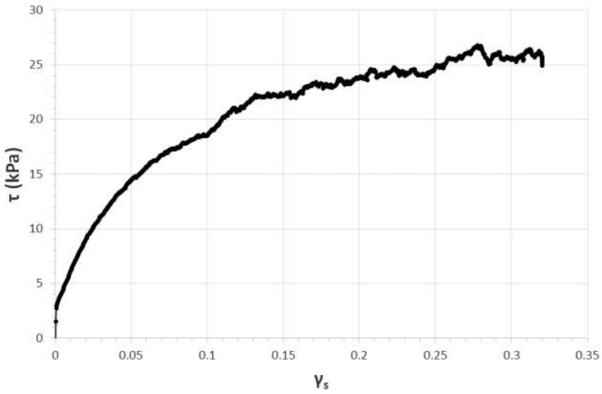


Figure 4. Shear stress (τ) *vs* shear strain (γ_s).

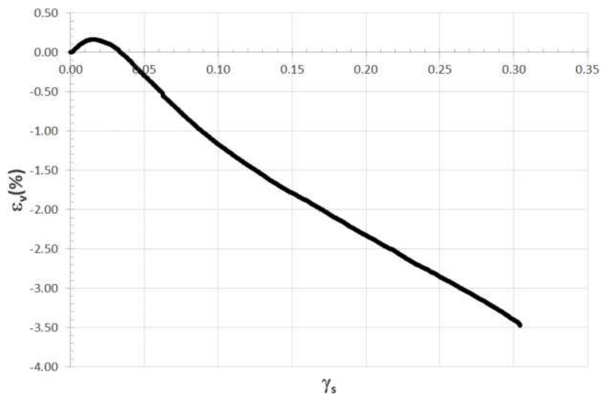


Figure 5. Volumetric strain (ε_v) *vs* shear strain (γ_s).

It can be noted that after a small amount of compression at the start of shearing the sample dilated for the remainder of the test (Figure 5). This was because although the material was placed in a loose state and was not compacted it was subjected to a relatively low vertical load (provided by the mass of the top lid and the self-weight of the material above the shear plane, as explained above). However, the variation of the shear stress (Figure 4) does not exhibit significant post peak softening and appears to represent the typical shearing behaviour of loose samples.

Figure 5 suggests that following an initial compression there was a continuous increase in dilation of the sample with shearing without reaching a point of zero volume change (i.e. critical state). However, part way through shearing, tilting of the top lid was observed towards the direction of shearing, as a consequence of which, full contact between the lid and the material was lost. The effect of this is that the top lid displacement cannot be considered as completely representing the real volumetric strain of the material. In order to identify where this movement of the top lid became more significant, the variation of the difference in height between opposite sides of the top lid during the test was calculated, as plotted in Figure 6 (essentially a measure of the rotation of the lid).

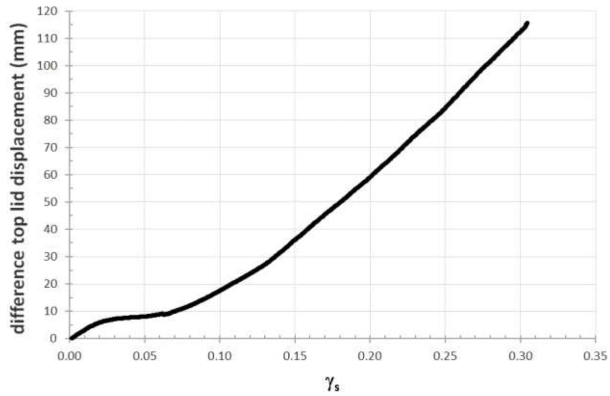


Figure 6. Difference in top lid height *vs* shear strain (γ_s).

From Figure 6 it can be observed that the difference in displacement between the two sides of the top lid starts to significantly increase as the shear strain approaches 10%. As a consequence, the results obtained cannot be considered as reliable after this level of shear strain. It can be observed from Figure 4 that for values of strain larger than 10% even the values of shear stress seem to be characterised by some level of distortion.

6 COMPARISON WITH SMALL SCALE TESTING

A series of standard (small) shear box tests was conducted on reduced scale limestone samples and the derived friction parameters of the material obtained are summarised in Table 1. In these tests, plots of the volumetric strain against shear strain were produced and the critical state identified as the point at which there was no further volumetric strain. What is interesting to observe from these results is a decrease in both the critical and peak angle of friction of the sample (about 2°) when reducing the maximum particle size of the sample from 3 mm to 2 mm.

Table 1. Summary of the results obtained from direct shear tests conducted on the small-scale samples using a standard shear box

	d_{max}=3mm	d_{max}=2mm
Critical angle (°)	40.8	38.5
Av. shear strain at critical (%)	31	25
Peak angle (°)	49.6	47.8
Average shear strain at peak (%)	13	11

The results of the large-scale test presented in Figure 4 did not allow the calculation of a peak angle of friction with a certain level of confidence. However, in order to compare the results with those obtained from the small scale tests, it was possible to estimate the critical friction angle, ϕ'_c, using the stress-dilatancy approach based on Taylor (1948), which considers the work done to overcome both friction and volume change during shearing. Simplified equations based on this approach, which relate the stress ratio, τ/σ_v, during shearing to the critical friction angle and the angle of dilation, ψ, have been presented by, for example, Atkinson (2007), i.e.

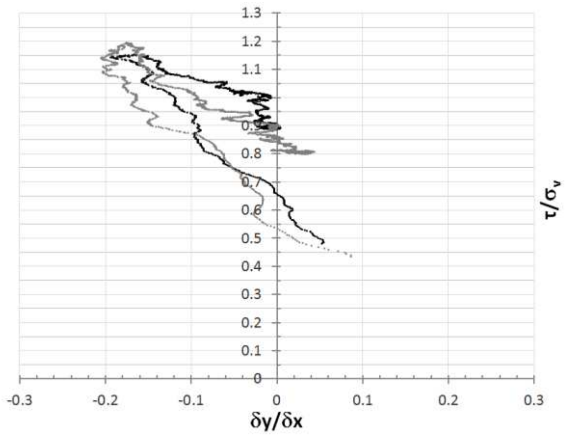
$$\tau/\sigma_v = \tan(\phi'_c + \psi) \quad (1)$$

Where, $\tan\psi = -\delta y/\delta x$

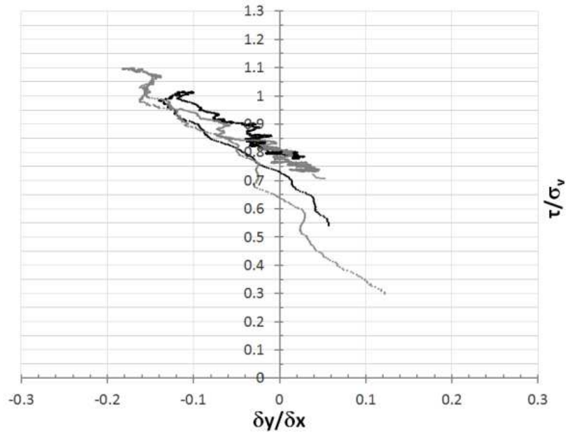
It can be seen from Eq.1 that when $\delta y/\delta x = 0$

$$\tau/\sigma_v = \tan \phi'_c \quad (2)$$

This condition will occur at both: (i) pre-peak strength - at the point of maximum compressive strain during shear, when the change in soil volume moves from compression to dilation, and (ii) post-peak strength - at the critical state, when the soil is shearing at constant volume and shear stress. Whilst Wood (1991) has indicated that in the former case work, not accounted for in the derivation of Eq.1, is probably being done by the shear load in causing elastic deformation of the soil particles, this effect will be similar for tests using the same limestone but with different particle sizes. Therefore, Eq.2 may be used to allow a comparison of the shear box test data at the pre-peak strength stage of shearing when $\delta y/\delta x = 0$.



(a)



(b)

Figure 7. Stress ratio (τ/σ_v) *vs* dilatancy ($\delta y/\delta x$) of dense samples with maximum particle size equal to 3 mm (a) and 2 mm (b), normal stress equal to 100 kPa.

Following the method proposed by Wood (1991), plots of τ/σ_v v. $\delta y/\delta x$, are plotted for data obtained from the shear tests conducted with the 3mm and 2 mm maximum particle size samples (two tests for each sample size) in Figure 7 and the full-scale (60 mm maximum particle size) sample in Figure 8. In these plots, the point at which the curves cross the stress ratio (τ/σ_v) axis at $\delta y/\delta x=0$ indicates a stress ratio at which only frictional forces between the soil grains are resisting shearing.

Considering the higher points of interception of the curves with this axis in Figures 7 (a) and (b), which represent the post-peak strength stage of the tests, the values obtained are approximately 0.9 and 0.8 for the 3mm and 2mm maximum particle size samples, respectively. These stress ratios correspond to a critical state angle of friction of 42° for the sample with maximum particle size equal to 3 mm and 39° for the 2 mm sample. The values are very close to those reported in Table 1 and confirm the validity of the method for identifying the critical state angle of friction.

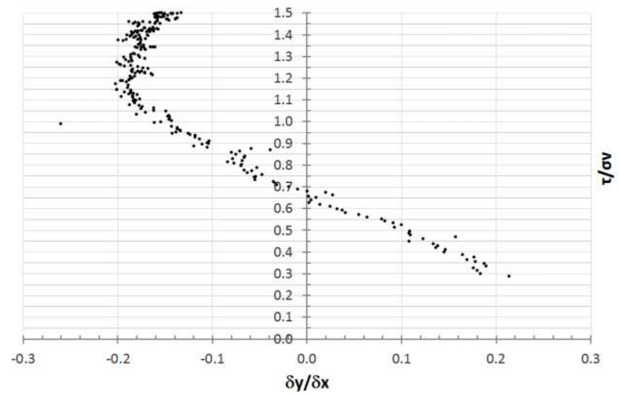


Figure 8. Stress ratio (τ/σ_v) *vs* dilatancy ($\delta y/\delta x$) of the full-scale sample tested using the large shear box apparatus, normal stress equal to 11 kPa.

In Figures 7 (a) and (b), the values of interception of the curves with the stress ratio axis at maximum compression are lower than those at critical state. This observation is consistent with experimental data presented by Wood (1991), which led to the explanation, above, regarding work for elastic deformation of the soil particles. The values of stress ratio are equal to 0.55 - 0.65 for the samples with a maximum particle size equal to 3 mm and 0.65 - 0.72 for the 2 mm maximum particle size samples. For the large-scale sample, Figure 8, data has been plotted up to the stage in the test where the stress ratio reached a value of 1.4. This was at a shear strain of 7%. Data for shear strains greater that this are not plotted because, as discussed above, rotation of the top lid during the test (Figure 6) resulted in the data being unreliable beyond this point in the test. Nevertheless, the data collected before that point were considered to be suitably reliable to be able to compare the results with those obtained from the small-scale tests. The value of stress ratio at which the curve in Figure 8 crosses the vertical axis is approximately 0.65, which is equivalent to an angle of friction of 33°. Therefore, a good agreement exists between the values of stress ratio reached by the three samples at the point of maximum compression.

Despite the encouraging outcome of the first test conducted at full scale, the results obtained show some irregularities which seem to be associated with the absence of an applied vertical pressure on top of the sample and the resulting excessive rotation of the top lid. Because of this it was not possible to obtain a comparison of small and full-scale test results at peak shear stress. However, the results published herein provide confidence in the ability of the large shear box to be used for testing full scale samples and a programme of further testing is planned. To permit shear behaviour at a range of stress levels to be investigated, in these tests a normal vertical pressure of up to 200 kPa will be applied to the top of the sample using an hydraulic jack. This will also have the effect of minimising rotation of the top lid.

7 CONCLUSIONS

From a series of standard shear box tests conducted at two different scales using a standard shear box apparatus it was possible to identify the critical and peak angle of friction of a limestone material with samples having a maximum particle size equal to 2 mm and 3 mm. The result of these tests indicated a consistent difference in the values of the peak and critical angles of friction (of about 2°) when increasing the particle size of the tested material. The same material was tested at full scale (maximum particle size equal to 60 mm) using a large shear box (shear plane equal to 2.25 m^2) that was designed and fabricated at City, University of London. Comparison of the results of a test using the new apparatus with the tests using the smaller particle size samples in standard apparatus yielded similar results at small displacements. However, excessive rotation of the top lid of the large shear box at large displacements, due to the absence of any vertical load, did not allow direct comparison between results from the standard and large-scale apparatus at larger displacements. Nevertheless, the comparison provided confidence in the use of the large shear box for testing large (≤ 100 mm) particle size materials at higher values of normal stress, which are consistent with the range of stresses typically encountered in engineering practice.

8 ACKNOWLEDGEMENTS

The work presented here forms part of the doctoral studies of the first author which are generously supported by the Temporary Works Forum (www.twforum.org.uk).

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