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On the use of elasto-plastic models with hardening for excavations on the gravel of Lima

Sur l'utilisation de modèles élasto-plastiques avec durcissement pour les excavations sur le gravier de Lima

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ABSTRACT: In recent years, the construction of large buildings has increased in the city of Lima, mainly for homes and offices. This in turn, has brought about the need to generate more space vehicles parking. Consequently, it is necessary to carry out excavations of greater depth, which very often exceeds 30m. The construction of these excavations requires specialized support systems, whose objective is to guarantee the stability of the excavation and limit the level of deformations that may occur as a result of the excavation. Anchored walls are the preferred system in Peru, which are mainly designed by limit equilibrium analysis. However, some accidents have been reported recently which has forced the revision of the methods of design, including those using FEM. The main limitation for the use of FEM is that more research is required in relation to the geotechnical parameters of the Lima gravel. This paper presents a review of the available parameters for the gravel soil of Lima, as a result of large scale in situ tests analysis and backanalysis of excavations, in relation to an elasto-plastic model with hardening, highlighting its advantages over the conventional Mohr-Coulomb model for excavation projects in gravels.

RÉSUMÉ : Ces dernières années, la construction de grands bâtiments a augmenté dans la ville de Lima, principalement pour les maisons et les bureaux. Cela à son tour, a entraîné la nécessité de générer plus de stationnement de véhicules spatiaux. Par conséquent, il est nécessaire de réaliser des fouilles de plus grande profondeur, qui dépassent très souvent 30 m. La construction de ces excavations nécessite des systèmes de support spécialisés, dont l'objectif est de garantir la stabilité de l'excavation et de limiter le niveau de déformations pouvant survenir à la suite de l'excavation. Les murs ancrés sont le système préféré au Pérou, qui sont principalement conçus par analyse d'équilibre limite. Cependant, certains accidents ont été signalés récemment, ce qui a forcé la révision des méthodes de conception, y compris celles utilisant FEM. La principale limitation de l'utilisation du FEM est que des recherches supplémentaires sont nécessaires en ce qui concerne les paramètres géotechniques du gravier de Lima. Cet article présente une revue des paramètres disponibles pour le sol graveleux de Lima, à la suite d'essais in situ à grande échelle d'analyse et de contre-analyse des fouilles, en relation avec un modèle élasto-plastique à durcissement, mettant en évidence ses avantages par rapport au Mohr conventionnel. Modèle coulombien pour les projets d'excavation en graviers.

KEYWORDS: constitutive models, elasto-plasticity, hardening, excavations.

1 INTRODUCTION.

The city of Lima is mainly settled on gravelly soils deposits corresponding to the dejection cone of the Rimac river. These highly thickness and homogeneous deposit, with a water table at depths greater than 50m, have good bearing capacity for the foundations of buildings. Under normal conditions it should not be a major problem to construct on the soil of Lima. However, in recent years the growth of the city has forced the construction of large buildings, which require deep excavations for the construction of basements, many of which exceed 20m in depth. This type of project is usually executed with the anchored wall technique. The design of this type of structures requires knowledge not only of the resistant characteristics of the soil, but also of the stiffness characteristics. However, engineering practice, both by designers and contractors, has been to ignore deformation characteristics, and instead adopt conservative safety factors in calculation models that do not take deformation into account (typically limit equilibrium methods). In other words, the support of the excavations is calculated and designed based on the principles of collapse of the structure, without taking into account that the possible effects on neighboring buildings are not necessarily associated with the failure of the excavation. A large number of projects have been reported, where neighboring buildings have been affected by settlements,

fissures and cracks, and in more critical cases, collapse of structures. This problem forces the need to have more precise calculation models that combine both the resistant characteristics and the deformation characteristics of the ground. The main limitation for the use of advanced models in the gravelly soil of Lima is the lack of laboratory and field equipment, adequate for the large particle size of the gravels of Lima ($>4''$), that allow obtaining reliable parameters. Faced with this limitation for years, in practice, different extrapolation methods have been used from the results of tests on the gravel matrix (particles smaller than 4.75mm), tests on scaled granulometries, correlations with index properties, or more recently correlation with geophysical tests, all of them with little or no research.

In the framework of large-scale projects, from the 70s to the present, some large-scale in situ tests have been carried out, mainly direct shear tests (De la Rosa, 1974; Humala, 1982; Shuan, 1997; Cañari, 2001), aimed at determining the resistance parameters of Lima gravel: friction angle and soil cohesion. Non data about stress-strain relationships of the gravelly soil was reported. As a result of the recent construction projects in Lima, the underground 'Lima Metro' and the viaduct 'Yellow Line', different tests have been carried out, among which the geophysical and pressurometer tests stand out. These tests provide results that allow estimating the modulus of deformation of the soil in small deformations, and in a state of failure,

respectively. This paper presents a review of the available parameters for the gravel soil of Lima, as a result of new large-scale laboratory and in situ tests and back analysis of excavations, in relation to an elasto-plastic model with hardening, highlighting its advantages over the conventional Mohr-Coulomb model for excavation projects in gravels.

2 PROCEDURE OF ANCHORED WALL EXCAVATION

The typical basement excavation process for buildings projects in Lima involves the following steps:

- Excavation to form the first line of anchors. Intermediate berms are left between panels.
- Drilling and installation of temporary anchors according to the paneling sequence.
- Execution of the reinforced concrete wall panel in a traditional way, including the use of industrialized formwork.
- Tensioning the anchors after curing the executed concrete panels.
- Construction of the adjoining panel.
- Tensioning the anchors of the adjoining panels.
- The procedure indicated above is repeated until the foundation level (See figure 1).

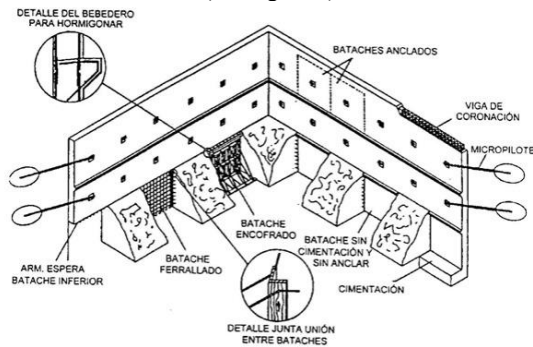


Figure 1. Excavation sequence of a typical building project

3 ELASTO-PLASTIC MODELS WITH HARDENING

Several elasto-plastic models are available for engineering practice. The most widely used is probably the Mohr-Coulomb model (MC), due to its simplicity, but it has some drawbacks to computed displacements in excavations. The more advanced hardening soil model (HS) and hardening soil model with small strain stiffness (HSS) implemented in Plaxis FE code, incorporates the Mohr-Coulomb failure criterion to define the resistance of the soil (friction angle, ϕ and cohesion, c) but unlike the perfect elasto-plasticity MC model, the yield surface of the HS and HSS model can expand in the principal stress space due to plastic deformations. Likewise, this model distinguishes between two types of hardening: shear hardening used to simulate irreversible deformations due to a deviatoric load and compression hardening to simulate irreversible plastic deformations due to isotropic compression (Schanz et al, 1999; Brinkgreve et al, 2019). The plastic deformations are calculated by means of the surface of plastic potential defined through the angle of dilatation, using a variant of the relation proposed by Rowe (1962).

Soils when subjected to deviating stress show decreasing stiffness and at the same time develop irreversible plastic deformations. In the case of a drained triaxial test the relationship between the axial strains and the deviating stress approximates the shape of a hyperbola. This relationship was used in the hyperbolic model of Duncan & Chang (1970). The 'Hardening Soil' model represents an improvement to the hyperbolic model in the sense that it uses the theory of plasticity instead of the

theory of elasticity, includes the dilation of the soil and introduces a yield surface.

An important characteristic of this model is the dependence of the stiffness with the stresses, this characteristic allows to better simulate the interaction of the soil with the structural elements, which is convenient in case of anchored wall excavations (Gouw, 2014).

Additionally, the HSS model allows to simulate the non-linear behaviour of the soil in the small strain range, which is the main different with the HS model. HSS model incorporates a non-linear law that defines the degradation of soil stiffness with the increase in deformations (Benz, 2007). Likewise, the hysterical behaviour of the soil is defined through the rules of Masing (1926). The model is defined through 9 parameters, as shown in Table 1. The first 3 parameters define the shear strength. The following 4 parameters define the deformation behaviour of the soil. E_{50} is the secant modulus corresponding to 50% of the soil resistance in a drained triaxial test, while E_{ur} is an elastic modulus associated with unloading-reloading processes. E_{edo} is the module associated with primary loading processes and is obtained from oedometric tests. Finally, the parameter 'm' allows to calibrate the level of modules dependence with the confining stress. Unfortunately, no laboratory equipment is available to determine these parameters in the Lima gravel. However, they can be estimated from back analysis of in situ tests, as described below.

On the other hand, during the loading and unloading processes, typical of a seismic event, cyclic shear deformations occur, and the shear moduli decrease, while the amount of energy dissipated increases (Seed & Idriss, 1970). The 'HS Small' model allows simulating this behaviour, adjusting the parameter $\gamma_{0.7}$, which represents the shear deformation corresponding to shear modulus degraded to 70% of their initial value in small deformations. Using this parameter, the stiffness and damping modulus curves are adjusted as a function of shear deformation (Brinkgreve et al, 2019). In the case of the gravels of Lima, as laboratory measurements are not available, the work carried out by Rollins et al. (1998) for gravels is considered.

Table 1. Summary of selected parameters for HSS model (Lima gravel).

Symbol	Description	Unit	Min.	Max.
c	Cohesion	kN/m ²	20	40
ϕ	Angle of internal friction	°	40	42
ψ	Angle of dilatancy	°	6	12
E_{50}^{ref}	Secant stiffness in standard drained triaxial test	MPa	80	80
E_{oed}^{ref}	Tangent stiffness for primary oedometer loading	MPa	80	80
E_{ur}^{ref}	Unloading / reloading stiffness from drained triaxial test	MPa	240	240
m	Power for stress-level dependency of stiffness	-	0.80	0.80
G_0^{ref}	Reference shear modulus at very small strains	MPa	600	850
$\gamma_{0.7}$	Threshold shear strain at which $G_s=0.722G_0$	-	1.0E-4	1.0E-4

4 CHARACTERISTICS OF LIMA GRAVEL

The gravel of Lima is classified mainly as GP, GW or sometimes GM. The granulometric characteristics show percentages of boulders (particles > 3") that vary between 2 to 62%, the gravels vary between 33 to 62%, while the sands vary between 5 to 24% and the fines between 0 to 12%. According to the work of Sanchez et al. (2016), the specific weights present a variation between 20 to 24 kN/m³, showing a greater dispersion in the first 10m of depth.

4.1 Shear resistance

Soil shear strength of granular soils is generally defined through the Mohr-Coulomb failure criterion, which for common load conditions in engineering projects is usually defined by a straight line:

$$\tau = c + \sigma' \tan \phi \quad (1)$$

Where: τ , the shear strength; σ' , the normal effective stress in the failure plane; ϕ , the angle of friction and c , the cohesion. The tests most used to determine the resistance parameters (c and ϕ) are the direct shear test and the triaxial test. However, in the case of gravelly soils, the main limitation to carry out these tests is the limited availability of large-scale equipment. For the study of the gravelly soils in Lima, only the in situ direct shear test equipment belonging to UNI (De la Rosa, 1974) was available for a long time and recently the Geotechnical laboratory of CISMID-UNI, has implemented a laboratory large-scale direct shear equipment that allows to test gravels with a maximum size of 4" (Bazurto, 2010). Large-scale direct shearing tests have been carried out with these equipment's in situ (De la Rosa, 1974; Humala, 1982; Shuan, 1997; Cañari, 2001) and in the laboratory (Bazurto, 2010). In recent years, the megaprojects that are being developed in Lima (electric train, subway train, yellow line viaducts) have required the execution of several large-scale direct-shear tests allowing a larger database of results from this type of tests. Figures 2 and 3 show a compilation of results of laboratory shear tests, carried out in the gravel of Lima where the results have been interpreted based on peak shear strength (maximum shear stress) and ultimate strength (shear stress at large displacements). A summary of the direct in situ shear tests, compiled by the authors, is presented in Figure 4. It can be clearly seen how the friction angle and cohesion values of laboratory tests are notably higher than those corresponding to in situ tests. An explanation for this difference results from the examination of the test procedures performed, in both cases they are tests in controlled tension, where incremental loading steps and displacement measurement have been carried out. In the case of the laboratory tests, the reported tests reached accumulated horizontal displacement values during the shearing phase of the order of 60-70 mm, while in the in-situ tests, the reported displacements vary between 5 and 20 mm, therefore, in this case, it is not possible to mobilize the total available resistance of the soil. Laboratory tests will be used to define the resistance of the Gravel of Lima.

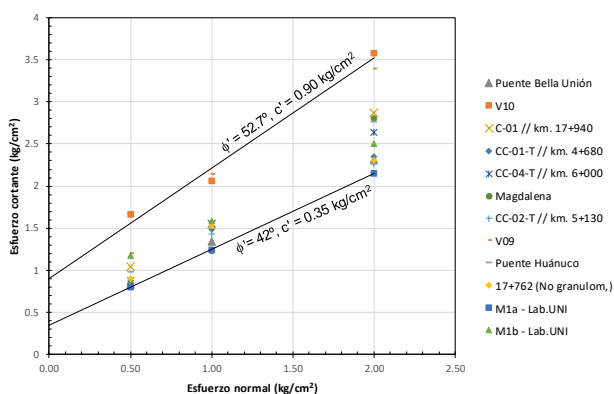


Figure 2. Peak resistance, in large-scale laboratory tests

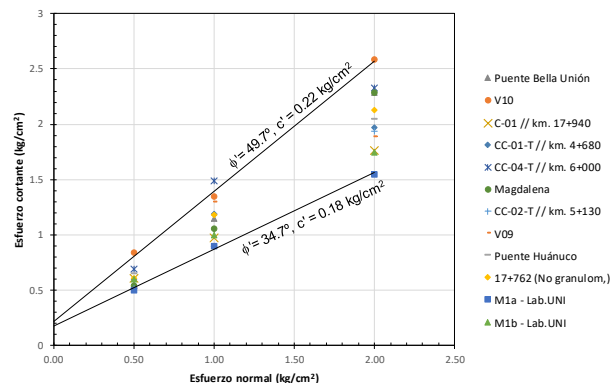


Figure 3. Ultimate resistance in large-scale laboratory tests

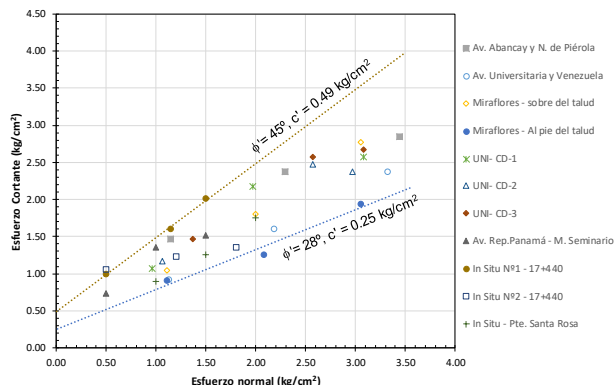


Figure 4. Shear resistance in in-situ tests

One of the most important aspects in the behaviour of granular soils is related to the phenomenon of dilation during shearing. Aspect that to date has not been treated in relation to the soil of Lima. As a reference, the value of the absolute dilation of direct shear tests in the laboratory has been calculated (see figure 5). A large dispersion is observed in the dilation values, probably due to the different granulometries tested. Even so, a trend of reduction in the value of the dilation is observed as the value of the vertical stress increases. For design purposes it is suggested not to use a dilation value greater than 6°, under typical stress conditions.

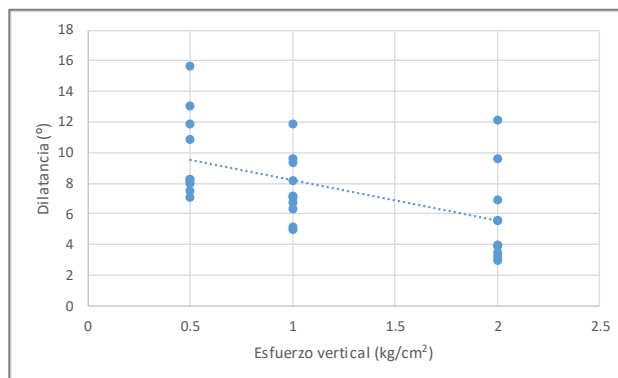


Figure 5. Lima gravel dilation calculated from large-scale direct shear tests.

4.2 Stress-strain characteristics

Unfortunately, to date there are no large-scale tests (triaxial tests or oedometric tests) that allow determining the stress-strain characteristics of the Lima gravelly soil, under controlled conditions. However, valuable information can still be obtained from field tests, such as plate load tests. In this type of test, a hydraulic jack is used to apply a vertical force to the ground

through a metal plate generally with a diameter of >30 cm and the settlement is measured as the force on the ground progressively increases. This type of test has two phases, a primary loading phase and an unloading phase. The primary load is related to the E_{edo} module, associated with plastic deformations and the unloading phase is related to the E_{ur} module, associated with elastic deformations. Both modules are defined in the constitutive model 'Hardening soil with small-strain stiffness (HSS)' by oedometric and triaxial tests. However, the values of the modules can be estimated indirectly, through a process of iterative trial and error simulations, until the best fit between the load-settlement curve of the plate test and a simulation of the test with the HSS model is achieved. During the simulations, the relationship established by Schanz and Vermeer (1998) is maintained: $E_{edo} = E_{50}$ and $E_{ur} = 3E_{50}$. Figure 6 shows the results of the adjustment carried out, using the HSS model and compared with two plate tests performed. The values of the modules used in the simulation are indicated in Table 1.

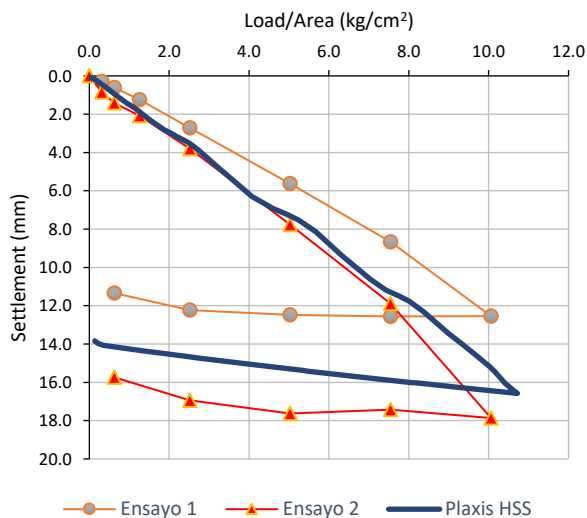


Figure 6. Comparison between plate load test simulation with the HSS model and in situ plate tests results.

4.3 Shear modulus (G_{max})

In general, the moduli of soil deformation vary with the confining pressure, this is an essential characteristic when it is necessary to calculate deformations in excavations. Likewise, it is known that the deformation moduli of the soil are progressively reduced as the shear deformation increases, from a maximum value G_{max} corresponding to small deformation levels ($<10^{-5}$). This maximum value can be established from geophysical tests that produce small deformations in the ground. Being the MASW test, which measures the propagation velocities of shear waves (V_s), the most widely used test in Peru. Figure 7 shows the fit achieved between the values of shear wave velocities V_s measured in distinct projects in Lima, and those computed with the HSS model. The lower limit and the upper limit of these tests can be achieved with values of modulus G_{max} between 600 to 850 MPa, and the parameter 'm', which allows defining the variation of the modulus with depth, established to $m = 0.8$. Finally, the degradation curve of the shear modulus with the deformation for the gravel of Lima, has been estimated using two characteristic points, one corresponding to G_{max} value (from geophysical tests) and the other obtained from pressuremeter tests carried out in the project of Line 2 of the Lima underground as reported by Sanchez et al. (2016). Figure 8 shows the curve obtained with the HSS model for Lima gravelly soil compared to the degradation curves for gravels obtained by Rolling et al. (1998). The same figure shows the modulus ratio obtained with the pressuremeter test.

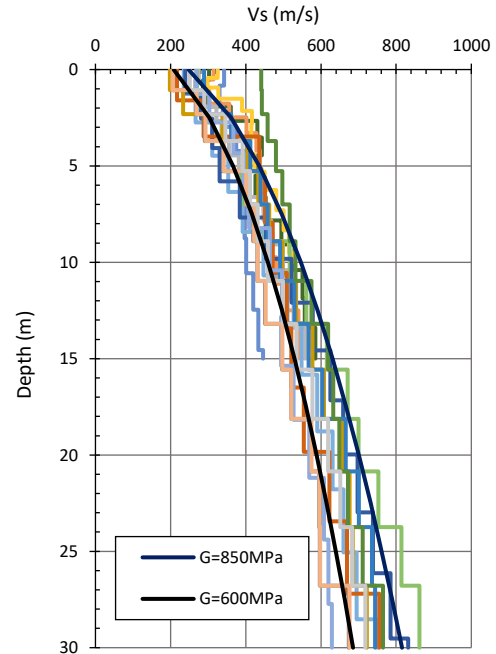


Figure 7. Summary of MASW tests performed in the Lima gravelly soil.

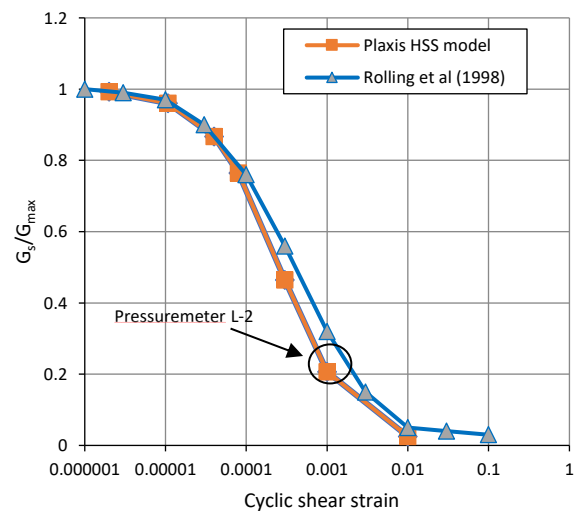


Figure 8. Modulus reduction curve for Lima gravelly soil.

5 APPLICATION OF ELASTO-PLASTIC MODELS FOR DEEP EXCAVATION IN LIMA

A finite element model (FEM) of a 32 m deep excavation with concrete wall and anchors has been prepared, using the recommended Lima soil parameters (see Table 1). The wall is 0.35 m thick up to 18 m deep, then increases to 0.50 m thick up to 32.2 m deep. The anchor cable is 16 m long in the upper levels, reducing to 4.5 m in the lower level, while the bulb has an average length of 7 m. A load of 10 kPa is included on the ground surface to represent the neighboring building. The model is developed under plane strain condition and formed by 2086 fourth-order triangular elements. The constitutive MC, HS and HSS models have been used. The elastic modulus in the case of the MC model has been adopted equal to the E_{ur} modulus. The upper soil layer adopts the parameters indicated in Table 1 as Min, while the lower stratum used the parameters indicated as Max. The results of the model are shown in Figure 9, where the

horizontal displacement profiles calculated with the MC, HS and HSS models are compared to the inclinometer readings during the excavation process. These results clearly show that the horizontal displacements calculated with the HSS model better fit the field measurements. The displacement pattern calculated with the MC model shows a negative inclination (towards the ground) at about 15m deep, which is not observed in HS and HSS models results. Note that the results obtained with the HS model are remarkably different from those obtained with the HSS model, even when they keep the same parameters. Also note, the displacement pattern computed with HS and HSS models is positive, towards the excavation, as was observed in excavations in Lima (Saucedo et al., 2012, Chavez and Correa, 2015).

The effect of dilation is investigated with the simulation of a 20 m excavation (see Figure 10). Two models were executed, one with an angle of dilation 0° and the other with 6° . It is observed that including the dilation has a considerable effect on the amount of horizontal displacement, especially in the HSS model.

Figure 11 shows the profile of settlements calculated on the ground surface. In this case, the MC model, regardless of the value of the dilation, predicts an uplift of the ground, which is improbably. On the contrary, the HSS model in all cases predicts settlement of the building.

Figures 12 show the horizontal displacement contours calculated with the MC and HSS models, respectively. These graphs clearly show how the use of an advanced model allows a more realistic prediction of the behavior of the wall system anchored with the ground.

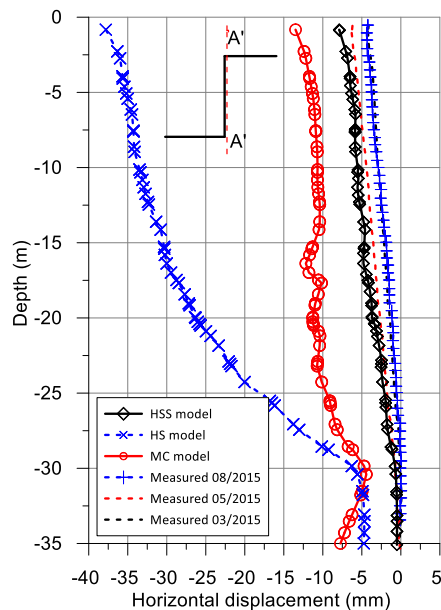


Figure 9. Horizontal displacement profile in the back of wall.

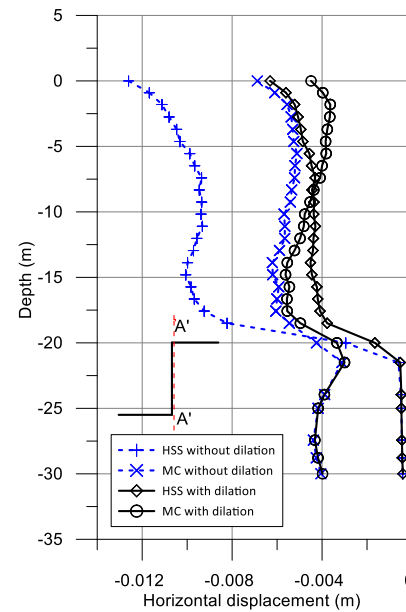


Figure 10. Horizontal displacement profile in a 20m deep excavation.

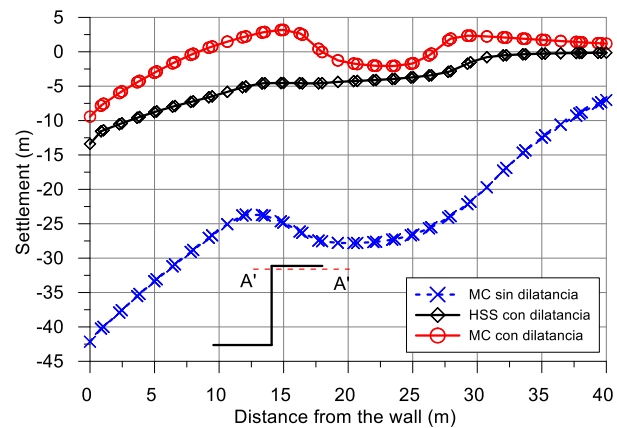


Figure 11. Computed surface settlement in the back of the wall.

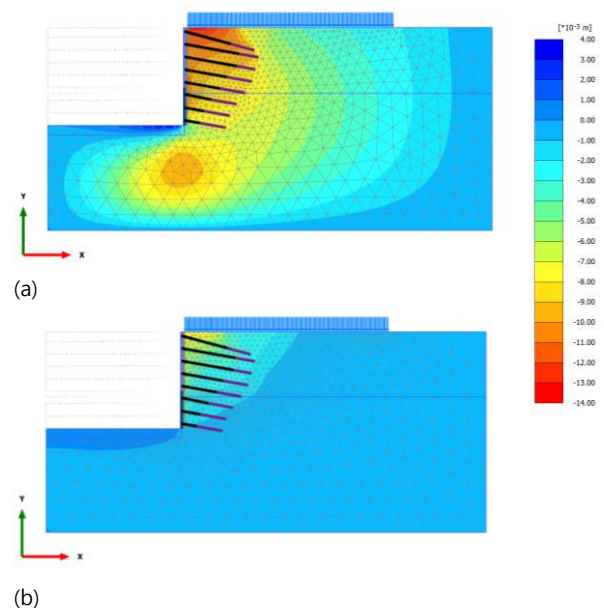


Figure 12. Contours of horizontal displacement determined: (a) MC model and (b) HSS model.

6 CONCLUSIONS

One of the difficulties in determining the geotechnical parameters of the Lima soil, which can be used in advanced models for calculating excavations, is the lack of large-scale equipment, essentially necessary are the oedometric and triaxial type tests. Even having this large-scale equipment for gravels, these will hardly be part of the engineering practice, due to the high cost and time of execution of these tests, therefore, it will be necessary to establish parameters through alternative well-supported tests. The paper established a range of parameters based on large-scale direct shear test, plate load tests and geophysical MASW tests.

The proposed set of parameters for the gravelly soil of Lima was tested in a finite element model of a deep excavation project. The calculated horizontal displacements show good results when compared to field measurements, in the case of using a hardening law into the elastoplastic model, and a hyperbolic relationship to simulate stiffness degradation. Also, dilating behavior must be well simulated in the model. Therefore, in the case of excavations in gravelly soils, where there is a potential risk of affecting neighboring buildings, it is necessary to use an advanced, such as the HSS model.

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