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A fully coupled absorbing boundary to simulate wave propagation through saturated partially drained porous half space using the Scaled Boundary Finite Element approach

Une frontière absorbante entièrement couplée pour simuler la propagation des ondes à travers un demi-espace poreux saturé partiellement drainé à l'aide de l'approche par éléments finis de limite à l'échelle

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ABSTRACT: Natural hazards associated with earthquakes occurring in saturated soils signify the analysis of wave propagation through soil layers. Defining a reliable absorbing boundary to simulate radiation damping effects is one of the most challenging issues to be addressed in problems of soil-structure interaction. Thus far, most of such studies are performed considering the soil as a solid medium under either a fully drained or a fully undrained condition. A partially drained analysis, where soil is modeled as a fully coupled porous medium with an absorbing boundary essentially providing a realistic pore water flow between bounded and unbounded media is yet to be developed. In this study, a fully coupled absorbing boundary is developed to simulate wave propagation through saturated porous half space by extending the original formulation of the Scaled Boundary Finite Element Method (SBFEM). The coupled flow and deformation equations of poroelasticity are employed for the unbounded solid media. First, a semi-analytical solution is developed for the response of unbounded domain governed by the coupled equations. Then numerical solutions are obtained to validate the semi-analytical results, which are found to be in sufficiently good agreement. The proposed method is capable of modeling the partial drainage condition along interaction boundaries between near and far-field as well as of analyzing fully drained and undrained cases. Hence, it is quite useful for modeling soil -structure interaction problems in earthquake engineering.

KEYWORDS: Absorbing boundary, partially drained half space, saturated porous media, scaled boundary finite element method, wave propagation

RÉSUMÉ : Les risques naturels associés aux séismes se produisant dans les sols saturés signifient l'analyse de la propagation des ondes à travers les couches du sol. La définition d'une limite absorbante fiable afin de simuler les effets d'amortissement du rayonnement est l'un des problèmes les plus difficiles à résoudre dans des problèmes tels que l'interaction sol-structure. Jusqu'à présent, la plupart de ces études sont réalisées en considérant le sol comme un milieu solide dans des conditions soit entièrement drainées, soit entièrement non drainées. Une analyse partiellement drainée, où le sol est modélisé comme un milieu poreux entièrement couplé avec une limite absorbante fournissant essentiellement un écoulement d'eau interstitielle réaliste entre les milieux limités et non liés, doit encore être développée. Dans cette étude, une frontière absorbante entièrement couplée est développée pour simuler la propagation des ondes à travers un demi-espace poreux saturé en étendant la formulation originale de la méthode des éléments finis à l'échelle (SBFEM). Les équations couplées d'écoulement et de déformation de poroélasticité sont utilisées pour les milieux solides non bornés. Premièrement, une solution semi-analytique est développée pour la réponse du domaine non borné régie par les équations couplées. Ensuite, des solutions numériques sont obtenues pour valider les résultats semi-analytiques, qui s'avèrent en accord suffisant. La méthode proposée s'avère capable de modéliser le drainage partiel le long des limites d'interaction entre les champs proches et lointains ainsi que d'analyser les conditions entièrement drainées et non drainées. Par conséquent, il est très utile pour modéliser les problèmes d'interaction sol-structure en génie parasismique.

MOTS CLÉS: Limite absorbante, demi-espace partiellement drainé, milieux poreux saturés, méthode des éléments finis aux limites échelonnées, propagation des ondes

1 INTRODUCTION

Wave propagation in porous media has been studied by many researchers in various problems including poroelasticity and liquefaction, where a Newtonian pore fluid interacts with a deformable solid such as soils. A variety of numerical and analytical approaches are utilized to investigate the behavior of these materials. The dynamic solid-fluid coupling is first formulated by Biot (1962, 1955, 1941). Later, the formulations are idealized based on inertial and drainage conditions (Ulker and Rahman 2009, Zienkiewicz et al. 1980) yielding three possible formulations for dynamic poroelasticity, i.e., fully dynamic (FD), partly dynamic (PD), and quasi-static (QS), where inertial terms associated with solid and fluid phases are considered in the FD form but neglected partly in the PD formulation and entirely in the QS solution (Ulker and Rahman 2009, Ulker et al. 2009). A plethora of investigations in this field have been performed using various numerical methods. Conventional finite element method (FEM) is widely used to solve poroelasticity problems, some of which are also performed by the second author (Ulker et al. 2009; Ulker et al. 2010; Ulker et al. 2012; Ulker 2012; Ulker 2014). Finite difference method (FDM) is another powerful tool to analyze engineering problems along with the control volume method (CVM) to evaluate wave propagation phenomena in a three dimensional (3-D) spherical half space (Zhang et al. 2014). Other numerical methods, meshless methods (Navas et al. 2016), boundary element method (BEM) (Igumnov et al. 2019), finite volume method (Nordbotten 2016) are also utilized to study such problems. Given the wave propagation analyses being performed in half space, satisfying the radiation conditions are of great importance. Moreover, in most studies, only the domain of the problem is considered, while the boundaries are considered either fully drained or fully undrained signifying a rather unrealistic boundary condition. In reality, essentially a partly drained boundary condition exists. In order to satisfy the radiation condition, several techniques are used as artificial absorbing boundary (Xu et al. 2017), BEM (Schanz 2001), and the scaled boundary finite element method (SBFEM) (Chen et al. 2015). Artificial absorbing boundaries are efficient methods to replicate the semi-infinite media. The BEM requires a fundamental solution, which must satisfy the governing equations in the domain of the problem (Wolf 2003). However, calculating the fundamental solution for different problems can be a very time consuming procedure. Wolf and Song (1996) present SBFEM for the first time for wave propagation problems in unbounded domains. SBFEM is capable of modeling radiation damping condition at the infinite boundary. There are some advantages that SBFEM possesses over the conventional FEM, FDM and BEM. Unlike BEM, no fundamental solution is required and unlike the FEM and FDM, only the boundaries of the problem are discretized in SBFEM. Then the convergence is satisfied through refining the mesh along the boundaries. Material anisotropy can also be applied without additional computational effort (Bazyar and Song 2008).

With these capabilities, the SBFEM is now generally considered to be an extended analysis approach applied to various problems in geo-engineering. While, SBFEM formulations for fully drained unbounded media are developed by several researchers, to the best of our knowledge, it is not the case for saturated unbounded media and such SBFEM formulation will be an original contribution. As far as the frequency-domain approach, which is the main objective of this research, dynamic stiffness of dry unbounded media can be developed using a fourth order Rung-Kutta method (Wolf and Song 1996), through a Pade approximation (Bazyar and Song, 2010), or by using the continued fractions method (Bazyar and Song 2008). In the case of two-phase infinite domains, however, Biot's coupled equations should be solved.

In this study, we consider a soil media, which is horizontally extended towards infinity and vertically restricted to bedrock, where the coupled flow-deformation equations are solved for a 1-D case. Moreover, formulation of the SBFEM for the coupled dynamic matrix of saturated unbounded media is presented and the accuracy of the proposed approach is evaluated by solving a number of benchmark examples.

2 THE PROPOSED FORMULATION

As mentioned previously, it is possible to have various idealizations (formulations) for the coupled flow and deformation problems depending on the motion of the pore fluid and the solid skeleton as well as the permeability of the porous medium. In this paper, the PD formulation is considered, therefore the below system is written in terms of its momentum and mass balance as,

$$L^T D L u + L^T p - \rho \ddot{u} = 0 \qquad (1)$$

$$L_p{}^T D_p L_p p + L_p{}^T \dot{u} - \frac{1}{Q}\dot{p} = 0 \qquad (2)$$

where, L and D are the differential operator and constitutive matrix for displacement, u and L_p and D_p are the differential operator and constitutive matrix for pore water pressure, p, respectively, and ρ is the mass density of solid particles. In addition, the parameter Q is defined as,

$$Q = K_f / n \qquad (3)$$

where, K_f is the compressibility modulus of pore water and n is the porosity of the soil. In this study, an example for the considered soil media is shown in Fig. 1a. In order to adopt the SBFEM formulation for such a problem, a scaling center located at infinity is assumed. The scaled boundary mesh used for discretizing the interface between near and the far-field is illustrated in Fig. 1b.

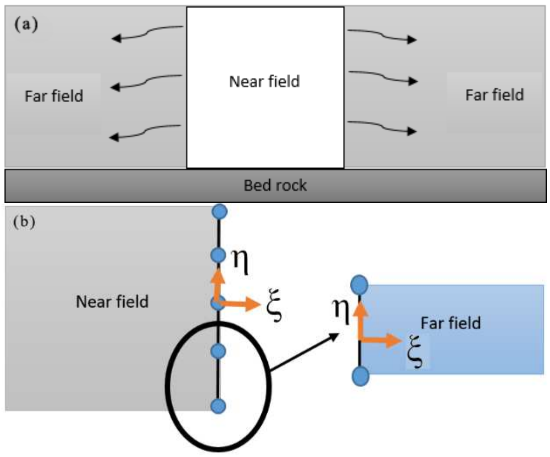

Figure 1. (a) The semi-infinite porous media and (b) Near and far field interface

The geometry in the scaled boundary coordinate system can be defined using Eq. 4 and 5.

$$x(\xi) = x_b + \xi \qquad (4)$$

$$y(\eta) = N(\eta) y_b \qquad (5)$$

Here, x_b and y_b are the coordinates on the boundary in the Cartesian system, and ξ and η are horizontal and vertical coordinates of the scaled boundary. $N(\eta)$ is the vector of shape functions. The relationship between spatial derivatives concerning Cartesian and scaled boundary coordinates can be stated with the Jacobian matrix. Similar to the original SBFEM formulation, the Jacobian can be calculated as,

$$J(\eta) = \begin{vmatrix} x(\xi)_{,\xi} & y(\eta)_{,\xi} \\ x(\xi)_{,\eta} & y(\eta)_{,\eta} \end{vmatrix} \qquad (6)$$

$$J(\eta) = \begin{bmatrix} 1 & 0 \\ 0 & y(\eta)_{,\eta} \end{bmatrix} \tag{7}$$

Using the definition of the Jacobian matrix, and considering Eq. 8 and 9, differential operators can be defined using Eq. 10 and 11.

$$b_1 = 1 \tag{8}$$

$$b_2 = \frac{1}{|J|} \tag{9}$$

$$\frac{\partial}{\partial x} = b_1 \frac{\partial}{\partial \xi} \tag{10}$$

$$\frac{\partial}{\partial y} = b_2 \frac{\partial}{\partial \eta} \tag{11}$$

For 1D wave propagation in the x-direction, and substituting Eq. 10 and 11 in Eq. 1 and 2 and by considering L_p=L, governing differential equations can be transferred to scaled boundary coordinate system. Resulting equations are defined below,

$$\left(b_1 \frac{\partial}{\partial \xi}\right)^T D(b_1 \frac{\partial}{\partial \xi})u + (b_1 \frac{\partial}{\partial \xi})^T p - \rho \ddot{u} = 0 \tag{12}$$

$$\left(b_1 \frac{\partial}{\partial \xi}\right)^T D_p \left(b_1 \frac{\partial}{\partial \xi}\right) p + \left(b_1 \frac{\partial}{\partial \xi}\right)^T \dot{u} - \frac{1}{Q}\dot{p} = 0 \tag{13}$$

The responses of displacement and pore water pressure can be approximated by interpolating nodal values using the equations,

$$u = u(\xi, \eta) = N_u(\eta) u(\xi) \tag{14}$$

$$p = p(\xi, \eta) = N_p(\eta) p(\xi) \tag{15}$$

To avoid numerical issues, the number of nodes selected per element for determining displacements can be more than those for pore water pressure, consequently, $N_p \neq N_u$. By substituting Eq. 14 and 15 into Eq. 12 and 13, respectively, and simplifying, governing differential equations can be stated as,

$$\left(b_1 \frac{\partial}{\partial \xi}\right)^T D\left(B_1 \mathrm{u}_{,\xi}\right) + \left(1 \frac{\partial}{\partial \xi}\right)^T N_P p + w^2 \rho N_u u = 0 \tag{16}$$

$$\left(b_1 \frac{\partial}{\partial \xi}\right)^T D_p \left(B_{p_1} \mathrm{p}_{,\xi}\right) + iwN_u \mathrm{u}_{,\xi} - \frac{1}{Q} iwN_p p = 0 \tag{17}$$

In these equations, ω is the angular frequency and B_1 and B_{p1} are defined as follows,

$$B_1 = b_1 N_u(\eta) \tag{18}$$

$$B_{p1} = b_1 N_p(\eta) \tag{19}$$

Using the Galerkin method the weak forms of Eq. 16 and 17 are obtained as,

$$E^0 \mathrm{u}_{,\xi\xi} + G^0 p_{,\xi} + w^2 M^0 u = 0 \tag{20}$$

$$H^0 \mathrm{p}_{,\xi\xi} + iw(G^0)^T u_{,\xi} - iwM^1 p = 0 \tag{21}$$

which are coupled scaled boundary equations for displacement and pore pressure used in 1D wave propagation problems. Coefficient matrices, which are used in these equations can be determined as follows,

$$E^0 = \int_{-1}^{+1} B^{1T} D B^1 |J|\, d\eta \tag{22}$$

$$G^0 = \int_{-1}^{+1} N^T{}_u N_p |J|\, d\eta \tag{23}$$

$$M^0 = \int_{-1}^{+1} N^T{}_u \rho N_u |J|\, d\eta \tag{24}$$

$$M^1 = \int_{-1}^{+1} N^T{}_p \frac{1}{Q} N_p |J|\, d\eta \tag{25}$$

$$H^0 = \int_{-1}^{+1} B_p{}^{1T} D_p B_p{}^1 |J|\, d\eta \tag{26}$$

Dynamic stiffness for displacement and pore pressure can be divided into four parts: pure displacement or dry soil, S_{uu}, pure pore pressure, S_{pp}, soil-pore water interaction, S_{up}, and pore water-soil interaction, S_{pu}. The coupled stiffness relationship for a saturated unbounded media in terms of pure displacement and pure pore pressure can be assumed as,

$$\begin{bmatrix} S_{uu} & 0 \\ 0 & S_{pp} \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} R_u \\ R_p \end{Bmatrix} \tag{27}$$

which can be converted to below equations by matrix multiplication as,

$$S_{uu} u = R_u \tag{28}$$

$$S_{pp} p = R_p \tag{29}$$

Utilizing Eq. 28 and 29, and considering Eq. 30 and 31 for internal nodal forces and internal nodal flow (Wolf 2003), displacement-pore pressure relationships can be converted into dynamic stiffness-degree of freedom (DOF) relations as Eq. 32 and 33.

$$R_u = -(E^0 u_{,\xi}) \tag{30}$$

$$R_p = -(H^0 p_{,\xi}) \tag{31}$$

$$S_{uu} u = -E^0 u_{,\xi} \tag{32}$$

$$S_{pp} p = -H^0 p_{,\xi} \tag{33}$$

Considering Eq. 32 and 33, dynamic stiffness of saturated unbounded media for the coupling term can be easily extracted by eliminating u and p from both sides of these equations. Note that, G^0 is the interaction coefficient between displacement and pore pressure. The resulting terms of dynamic stiffness for interaction between deformation and flow are expressed as,

$$S_{pu} = -iw(E^0)^{-1}S_{uu}(G^0)^T \tag{34}$$

$$S_{up} = -(H^0)^{-1}G^0S_{pp} \tag{35}$$

By substituting the derivative of Eq. 32 with respect to ξ into Eq. 20 and eliminating the second term of Eq. 20, Eq. 36 can be determined,

$$-S_{uu}u_{,\xi} + w^2M^0u = 0 \tag{36}$$

Using Eq. 32 and substituting resulting equation for $u_{,\xi}$ in Eq. 36, Eq. 37 is obtained,

$$S_{uu}(E^0)^{-1}S_{uu}u + w^2M^0u = 0 \tag{37}$$

Then by eliminating nodal displacement vector from all terms and using the same change of variables procedure for w proposed by Wolf (2003) for diffusion formulation of SBFEM, the scaled boundary differential equation for pure elasto-dynamics in frequency domain can be extracted as,

$$S_{uu}(E^0)^{-1}S_{uu} + w^2M^0 = 0 \tag{38}$$

The same procedure can be performed for the S_{pp} part and a SBFE equation for pore water pressure in frequency domain can be achieved as,

$$S_{pp}(H^0)^{-1}S_{pp} - iwM^1 = 0 \tag{39}$$

This equation expresses the dynamic stiffness differential equation of saturated unbounded media for the *pp* term. For pure diffusion problems, Eq. 39 define the governing equation of the problem.

Asymptotic expansion of the dynamic-stiffness matrix in high frequency components for "*uu*" and "pp" terms of the saturated unbounded media can be done using the same procedure proposed by Wolf and Song (1996) for elastodynamics and diffusion by setting E^1 and E^2 equal to zero. Subsequently, the SBFEM equation in dynamic stiffness (Eq. 38 and 39) is integrated for decreasing ω using the asymptotic expansion of the dynamic-stiffness matrix as the starting value. This establishes the dynamic-stiffness matrix over the entire frequency range. In this paper the Pade approximation is used for integrating Eq. 38 and 39.

3 NUMERICAL VERIFICATION

An elastic semi-infinite media, as seen in Fig. 2, is used to measure the efficiency of the proposed methodology. A Ricker wavelet type horizontal dynamic load function is applied to the domain of interest. The elasticity modulus of the media is E=2×10^5 kN/m^2, with constant Poisson's ratio of ν=0.3, and constant density of ρ=2×10^3 kg/m^3 in this example. In Fig. 2, Green nodes define the unbounded domain solved with SBFE method, and black and blue nodes have pore pressure and displacement DOFs, respectively.

Dynamic load is applied at point A (Fig. 2) with the frequency content of the load function plotted in Fig. 3. For the purpose of verification, an extended mesh is created, which discretizes a domain with a length of 3500m with a depth of 30m. Extended mesh method (EMM) is one of the common methods that is used to solve and verify wave propagation problems (Bazyar 2007). In this method, a large domain is modelled, which ensures that there is enough time between the wave hitting the model boundary and its return to the observation points. Then the obtained response can be used to evaluate the accuracy of other methods. Fig. 4 shows the extended mesh used to solve this problem. The FE-SBFEM coupled model employs 25 nodes, while the classical extended mesh approach contains 245 nodes.

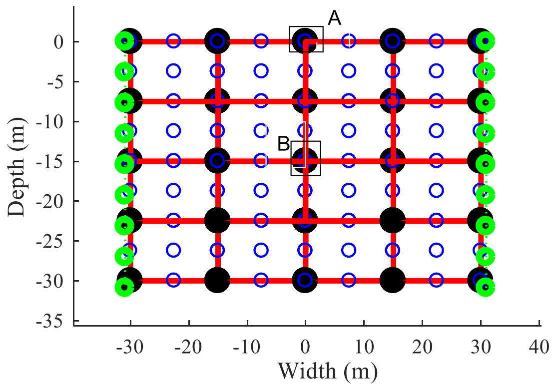

Figure 2. Considered semi-infinite media and the used FEM-SBFEM mesh

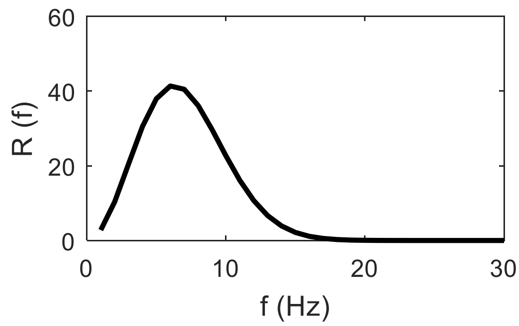

Figure 3. Applied dynamic load function in frequency domain

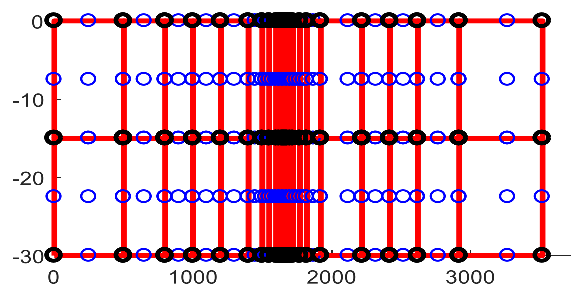

Figure 4. The semi-infinite media with an extended FE mesh (numbers are in meters)

The SBFEM and the EMM are used to calculate the displacement time history of point A and the excess pore water pressure of point B. The responses are shown in Fig. 5.

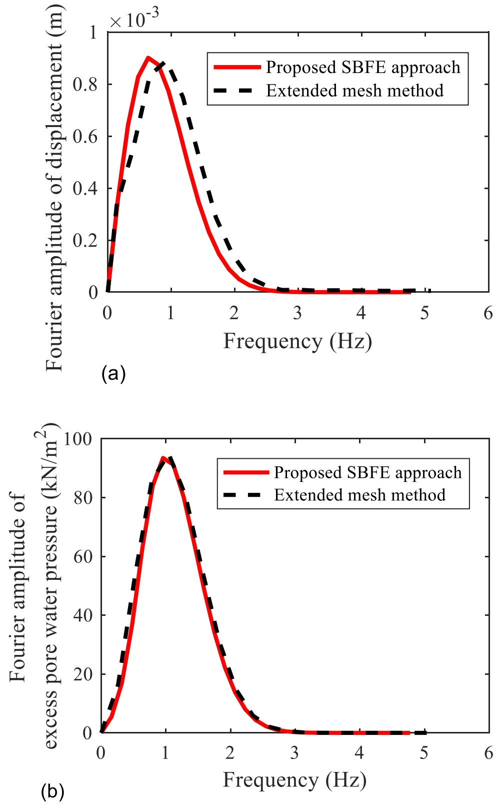

Figure 5. (a) Amplitude of horizontal displacement at point A (b) Amplitude of excess pore water pressure at point B.

The applied methods converge to the same response, as seen in Figure 5 indicating that the modified SBFEM can effectively model this problem. The excess pore pressure near the boundaries using both methods also demonstrate similar response as shown in Fig. 5a. This is due to the fact that propagating wave continues to travel horizontally through the boundaries without disturbance. The pore pressure and displacements are attenuated towards infinity.

4 CONCLUSION

In this paper, the poroelasticity formulation of the scaled boundary finite element method is developed for the problem of 1D wave propagation through porous media. A semi-infinite saturated porous media is considered with a physical bottom boundary as bedrock. Pore water flow between bounded and unbounded media is enabled by using SBFEM formulation, thus, a partially drained boundary condition is provided. Moreover, by using this approach, radiation damping condition at infinity is satisfied. Also with this method, displacements and excess pore pressures vanish at infinite domain. This is particularly important for soil-structure interaction problems in earthquake engineering. The proposed formulation is compared with the results of the extended mesh method showing a good agreement, which provides confidence for further use of the proposed absorbing boundary condition.

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