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Analysis of a level site liquefaction using a centrifuge model subjected to biaxial shaking

Analyse de la liquéfaction d’un site plat à l'aide d'un modèle centrifuge soumis à une excitation biaxiale

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ABSTRACT: This study investigated the dynamic response and liquefaction of saturated-sand level deposits subjected to biaxial shaking using a number of centrifuge tests. Loose and dense sandy soil deposits were built in a 2D laminar container and subjected to a series of biaxial base excitations that approximate in a fairly realistic fashion the conditions of a site subjected to earthquake shaking. A dense array of accelerometers and pore pressure sensors were used to monitor the deposit response. The recorded accelerations and pore pressures were used along with non-parametric identification procedures to estimate the corresponding dynamic shear stress-strain histories. In turn, these histories were employed to assess the effects of non-proportional loading on soil contraction and dilation mechanisms.

RÉSUMÉ : Cette étude a examiné la réponse dynamique et la liquéfaction de dépôts de sable saturé soumis à une excitation biaxiale à l'aide d'un certain nombre d’essais en centrifuge. Des dépôts de sol sableux lâches et denses ont été construits dans un conteneur 2D laminaire et soumis à une série d'excitations biaxiale de base qui se rapprochent de manière assez réaliste des conditions d'un site soumis à un tremblement de terre. Un réseau dense d'accéléromètres et de capteurs de pression interstitielle a été utilisé pour surveiller la réponse des dépôts. Les accélérations et les pressions interstitielles enregistrées ont été utilisées avec des procédures d'identification non paramétriques pour estimer les historiques correspondantes de contrainte-déformation dynamiques de cisaillement. Ensuite, ces histoires ont été utilisées pour évaluer les effets de charge non proportionnelle sur les mécanismes de contraction et de dilatation du sol.

KEYWORDS: Liquefaction; centrifuge; numerical simulation; physical modeling; biaxial shaking.

1 INTRODUCTION.

The ability to understand and predict soil response during earthquakes is an important aspect in the design and management of soil-structure systems. Generally, past research about the effect of shaking on soil dynamic properties and response was done using uniaxial excitations. Numerous studies were conducted over the last four decades. The simulation of base excitation progressed from single frequency sine excitation, to more complex motions with variable amplitude and multi-frequency content. The vast majority of research was conducted using uniaxial excitations. Recently, the importance of using biaxial excitations has been acknowledged and a number of studies used multiaxial shaking.

Kammerer et al. (2002) undertook at an early effort. Multiaxial cyclic tests were performed on Monterey sand using a shear box and triaxial apparatus. Ng et al. (2003) conducted the first biaxial shaking centrifuge test at Hong Kong University to study the response of saturated embankments. Su and Li (2006, 2008) also used the Hong Kong centrifuge to investigate the effect of multiaxial shaking on soil-pile interaction, and the effects of biaxial shaking on Toyoura sand and compared the results to constitutive model predictions. Su (2012) tested piles under multiaxial lateral loading using the biaxial shaking table at Shenzhen University (China). In spite of these efforts, the effects of multiaxial non-proportional loading on soil response (e.g., influence on contraction rate and pore water pressure buildup) remain largely unexplored.

An important aspect in previous research is how a uniaxial shaking can be used to represent the consequences of biaxial shaking. The common practice is to increase the amplitude of the uniaxial shake by 10-15% to account for the perpendicular component of the shake omitted in the analysis. This common practice concept is based on the work done by Seed et al. (1978) using cyclic simple shear testing under undrained conditions. They found that shear stresses causing liquefaction under biaxial shaking is about 10-20% less than needed to cause liquefaction under uniaxial shaking, so they recommended to increase the uniaxial shake by 10% due to the low probability of having two shaking components with the same maximum amplitude.

Soil contraction is an important factor that affects the rate and total excess pore water pressure buildup, especially under multiaxial conditions. This factor was used by a number of researchers to directly model the mechanism of pore pressure buildup under dynamic loading, (e.g., Martin et al. 1975 and Seed et al. 1978). Ishihara and Towhata (1980) and Yamazaki et al. (1985) used a stress path integration and two-phase continuum formulation to compute the pore pressure buildup in sands under uniaxial and multiaxial shakings. Ghaboussi and Dikmen (1981) conducted simulations that showed that shaking using two horizontal components increases the liquefaction potential compared to a uniaxial case, even if the amplitude of the uniaxial input is equal to the resultant of the two horizontal components.

The work presented in this paper examines the validity of the assumptions used to simulate soil response during multiaxial excitations, such as during earthquake shaking. Physical models are used to assess the behavior and response of saturated soil deposits subjected to biaxial base excitations. Also, two uniaxial shaking tests were used to evaluate the validity of using equivalent uniaxial base shaking soil models to simulate biaxial shaking conditions. The following sections give a brief description of the analyzed level deposit models, the associated input motions, and the obtained experimental results. A simple identification procedure is used to evaluate the associated stress-strain histories, effective stress path and contraction characteristics of the (centrifuge) models from the recorded accelerations and pore pressures.

2 TESTING PROCEDURE AND INSTRUMENTATION

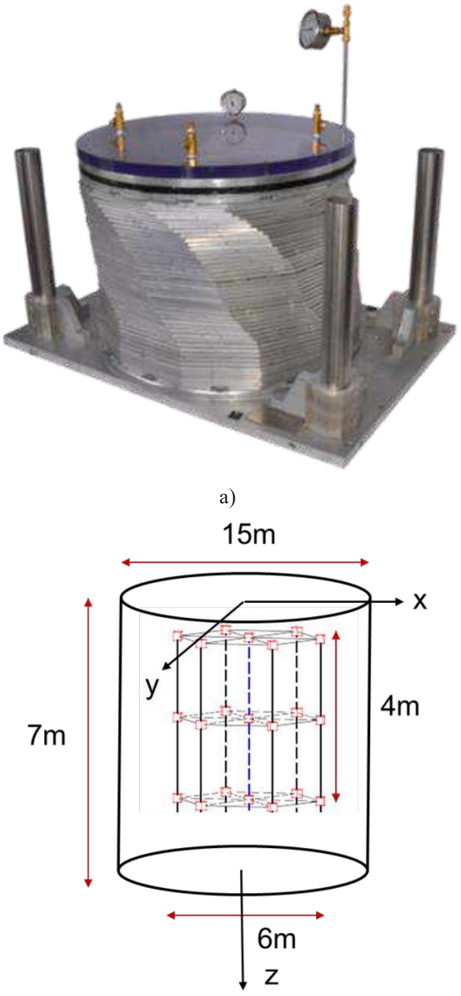
A number of centrifuge model tests were conducted at the Rensselaer geotechnical NEES (Network for Earthquake Engineering Simulation) center to assess the effects of biaxial base excitation on the response and liquefaction of level deposits of saturated granular soils. A 2D laminar container (Figure 1a) was used to approach free field site conditions. The container consists of twelve-sided rings (made of lightweight aluminum alloy). Each ring is separated from the ones above and below by roller bearings, specifically designed to permit translation in the two horizontal directions with minimal frictional resistance (Sasanakul et al. 2014, El-Shafee 2016, El-Shafee et al. 2018 and 2019). Thus, the soil model within the container can translate in the x and y horizontal directions and rotate in the associated plane. A biaxial shaker (Sasanakul et al. 2014) capable of producing realistic in-flight synthetic and earthquake excitations (in the X and Y directions) was used to subject the models to uniaxial and biaxial base excitations. The employed excitations did not include any torsional component and consequently the container rotational degree-of-freedom had no effect on the study presented herein. Two soil models with a 280 mm height were tested under a 25g centrifugal field, corresponding to a 7 m high prototype. Hereafter, all quantities are presented exclusively in prototype units. The models consisted of Nevada 120 sand deposits having a relative density of 45%. Nevada 120 is a clean uniform fine sand with a specific gravity of 2.65, grain size distribution characterized by D10=0.09 mm, D50=0.3 mm, and a uniformity coefficient of 2.07. The minimum and maximum void ratios are emin=0.55, and emax=0.751. The model was built using dry pluviation technique and then saturated under vacuum with a viscous fluid (25 times the viscosity of water to ensure a proper scaling of soil permeability).

The tested models were equipped with 53 accelerometers and 6 pore pressure sensors. The accelerometers were installed at three levels within the sand model to monitor the x and y responses at (1m, 3m and 5m depths) and arranged in a 3D configuration, as shown in Figure 1b. Specifically each level had seven locations with sensors measuring accelerations in the horizontal X and Y directions. The accelerometer configuration was intended to capture the biaxial soil response in the horizontal directions as well as any evidence of variation of this response with location within horizontal planes of the deposits. Each sensing level included also 2 pore water pressure transducers to measure the soil excess pore pressure buildup during shaking. Two vertical accelerometers and Linear Variable Differential Transformer (LVDT) sensors were used to monitor the corresponding accelerations and settlements induced by the input base excitations, more about testing details can be found in El-Shafee et al. (2016).

Different tests were conducted with models subjected to biaxial or uniaxial shakings. The recorded accelerations and pore water pressure were studied to evaluate the difference between the aforementioned testing scenarios

3 INPUT MOTIONS

The input motions consisted of synthetic accelerations with 1, 2, 3 Hz dominant frequencies and varying amplitude. The biaxial input motions were obtained by varying the phase angle θ of the Y and X accelerations ($\theta = \arctan(a_y/a_x)$), to give phase angle time history equivalent to real earthquake records. The XY acceleration path for the base shake is shown in Figure 2 where the blue represents the input shake and the red represents the recorded unfiltered shake, and the time histories for X and Y components are shown in Figure 3. The corresponding phase angle time history for this shake is provided in Figure 4, and shows that although the used shakes are synthetic motions, the phase angle has some randomness, and similarity to typical earthquake phase angle time history.


a)

b)

Figure 1. a) RPI 2D laminar container, b) overall sensor schematic configuration

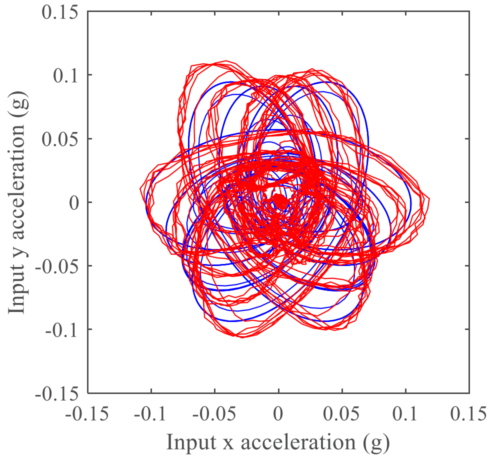


Figure 2 Acceleration (XY) path of the input motion.

The uniaxial shakes used in these tests have the same profile of the X component in the biaxial shakes (Figure 3a). Two uniaxial tests were used to examine the assumption widely used by practitioners (i.e., increasing the uniaxial input shake amplitude by 10% to account for biaxial input shaking). For the second uniaxial test, the arias intensity defined by Arias intensity (Arias 1970) is used to ensure that the biaxial and uniaxial shakes used have the same energy content, which resulted in about 40% increase in the amplitude of the uniaxial input shake. The two uniaxial shaking profiles were developed to be compared with the biaxial shaking profiles. The Arias intensity defined by Arias (1970) is the total energy per unit weight stored by a set of simple oscillators evenly spaced in frequency. The Arias intensity for uniaxial base motion in the X-direction (I_{1Dx}) can be written as:

$$I_{1Dx} = \frac{\pi}{2g}\int_0^{T_d} a_x(t)^2\, dt \qquad (1)$$

Where: a_x *(t)* is the acceleration time history in the X-direction in (m/s^2), g is the acceleration due to gravity in (m/s^2), T_d is the total duration of motion in seconds.

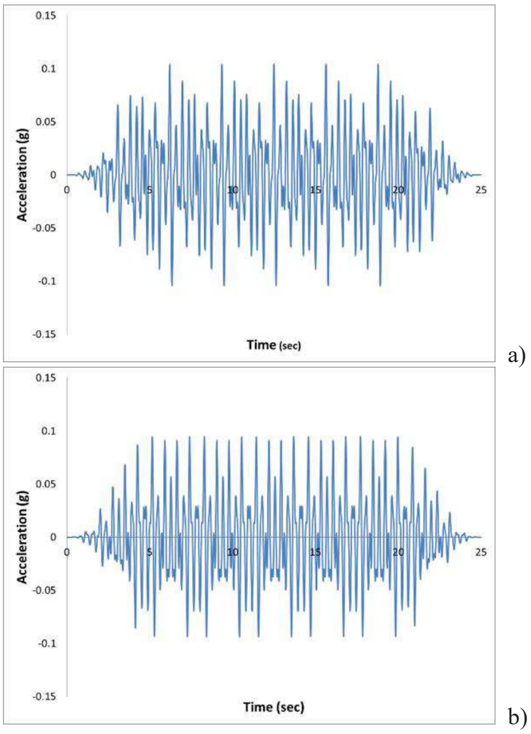


Figure 3. Shake time history (a) biaxial X acceleration, (b) biaxial Y acceleration

The Arias intensity of the biaxial shake in x and y directions and can be written as:

$$I_{xy} = \frac{\pi}{2g}\int_0^{T_d}(a_x(t)^2 + a_y(t)^2)\, dt \qquad (2)$$

Where: I_{xy} is Arias intensity associated with a_x *(t)*, the acceleration time history in the X-direction in (m/s^2), and a_y *(t)*, the acceleration time history in the Y-direction in (m/s^2).

By Equating Equations 1 and 2, the uniaxial equivalent energy shake was found to be about 40% larger in amplitude. The uniaxial shake used was obtained by amplifying the X component acceleration of the biaxial shake by about 40%. Arias intensity was computed for the biaxial shake and both uniaxial shakes and compared in Figure 5. It is clear that the uniaxial 10% shake has noticeably less energy than both the biaxial and equivalent uniaxial shakes which are almost identical.

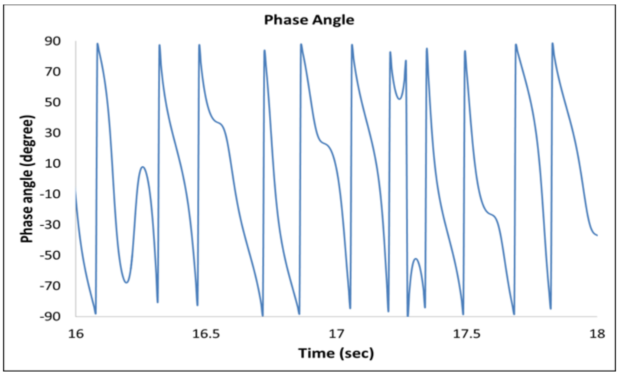


Figure 4. Phase angle of the XY acceleration for 16 s to 18 s time window

For each test, the model is spun up to 25g and then selected shaking events are applied to the base of the model using the RPI 2D shaker. The equivalent energy of uniaxial shakes with 40% increase in X component amplitude were applied in the same exact sequence. For the sake of simplicity the equivalent uniaxial shakes will be captioned with the biaxial shake amplitude (For example input shake of magnitude 0.1g in biaxial test is equivalent to a magnitude of 0.14g in uniaxial test but the later will also be called “0.1g”). For example, biaxial test will be called “Test 2D”, while the equivalent uniaxial test for the same amplitude will be called “Test 1D40%”, and finally the 10% amplified uniaxial test will be called “Test 1D10%”.

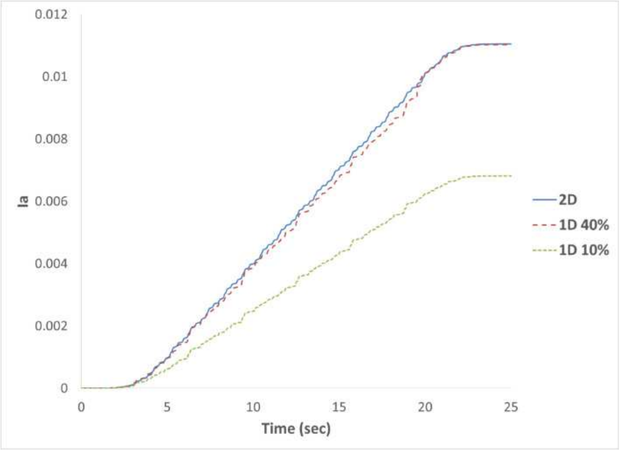


Figure 5. Input shake Arias intensity comparison

4 MEASURED SOIL RESPONSE

Sample accelerations measured inside the soil under the input base shake with amplitude 0.1g are shown in Figure 6. The results reveal that in both Test 2D and Test 1D40% the amplitude of accelerations decreased for the top 3 meters of the model. This decrease in recorded acceleration amplitude is indicative of soil softening apportioning full liquefaction up to this depth. A closer look at the accelerations time history shows that in biaxial shaking the decay was faster than both uniaxial shakes. This indicates faster liquefaction for the soil under biaxial shaking. Another observation is that uniaxial tests acceleration records shows large spikes for the top 3 meters of the model. This reflects higher dilative behavior than that observed in biaxial shaking, which can be attributed to concentration of shear strain in one direction. On the other hand, faster liquefaction in Test 2D is attributed to higher level of sand particle destabilization because of shearing is occurring simultaneously in two perpendicular directions. Results of the Test 1D10% is compared with the two tests with same input energy. For Test 1D10% the amplitude at shallow depth decreased much later than the other tests at 1 meter depth. It did not decrease at all at the mid-level, which is opposite to what happened in both Test 2D and Test 1D40%, which indicates slower and shallower liquefaction of soil.

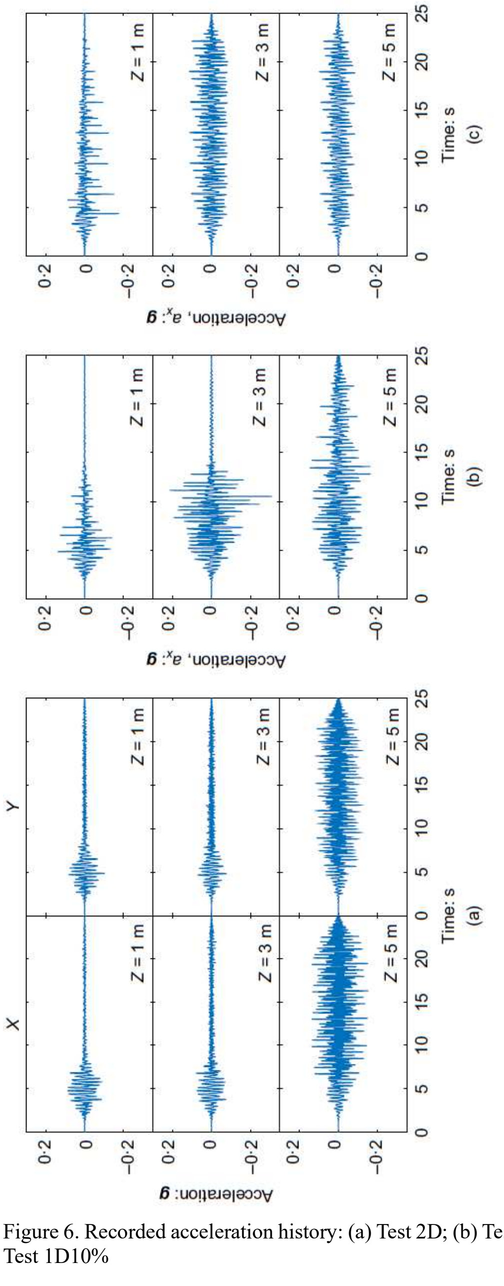


Figure 6. Recorded acceleration history: (a) Test 2D; (b) Test 1D40%; (c) Test 1D10%

The pore pressure response of the sand deposits to the 2D and 1D shakings showed similarities and discrepancies. For excess pore water pressure, the comparison has two aspects. First, comparison between Test 2D and Test 1D40%, then comparison of the three experiments all together. Figure 7 shows the pore pressure time history recorded during shake A with amplitude 0.1g. With the exception of marginal differences in how fast the excess pore pressure rises and slightly larger drops in Test 1D40% records, the two time histories are very comparable. This is rater expected given that the input energy for both tests is exactly the same. The small differences are within the experimental error range, yet the slightly larger drops observed in Test 1D40% is consistent with the larger spikes in the acceleration records. This consistency in different soil response measurements between the two experiments increases the confidence in each individual soil response measurement.

The comparison of pore pressures recorded in the three tests is shown in Figure 7. The results agree with the acceleration records observations. Test 1D10% reached full liquefaction at depth of 1 meter with similar trend to Test 1D40%. The buildup excess pore pressure was smaller and slower than Test 2D and Test 1D40% for 3 meter, in which the soil partially liquefied. Soil didn't liquefy at 5 meters for Test 1D10%, with much slower buildup rate than the other two tests. The drops in pore pressure records in Test 1D10% test are much smaller than the other two tests for all depths. Yet they are still noticeable at 1 meter depth, which agrees with the acceleration records.

5 STRESS AND STRAIN RESPONSE

The acceleration records (Figure 6) were used to assess the corresponding shear stress and strain vector time histories induced inside the soil. The evaluated stresses were at the mid-points locations between the three sensor levels. (i.e., at 2, 3 and 4 meters). These stresses and strains were obtained using the system identification technique that was developed by Zeghal et al. (1993) and (1995). This technique provides non-parametric estimates of shear stress-strain histories utilizing only accelerations records provided by vertical arrays of accelerometers.

For the tests discussed in this paper, the focus is on the comparison of the stress-strain curves for the two equivalent energy experiments (Test 2D and Test 1D40%). First each one of the two components of stresses and strains of Test 2D is compared independently to Test 1D40% curves. The magnitudes of the biaxial components of Test 2D are smaller than Test 1D40% which is clear in the case of strains. The reason is that in Test 1D40% all the input energy is concentrated in a single direction, while in the Test 2D it is distributed in two perpendicular direction which produces smaller stresses and strains in each component. It is also observed that for Test 2D, the stress strain curves in the X and Y directions have similar and independent behavior both in loops proportionality and in amplitude, as shown in Figure 8. On another hand, the biaxial base excitation leads to conditions of non-proportional loading where the principal directions are continuously changing (Bentachfine et al. 1996). Non-proportionality is also referred to as "non-coaxiality", "non-synchronous" or "non-coincidence" loading (Chang and Sture 2006). This conditions produces additional mechanisms of shear deformation, pore pressure buildup and settlement that are not observed under uniaxial loading. In engineering practice, these effects are handled by either using an "equivalent" uniaxial loading corresponding to the largest component multiplied by 1.1 (i.e., increased by10%) (Seed et al. 1978).

The effects of non-proportionality on the conducted centrifuge tests was assessed using the XY strain path, XY stress path, and the phase angle distribution for both strains and stresses. Under low level of biaxial excitations, the X and Y responses of the loose and dense deposits were found to be mostly uncoupled, as shown in Figure 9. This figure shows that the strain and stress paths in the X and Y planes are significantly similar, the main directions of stress and strain peaks are roughly the same. In other words, the associated stresses and strains are mostly coaxial and soil response in each of the x and y directions may be treated independently. In contrast, the large non-proportional loading of test 2D lead to coupled X and Y responses for both stresses and strains (Figure 10). The corresponding stresses and strains showed different pattern and the associated phase angle distribution confirmed that the stresses and strain orientation have a decreased level of correlation after liquefaction (Figure 9, see also Zeghal et al. 2018).

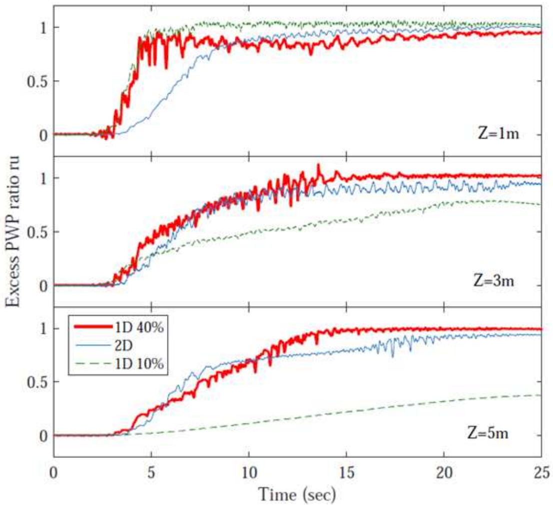


Figure 7. Pore water pressure–time history comparison for the three tests

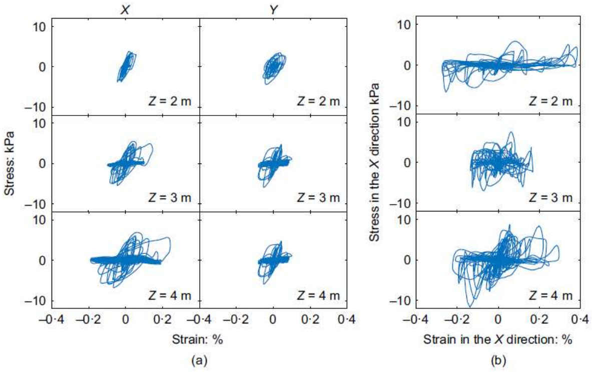


Figure 8. Stress–strain curves: (a) Test 2D; (b) Test 1D40%

Furthermore, the obtained stress and strain histories were used to compute the strain energy for the whole soil system for the three tests (Test 2D, Test 1D40% and Test 1D10%). This is done by integrating the sum of the products of stress and strain in X and Y directions over time. Then summing the energies computed at the three levels to compare the whole strain energy of the soil system, this can be written as:

$$E = \int_0^T (\gamma_x \tau_x + \gamma_y \tau_y) dt \qquad (3)$$

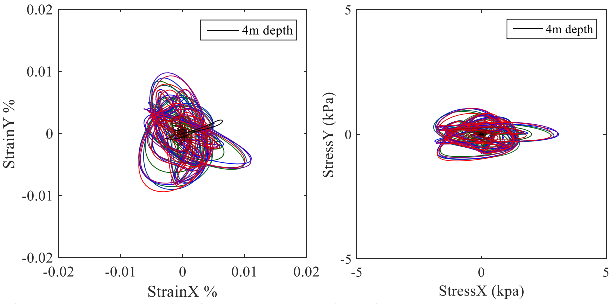


Figure 9. Low amplitude XY stress and strain

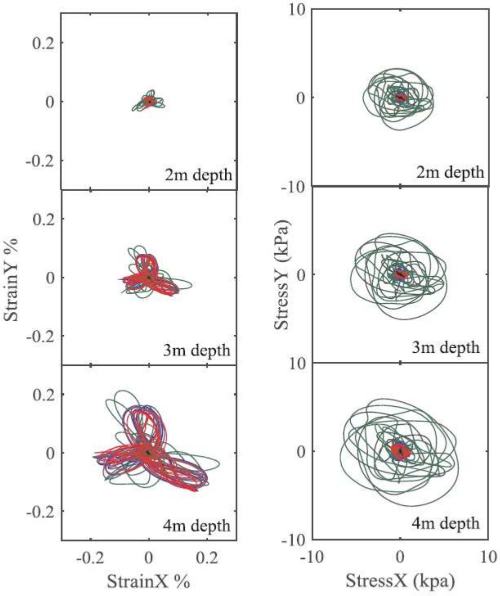


Figure 10. XY stress and strain loose deposit

Where E is strain energy, γ is shear strain, and τ is shear stress. The comparison of computed energies for the three tests is shown in Figure 11. It is clear that strain energy for Test 2D is larger than Test 1D40% and Test1D10% by about 15% and 28% respectively. Although Test 2D and Test 1D40% have the same Arias intensity, which represent the input energy to the soil, the strain energy computed for the soil system is different. This is strong evidence that biaxial shaking introduces more demand on the soil and produces different behavior in the soil than uniaxial shaking even if both shakes have exactly the same input energy, and the Arias intensity is not a good measure of this demand. Also, Test 1D10% has a noticeable difference in strain energy compared to the Test 2D and Test 1D40%. This finding proves that the current method of multiplying the uniaxial shake by 10% is underestimating the real response of the soil under biaxial shaking (El-Shafee et al. 2018).

6 CONCLUSIONS

This paper presented the outcome of a series of centrifuge tests conducted to assess the effects of bi-axial shaking on the response of level-ground deposits. The recorded accelerations were used to evaluate the horizontal shear stress and strain vectors at a number of depth locations of the deposits.

For the 2D and 1D equal Arias intensity tests (Test 2D and Test 1D40% respectively), the recorded soil response had differences in rate of acceleration amplitude attenuation, and presence of dilation spikes in pore pressure records. The slightly higher dilative behavior in Test 1D40% is mainly due to input energy concentration in one direction, versus distributing it on a plane in the case of Test 2D. For the input motion used in traditional practice method (which consists of multiplying the input acceleration by 1.1, Test 1D10%), the recorded soil response is noticeably smaller than the higher energy tests, especially at deeper soil layers. The observed response for this test is consistent with the fact that it has lower input energy, which leads to weaker impact on the soil (shallower liquefaction and less dilative behavior).

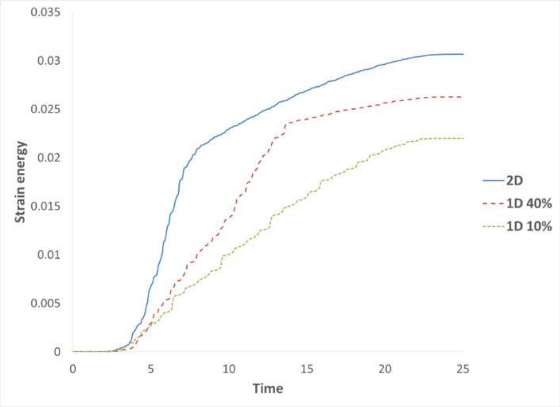


Figure 11. Strain energy comparison for the three tests

The evaluated stress and strain time histories and curves for the three analyzed tests showed that the two components of stresses and strains for biaxial shaking Test 2D are not independent and there is evidence of coupled behavior. Strain energy was used to compare the 1D and 2D tests. The calculated strain energies showed that Test 2D has higher strain energy compared to that of the two uniaxial tests even when the Arias intensities were comparable, and demonstrated the effects of a non-proportional biaxial base excitation. The findings show that modifications are needed to the conventional liquefaction analysis techniques that are based on load cycle counting. Also, the common practice method of increasing the uniaxial shake component by 10% does not represent the real soil response and is not capable of capturing the real soil behavior under multiaxial shaking.

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