Proceedings of the 20th International Conference on Soil Mechanics and Geotechnical Engineering– Rahman and Jaksa (Eds)

Binary-medium constitutive model for geological materials: a multi-scale approach

Modèle constitutif binaire-milieu pour les matières géologiques: une approche multi-échelle

Enlong Liu

State Key Laboratory of Hydraulics and Natural River Engineering, College of Water Resource and Hydropower, Sichuan University, Chengdu 610065, China

ABSTRACT: For granular materials with bonding effects among grains, such as structured soil, frozen soil, and rock material, they usually behave strain softening and initial contraction followed by dilatancy at relatively low stress level, and the non-uniform distribution of stress/strain within a representative elementary volume (REV) of these materials can be also observed, which can be modelled by Binary-medium constitutive model (BMCM) for geological materials combining homogenization theory and mesomechanical approach. For BMCM for geological materials, the REV can be idealized as a binary-medium material consisting of bonded elements and frictional elements, and upon loading the bonded elements may gradually break up and transform to be frictional elements of broken assemblies, and both of them bear the external loading simultaneously. By introducing structural parameters, the breakage ratio and local strain coefficient, the binary-medium constitutive model for geological materials can be formulated based on homogenization theory and mesomechanics-based method. Both the breakage ratio and local strain coefficient are internal variables and can be obtained by mesomechanics-based approach, through which the interactions of soil grains at micro-scale level can also be considered. Based on the Binary-medium model (BMM) concept, the elasto-plastic/elasto-visco-plastic constitutive models for geological materials soils including structured soil, frozen soil, rock material, are proposed and summarized here. Finally, the predicted results by the models for these geological materials having bonding actions are compared with tested results under compressional conditions, which demonstrate that the model can duplicate the main features of geological materials with bonding effects among grains well.

RÉSUMÉ : Pour les matériaux granulaires avec des effets de liaison entre les grains, tels que le sol structuré, le sol gelé et le matériau rocheux, ils se comportent généralement avec anti-écrouissage et avec une contraction initiale suivis d'une dilatance à un niveau de contrainte relativement faible. La distribution non uniforme des contraintes / déformations au sein d'un volume élémentaire représentatif (REV) de ces matériaux peut également être observée, qui peut être modélisée par le modèle constitutif binaire-milieu (BMCM) pour les matériaux géologiques combinant la théorie de l'homogénéisation et l'approche mésomécanique. En appliquant le modèle BMCM pour les matériaux géologiques, le REV peut être idéalisé comme un matériau binaire-milieu composé d'éléments des liaisons et d'éléments des frictions. Lors du chargement, les éléments des liaisons peuvent progressivement se briser et se transformer en éléments des frictions, et tous deux portent le chargement externe simultanément. En introduisant des paramètres structurels, à savoir le taux de rupture et le coefficient de déformation local, le modèle constitutif binaire-milieu pour les matériaux géologiques peut être formulé sur la base de la théorie de l'homogénéisation et de la méthode mésomécanique. Le taux de rupture et le coefficient de déformation local sont tous des variables internes et peuvent être obtenus par une approche mésomécanique, à travers laquelle les interactions des grains du sol au niveau micro-échelle peuvent également être considérées. Basés sur le concept du modèle binaire-milieu (BMM), les modèles constitutifs élasto-plastique/élasto-visco-plastique pour les matériaux géologiques, y compris les sols structurés, les sols gelés, les matériaux rocheux, sont proposés et résumés ici. Enfin, les résultats prédits par les modèles pour ces matériaux géologiques ayant des actions de liaison sont comparés aux résultats testés dans des conditions de compression, ce qui démontre que le modèle peut bien dupliquer les principales caractéristiques des matériaux géologiques avec des effets de liaison entre les grains.

KEYWORDS: Binary-medium constitutive model (BMM); multi scales; geological materials; strain softening

1 INTRODUCTION

Many constitutive models have been formulated to describe the stress-strain properties of soils, the widely used one of which is the Cam-clay model for normally consolidated or reconstituted clay (Schofield & Wroth 1968), behaving strain hardening and volumetric contraction. However, many soils in nature have cementation effects between soil particles, which usually behave strain softening, initially volumetric contraction followed by dilatancy. For example, structured/natural clays have bonding and big pore pores resulted from the deposit process; cemented sands have cementation between soil particles; rock has strong bonding between grain matrix; and frozen soils also have strong ice bonding. For these kinds of soils, the cementation will gradually break up upon loading, leading to strain softening and dilatancy.

For a soil element tested under triaxial compression conditions, the strain distribution within it is uniform, especially when shear band appears, companied by strain localization, and thus the stresses within the soil element are also distributed unhomogeneously. In order to consider the uniform distribution of stress/strain within a soil element, the multi-scale approach can be employed.

Currently, constitutive models for geological materials usually have many model parameters, in which some parameters have not clear physical meaning and cannot be directly determined by test methods. To reduce the model parameters, we can use the mesomechanics-based method, in which the informations on physical and geometrical parameters of soil particles can be upscaled, and thus the bridges between micro and macro scales can be established.

Even though some multi-scale constitutive models have been established for composite materials, few of them can be found for geological materials, one of which, Binary-medium constitutive model (BMCM) for geological materials, will be introduced here. In the paper, the concept of BMCM is introduced first, followed by the mathematical expression for the constitutive model; then the multi-scale approach for Binary-medium is formulated; and lastly some discussions are given.

2 THE GENERALIZED STESS-STRAIN RELATIONSHIPS FOR BINARY-MEDIUM MODEL

2.1 *The concept of Binary-medium model (BMM)*

For geological materials with cementation/bonding among soil particles, the bonds between soil particles may break up gradually, and thus the parts of the soil element with intact bonding effects and broken parts consisting of soil particles have different mechanical features, and both of them contribute to the bearing capacity of the soil element. Considering this deformational mechanism for geological materials with bonding/cementation effect, the Binary-medium model idealizes respectively the intact part of the soil element as the bonding element and the broken part of the soil element as the frictional element, as shown in Figure 1. For the breakage mechanism for geological materials, the readers can refer to the reference (Liu 2006; Shen 2002, 2006).

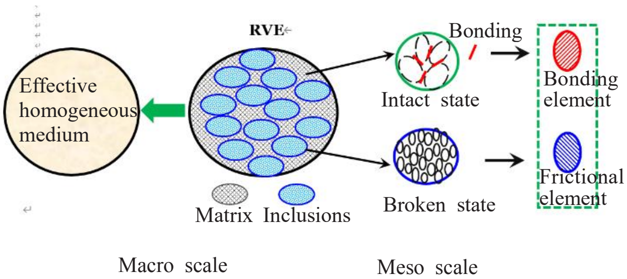


Figure 1. Schematic of Binary-medium model

Upon loading, the bonding elements within soil element will break up gradually and transform to be the frictional elements, when the breakage criterion is satisfied for the bonding element. For the soil element (RVE), initially the bonding elements mainly bear the external loading, and with the increasingly breaking process of bonding among soil particles, the frictional elements increasingly bear more external loads. And thus, the frictional elements/bonding elements can be assumed to be matrix or inclusions within the soil element, in which their interactions should be considered. From Figure 1, we see that the distribution of stress/strain within the soil element will be uniform due to the different stiffness of the bonding elements and frictional elements. Before the bonding element breaks up, it may behave elastic, linear or nonlinear, which is mainly resulted from the cementation/bonding effects among soil particles. The frictional elements, however, may behave elasto-palstic, which is mainly resulted from the slip and rotation among soil particles.

For geological materials with cementation/bonding actions among soil particles, they can be idealized as Binary-medium model, including structured soils (Liu & Shen 2005a,b; Liu et al. 2017), frozen soils (Zhang & Liu 2019; Zhang et al. 2019, 2020; Wang et al. 2019, 2020, 2021), rocks (Liu &Zhang 2013; Yu et al. 2020), cemented soils (Liu et al. 2020; Yu & Liu 2021), in which upon loading the bonding elements will break up gradually and transform to be frictional elements. Structured soils have bonding effects between soil particles during the process of sedimentation, frozen soils have ice bonding, rocks have strong bonds, and cemented soils have cementation within soils resulted from cementing materials or sedimentation. Subjected to the external loading, the bonding/cementation among soil particles may break up and transform to be assemblies of soil particles without bondings.

2.2 *The stress-strain relationship for Binary-medium Constitutive Model (BMCM)*

2.2.1 *One parameter-based BMCM*

For the RVE as shown in Figure 1, the stress and strain for BMM can be expressed as follows (Shen et al. 2002, 2006) by homogenization theory:

$$\sigma_{ij} = (1-\lambda)\sigma_{ij}^b + \lambda\sigma_{ij}^f \tag{1a}$$

$$\varepsilon_{ij} = (1-\lambda)\varepsilon_{ij}^b + \lambda\varepsilon_{ij}^f \tag{2b}$$

in which σ_{ij}, σ_{ij}^b, and σ_{ij}^f are the stresses of RVE, bonding element, and frictional element, respectively; ε_{ij}, ε_{ij}^b, ε_{ij}^f are the strains of RVE, bonding element, and frictional element, respectively; λ is the breakage ratio and equal to the ratio of the bonding elements to frictional elements within the RVE. At the initial loading stage, the RVE is mainly composed of the bonding elements, which break up gradually with the process of loading and transform to be the frictional elements, and at failure the frictional elements dominate the behavior of RVE. The uniform distribution of strain/stress can be realized by introducing the local strain/stress coefficient, C_{ijkl} or A_{ijkl}, linking the strain/stress of the bonding elements with that of RVE, expressed as follows,

$$\varepsilon_{ij}^b = C_{ijkl}\varepsilon_{kl} \ \text{ or } \ \sigma_{ij}^b = A_{ijkl}\sigma_{kl} \tag{2}$$

The tangential stiffness matrixes of bonded elements and frictional elements are represented by D_{ijkl}^b and D_{ijkl}^f, respectively, and we have

$$d\sigma_{ij}^b = D_{ijkl}^b d\varepsilon_{kl}^b \tag{3a}$$

$$d\sigma_{ij}^f = D_{ijkl}^f d\varepsilon_{kl}^b \tag{3b}$$

Expressing (1a) and (1b) in incremental form, and combining (2) and (3a), (3b), we can have the generalized stress-strain relationship for BMCM, expressed as follows,

$$d\sigma_{ij} = \left[(1-\lambda^0)\left(D_{ijkl}^b - D_{ijkl}^f\right)C_{klmn}^0 + D_{ijmn}^f\right]d\varepsilon_{mn} + (1-\lambda^0)\left(D_{ijkl}^b - D_{ijkl}^f\right)dC_{klmn}\varepsilon_{mn}^0 \tag{4}$$

in which λ^0 is the current breakage ratio; C_{klmn}^0 is the current local strain coefficient.

2.2.2 *Two parameters-based BMCM*

The stress and strain can be divided into spherical and deviatoric parts, and thus we have

$$\sigma_m = (1-\lambda_v)\sigma_m^b + \lambda_v\sigma_m^f \tag{5a}$$
$$\varepsilon_v = (1-\lambda_v)\varepsilon_v^b + \lambda_v\varepsilon_v^f \tag{5b}$$
$$s_{ij} = (1-\lambda_s)s_{ij}^b + \lambda_s s_{ij}^f \tag{5c}$$
$$e_{ij} = (1-\lambda_s)e_{ij}^b + \lambda_s e_{ij}^f \tag{5d}$$

in which $\sigma_m = \sigma_{kk}/3$, $\varepsilon_v = \varepsilon_{kk}$, $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_m$, $e_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon_m/3$, and δ_{ij} is the Kronecker symbol; λ_v and λ_s are the volumetric breakage ratio and area breakage ratio respectively, which are introduced to account for the breakage of bonding elements and slip along the shear band, commonly companying strain softening. Similarly, the stress and strain for both the bonding element and frictional element can be divided into the spherical and deviatoric parts, so we have

$$d\sigma_m^b = K^b d\varepsilon_v^b \tag{6a}$$
$$ds_{ij}^b = G_{ijkl}^b de_{kl}^b \tag{6b}$$
$$d\sigma_m^f = K^f d\varepsilon_v^f \tag{6c}$$
$$ds_{ij}^f = G_{ijkl}^f de_{kl}^f \tag{6d}$$

The strains of the bonding element and RVE have the following relationship:

$$\varepsilon_v^b = C_v\varepsilon_v \text{ and } e_{ij}^b = C_{ijkl}e_{ij} \tag{7}$$

As the same way for deriving (4), we can obtain the two parameters-based BMCM in the following general form:

$$\begin{aligned} d\sigma_{ij} = & \left[K^f + (1-\lambda_v^0)(K^b - K^f)B^v\right]d\varepsilon_v\delta_{ij} \\ & + \frac{\varepsilon_v^0}{\lambda_v^0}\left[(C_v^0 - 1)K^f + D_m^0 - K^{b0}C_v^0\right]d\lambda_v\delta_{ij} \\ & + \left\{G_{ijkl}^f + (1-\lambda_s^0)\left(G_{ijmn}^b - G_{ijmn}^f\right)B_{mnkl}^s\right\}de_{kl} \\ & + \left\{\left[(C_{ijmn}^{s0} - 1)G_{mnkl}^f + G_{ijkl}^{s0} - G_{ijmn}^{s0}C_{mnkl}^{s0}\right]\right\}\frac{e_{kl}^0}{\lambda_s^0}d\lambda_s \end{aligned} \tag{8}$$

in which $B^v = C_v^0 + \frac{\partial C_v}{\partial \varepsilon_v}\varepsilon_v^0$; $D_m^0 = \sigma_m^0/\varepsilon_v^0$; $K^{b0} = \sigma_m^{b0}/\varepsilon_v^{b0}$; $B_{ijkl}^s = C_{ijkl}^{s0} + \frac{\partial C_{ijkl}}{\partial e_{mn}}e_{mn}^0$; $s_{ij}^0 = G_{ijkl}^{s0}e_{kl}^0$; $s_{ij}^{b0} = G_{ijkl}^{sb0}e_{kl}^{b0}$; λ_v^0, C_v^0, λ_s^0, C_{ijmn}^{s0} are the current values.

When deriving the generalized stress-strain relationships (4) and (8), we used the local strain coefficient to link the strain of the bonding element and that of RVE. In addition, we can also use the local stress coefficient to derive the generalized stress-strain relationship for BMCM, referring to Shen et al. (2005), and Yu and Liu (2021).

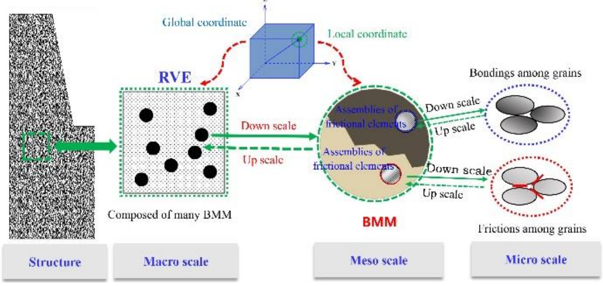
2.3 *The determination of model parameters*

There are four sets of model parameters in BMCM, including the parameters of the bonding elements, the frictional elements, the breakage ratios, and the local strain/stress coefficient.

From the definition of the bonding element, we know that its model parameters can be determined for the sample at the initial loading stage, on the moment the behavior of the sample is dominated by the bonding elements and can be described by elastic stress-strain relationship. For structured soils, the model parameters are determined within 0.25% of the axial strain (Liu et al. 2017); and for frozen sands having larger elastic stage, the model parameters are determined within 0.5% of the axial strain (Zhang et al. 2020).

From the definition of the frictional element, we know that its model parameters can be determined when the sample fails, which behaves elasto-plastic. For structured soils, the behavior of the frictional elements can be determined on the reconstituted sample with the same density of the intact sample. For rocks, the frictional elements can be modeled by crushed rocks or sand particles (Liu et al. 2013; Yu et al. 2020). According to the test results on reconstituted soils, the frictional elements can be described by the Cam-clay model (Liu & Shen 2005b), Lade-Duncan model (Liu et al. 2017; Yu & Liu 2021), and double hardening model (Zhang et al. 2020; Wang et al. 2021), elastic-perfectly plastic Mohr-Column model (Yu et al. 2020).

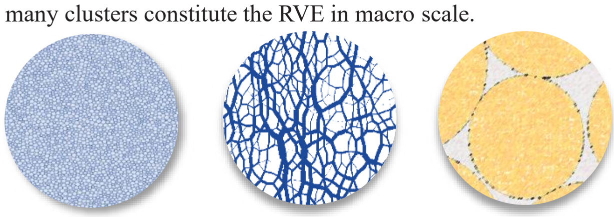
In the BMCM, both the breakage ratios and the local strain/stress coefficient are the structural parameters, which can be determined by the following approaches. The breakage ratio is an internal variable, having the similar meaning as the hardening parameter in plasticity and the damage variable in damage mechanics, which denotes the breakage of the bonding element in some degree. When the breakage ratio is expressed as the function of stress or strain of RVE, it has the evolving rule of Weibull distribution (Liu et al. 2017; Zhang et al. 2019; Yu et al.2020; Yu & Liu 2021). In addition, the energy conservation in meso and macro scales can be employed to solve the breakage ratio which is implied in the constitutive model (Wang et al. 2021).



The local strain/stress coefficient bridges the strain/stress of the bonding element to that of RVE, which reflects the uniform distribution of strain/stress in REV. There are two approaches can be used to determine it, one of which is to express it as the function of the breakage ratio, and the other of which is to use the mesomechanics-based method (such as Mori-Tanaka method), and thus the bonding element/frictional elements can be idealized to be matrix/inclusions transforming to inclusions/matrix (Zhang et al. 2020; Wang et al. 2021). The mesomechanical method used to determine the local strain/stress coefficient can consider the interactions between the bonding element and frictional element, and the local strain/stress coefficient can be implicitly solved.

3 THEORTEICAL FREAMWORK FOR MULTI-SCALE BINARY-MEDIUM MODEL(BMM)

In this section the definition of multi scales for geological materials is given, as shown in Figure 2. As we know that for metal materials, the size of material cam be used to define the micro, meso and macro scales, but for geomaterials, they are grains with/without cementation, and the slip and rotation are the deformation mechanism, and thus only the relative size is suitable to define their multi scales. The single particle which can deform or crush is defined at micro scale; the cluster of many particles, which can interact with each other, slip or rotate among them, and form the force chain, is defined at meso scale; and many clusters constitute the RVE in macro scale.



(a)Macro scale (RVE): many clusters
(b)Meso scale: clusters of particles. The interactions of particles.
(c)Micro scale: single particle. Breakage deformation.

Figure 2. Multi scales for geomaterials (taking DEM simulation as an example)

Considering that some geological materials have some constituents and some model parameters do not have physical meaning, we have to formulate multi-scale BMCM. For example, for saturated frozen soil, the bonding elements are composed of soil particles, ice crystals, and unfrozen water, which break up and transform to be the frictional elements consisting of crushed soil and ice particles (Wang et al. 2021). The soil particles cemented by ice crystals neglecting unfrozen water in meso scale are up scaled to form the bonding elements, and the broken soil/ice particles in meso scale are also up scaled to form the frictional elements, and thus this is two scales of BMM, also referring to Figure 1.

Figure 3 presents the schematic of multi-scale BMM, in which BMM is assumed to a meso scale bridging both the macro and micro scale. BMM in meso scale is composed of assemblies of bonding elements and frictional elements and upscales to form RVE in macro scale, in which the assemblies of the bonding elements are formed by upscaling soil particles with bonding actions and the assemblies of the frictional elements are formed by upscaling soil particles with frictional actions in micro scale. In micro scale, the basic parameters are bonding, the volumetric fraction, and frictional actions among particles.

Figure 3. Schematic of Multi-scale BMM

From the micro scale to macro scale, two steps of homogenization are needed to formulate the constitutive model for BBM. In Figure 3, the BMM can also be assumed to the macro scale, which can be selected according to the actual requirements or conditions of the material properties.

4 CONCLUSIONS AND DISSCUSIONS

The BMCM reviewed here have been used to model some kinds of geomaterials, including structured soil (Liu et al. 2017;), frozen soil (Zhang et al. 2019, 2020; Wang et al.2020, 2021), rock material (Liu et al.2012; Yu et al. 2020), cemented mixed soil (Yu & Liu 2021), tailing soil subjected to freeze-thaw cycles(Liu et al. 2020). The model parameters and validation results can be found in the corresponding articles, in which both the strain softening, initial contraction followed by dilatancy at low stress level, and the strain hardening and volumetric contraction at high stress level, can be modeled relatively well compared with the test results. It demonstrates that BMCM can grasp the salient features of cemented/bonding geomaterials.

Some discussions on BMM are done here.

(i)There are some differences between BMM and the existing constitutive models. In BMM, the breakage of the bonding element is the study focus, in which the breakage criterion and the evolutions of structural parameters, the breakage ratio and the local strain/stress coefficient, are new concepts to describe the transform of the bonding elements to the frictional elements and consider the uniform distribution of stress/strain in RVE. When determining the model parameters, we used the mesomechanics-based approaches, which can reduce the model parameters and let the parameters have clear meanings, and thus multi-scale BMCM have been formulated, which enrich the constitutive models for geological materials.

(ii)Even though the BMM can model several types of geological materials, there are many works to improve it. For example, some model parameters do not yet have physical meanings, which can be improved by multi-scale approaches; the breakage criterion for the bonding element can be investigated from micro scale to macro scale (Liu et al. 2013; Luo et al. 2019); the numerical simulation of BMM is another study field which can use the model to guide the design and construction of practical engineering. In addition, the BMM can also be extended to model the complex loading conditions for geological materials.

5 REFERENCES

Liu, E.L., and Shen, Z.J. 2005a. Research on the constitutive model of structured soils. *Proceedings of The International Conferences on Problematic Soils*, Famagusta, North Cyprus,373-380.

Liu, E.L., and Shen, Z.J. 2005b. Binary medium model for structured soils.*Journal of Hydraulic Engineering*,36(4),391-395.(in Chinese)

Liu, E.L. 2006. *Study on breakage mechanism of structural blocks and the binary-medium model for geomaterials*. Ph.D thesis, Tisnghua University, Beijing. (in Chinese)

Liu, E.L., Nie, Q., and Zhang, J.H., 2013. A new strength criterion for structured soils. *Journal of Rock Mechanics and Geotechnical Engineering*, 5, 156-161.

Liu, E.L., and Zhang, J.H., 2013. Binary medium model for rock sample. *Constitutive Modeling of Geomaterials: Advances and New Applications,* SSGG, Yang, Q. et al. (eds), 341-347.

Liu, E.L.,Yu, H.-S., Zhou, C., Luo, K.T., Nie, Q. 2017. A binary-medium constitutive model for artificially structured soils based on the disturbed state concept (dsc) and homogenization theory. *International Journal of Geomechanics*, 17(7), 04016154.

Liu, Y.N., Liu, E.L., and Yin, Z.Y. 2020. Constitutive model for tailing soils subjected to freeze-thaw cycles based on meso-mechanics and homogenization theory. *Acta Geotechnica*, 15, 2433-2450.

Luo, F., Liu, E.L., and Zhu, Z.Y. 2019. A strength criterion for frozen moraine soils. *Cold Regions Science and Technology*, 164:102786.

Schofield, A.N., and Wroth, P. 1968. *Critical state soil mechanics*, McGraw-Hill, United Kingdom.

Shen, Z.J. 2002. Breakage mechanics and double-medium model for geological materials. *Hydro-science and Engineering*, (4), 1-6. (in Chinese)

Shen, Z.J., Liu, E.L., and Chen, T.L. 2005.Generalized stress-strain relationship of binary medium model for geological materials. *Chniese Journal of Geotechncal Engineering*, 27(5), 489-494. (in Chinese)

Shen, Z.J. 2006. Progress in binary medium modeling of geological materials. *Modern trends in geomechancis*, Wu, W., and H. S Yu, eds., Springer, Berlin, 77–99.

Wang, P., Liu, E.L., Song, B.T., Liu, X.Y., Zhang, G., and Zhang, D. 2019. Binary medium creep constitutive model for frozen soils based on homogenization theory. *Cold Regions Science and Technology*, 162, 35-42.

Wang, P., Liu, E.L., Zhang, D., Liu, X.Y., Zhang, G., and Song, B.T. 2020. An elastoplastic binary medium constitutive model for saturated frozen soils. *Cold Regions Science and Technology*, 174, 103055.

Wang, P., Liu, E.L., and Zhi, B. 2021. An elastic-plastic model for frozen soil from micro to macro scale. *Applied Mathematical Modelling*, 91, 125-148.

Yu, D., Liu, E.L., Sun, P., Xiang, B., and Zheng, Q. 2020. Mechanical properties and binary-medium constitutive model for semi-through jointed mudstone samples. *International Journal of Rock Mechanics and Mining Science*, 132, 104376.

Yu, H.J., and Liu, E.L. 2021. A Binary-Medium Constitutive Model for Artificially Cemented Gravel-Silty Clay Mixed Soils. *European Journal of Environmental and CivilEngineering*, Published online. DOI: 10.1080/19648189.2021.1917455.

Zhang, D., and Liu, E.L.2019. Binary-medium-based constitutive model of frozen soils subjected to triaxial loading. *Results in Physics,* 12**,** 1999-2008.

Zhang, D., Liu, E.L., and Yu, D. 2020. A micromechanics-based elastoplastic constitutive model for frozen sands based on homogenization theory. *International Journal of Damage Mechanics*, 29(5), 689-714.

Zhang, D., Liu, E.L., and Huang, J. 2020. Elastoplastic constitutive model for frozen sands based on framework of homogenization theory. *Acta Geotechnica*, 15, 1831-1845.