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An alternative plasticity formulation for modelling of unsaturated soils

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ABSTRACT: A constitutive model is proposed to predict the response of partially saturated geomaterials showing collapsible behaviour under wetting paths. A bounding surface plasticity model for unsaturated soils is adopted and improved with alternative incremental elasto-plastic relationships. This formulation separates the effect of strain tensor and the matric suction on the developed effective stress and hence overcomes the deficiency of conventional elasto-plastic formulations with the effect of matric suction only embedded inside the stiffness tensor. The proposed incremental equations can predict the independent irreversible effect of effective stress and matric suction on the deformation response of material. The key feature of the novel incremental plasticity relationships is it can capture accurately the response of unsaturated soils subjected to soaking tests with a negligible volume change. The model also features a void ratio-dependent water retention curve (WRC) to capture hydraulic and mechanical coupling, in which the parameter identification is based only on the parameters of a reference WRC. The prediction capability of the model is validated through comparison with existing experimental tests on unsaturated soils, including constant void ratio oedometer tests. Specifically, in the latter test, unsatisfactory prediction would have been expected if conventional elasto-plastic relationships have been used for explicit integration of the constitutive model.

RÉSUMÉ : Un modèle constitutif est proposé pour prédire la réponse de géomatériaux partiellement saturés présentant un comportement pliable sous les chemins de mouillage. Un modèle de plasticité de surface limite pour les sols non saturés est adopté et amélioré avec des relations élasto-plastiques incrémentales alternatives. Cette formulation sépare l'effet du tenseur de déformation et de l'aspiration matricielle sur la contrainte effective développée et surmonte ainsi le déficit des formulations élasto-plastiques classiques avec l'effet de la succion matricielle uniquement noyée à l'intérieur du tenseur de rigidité. Les équations incrémentales proposées peuvent prédire l'effet irréversible indépendant de la contrainte effective et de l'aspiration matricielle sur la réponse à la déformation du matériau. La caractéristique clé des nouvelles relations de plasticité incrémentale est qu'elles peuvent capturer avec précision la réponse des sols non saturés soumis à des tests de trempage avec un changement de volume négligeable. Le modèle comporte également une courbe de rétention d'eau (WRC) dépendant du rapport de vide pour capturer le couplage hydraulique et mécanique, dans laquelle l'identification des paramètres est basée uniquement sur les paramètres d'une référence WRC. La capacité de prédiction du modèle est validée par comparaison avec des tests expérimentaux existants sur des sols non saturés, y compris des tests d'œdomètre à taux de vide constant. Plus précisément, dans ce dernier test, une prédiction insatisfaisante aurait été attendue si les relations élasto-plastiques conventionnelles avaient été utilisées pour l'intégration explicite du modèle constitutif.

KEYWORDS: unsaturated soils, bounding surface plasticity, water retention curve, effective stress, explicit integration

1 INTRODUCTION.

The hydro-mechanical behaviour of partially saturated soils can be predicted satisfactorily if the interaction of all phases is incorporated in the mathematical framework. It has been noted that the matric suction is a fundamental variable explaining the stiffness, strength and hydraulic conductivity of unsaturated soil. Also, the volume change even without the variation of the matric suction can influence on the water retention and mechanical properties of unsaturated soils(Toll 1990, Fleureau et al 1993). All the above sophisticated features need to be addressed in the constitutive models.

There have been a number of constitutive model intending to explain the essential characteristics of unsaturated soils. In the early models, individual effect of the state variables (e.g. matric suction and external loading) has been used to describe the behaviour of unsaturated soils(Alonso et al 1990, Wheeler 1995). These models can recover sufficiently many features of the behaviour of unsaturated soils such as excessive hardening due to the suction, recoverable deformation in drying events and the plastic collapse in wetting events(Toll & Ong 2003). However, extensive laboratory tests are required for the parameter identification of these models and their models usually suffer from the non-convexity of yield surface. The constitutive models based the single stress state have then been proposed to overcome these deficiencies (Loret and Khalili 2002, Gallipoli et al 2003, Sun et al 2007). The collapse of unsaturated soils upon wetting, which was a challenging issue in early models, has been properly addressed in recent models utilizing single stress state. This has been achieved via attributing collapse phenomenon to the irrecoverable plastic behaviour. The models described above are based on the conventional plasticity formulation and therefore are unable to explain the behaviour of unsaturated soils in complex loading(Gallipoli et al 2003). The bounding surface plasticity models can improve the capability of conventional plasticity model by predicting a smooth elasto-plastic response. Despite these achievements, there have been few constitutive models utilizing this framework for modelling of unsaturated soils(Khalili et al 2008, Liu & Muraleetharan 2011).

The void ratio dependent WRCs have been incorporated in a number of constitutive models(Sun et al 2007, Masin 2010, Liu & Muraleetharan 2011, Pasha et al 2017). The overwhelming majority of these models introduced an empirical relationship for WRCs, requiring time-consuming laboratory tests to determine the model parameters. In few studies, coupling of various phases in porous media and the validity of effective stress principle has been used to establish the void ratio dependent WRCs(Masin 2010, Pasha et al 2017). In these models, no material parameters are required to describe the volume change dependency of WRCs since the continuum macromechanics approach are involved.

The aim of the presented study is to examine an alternative approach for elasto-plastic modelling of unsaturated soils. For the mechanical model, the bounding surface plasticity model is adopted(Dafalias, 1986, Khalili et al 2008, Moghaddasi et al 2021a) and reformulated to capture individual effects of matric suction and the volume change. This has been achieved in the incremental elasto-plastic relationship isolating the effects of matric suction and strain tensors. The model can predict full coupling of hydraulic and mechanical behaviour since the volume change dependent WRC is incorporated in the constitutive model. The performance of the model has been validated through comparison to the results of unconventional experiments.

1 *Water retention curve and effective stress parameter*

The effective stress principle for saturated soil can be extended to the domain of partially saturated porous media as

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \chi p_w \boldsymbol{\delta} + (1-\chi) p_a \boldsymbol{\delta} = \boldsymbol{\sigma}_{\boldsymbol{net}} - \chi s \boldsymbol{\delta} \tag{1}$$

In the above equation, p_w and p_a are water and air pressure and χ is the effective stress parameter to emphasize the effect of matric suction($s = p_a - p_w$) on the excess stiffness of the unsaturated soil and $\boldsymbol{\sigma}_{\boldsymbol{net}} = \boldsymbol{\sigma} + p_a \boldsymbol{\delta}$ is the net stress. A general equation based on matric suction is then adopted for the effective stress parameter in this study

$$\chi = \begin{cases} 1 & {}_{e} \\ \left(\dfrac{s_e}{s}\right)^{\Omega} & s > s_e \end{cases} \tag{2}$$

Where s_e is a suction value at the air-entry or air-expulsion,(s_{ae} or s_{ex} , depending on the direction of hydraulic loading), Ω is a material parameter which has been suggested to be constant as 0.55 (Khalili et al 2008).The capability of porous soil media to maintain the quantity of the water is measured through water retention curve. Various mathematical expressions have been proposed for WRC as a function of matric suction. Here equation proposed by Brooks and Corey 1964 is adopted

$$S_{eff} = \begin{cases} 1 & s < s_e \\ \left(\dfrac{s_e}{s}\right)^{\lambda_p} & s > s_e \end{cases} \tag{3}$$

where λ_p is the pore size distribution index or the slope of WRC in a $\ln(S_{eff}) - \ln(s)$ plane and S_{eff} is the effective degree of saturation defined as $S_{eff} = (S_r - S_{res}) / (1 - S_{res})$ in which S_{res} is the residual degree of saturation. This WRC can be extended to include the effect of the volume change. One possible extension is to use the coupling between the effective stress parameter and fluids' governing equation. This approach will offer a model, which no empirical parameter are required to define the volume change dependency of WRC. Following this method, the main parameters of WRC can be updated upon the small change of void ratio(de) as(see Pasha et al 2017 and Moghaddasi et al 2021b)

$$s_e^* = s_e \left[1 - \frac{\Omega}{(1 - S_{res}) \lambda_{psu}} \frac{de}{e} \right] \tag{4}$$

$$\lambda_p^* = \lambda_p \left\{ 1 - \frac{3\left[(1-\Omega)(2^{(1-\Omega/\lambda_p)} - 1) - S_{res} \right]}{2(1 - S_{res})} \frac{de}{e} \right\} \tag{5}$$

In which λ_{psu} is the pore size distribution index at the point of transition from the saturation to unsaturation. The void ratio dependent WRC can then be obtained from the integration of a reference WRC where initial void ratio e_0, s_{e0} and λ_{p0} are already available.

2 BOUNDING SURFACE PLASTICITY MODEL

The bounding surface plasticity formulation is used for the constitutive modeling of unsaturated soils. The following general equation is considered for the bounding surface

$$F\left(\bar{p}', \bar{q}, \bar{\theta}, \bar{p}'_c\right) = \left[\frac{\bar{q}}{M_{cs}\left(\bar{\theta}\right) . \bar{p}'} \right]^N - \frac{ln\left(\bar{p}'_c / \bar{p}' \right)}{ln(R)} = 0 \tag{6}$$

Where $\bar{p}'$, $\bar{q}$, $\bar{\theta}$ represents the mean effective stress, deviatoric stress and lode angle on the bounding surface, $\bar{p}'_c$ is the size of the bounding surface and N and R are material parameters. $M_{cs}\left(\bar{\theta}\right)$ is the slope of the critical state line (CSL) in the stress space, which varies with the Lode angle as

$$M_{cs}\left(\bar{\theta}\right) = M_{max} \left[\frac{2\alpha^4}{1 + \alpha^4 - \left(1 - \alpha^4\right) sin\left(3\bar{\theta}\right)} \right]^{1/4} \tag{7}$$

in which $\alpha = M_{max} / M_{min}$ with M_{max} and M_{min} are the slope of CSL at triaxial compression and extension. The slope of CSL in extension can be linked to the same slope in compression as $M_{min} = 3M_{max} / (M_{max} + 3)$. The movement of the stress can be traced on a loading surface, assumed to have a similar shape to the bounding surface. Also the mapping rule should be proposed to define an image point on the bounding surface. In this study, the origin of stress is selected as the projection centre and a simple radial mapping rule is utilized. A non-associated flow rule is assumed for the dilatancy parameter (d) as

$$d = \frac{\dot{\varepsilon}_p^p}{\dot{\varepsilon}_q^p} = A(M_{cs}(\theta) - \frac{q}{p'}) \tag{8}$$

Where A is a material parameter, $\dot{\varepsilon}_p^p$ and $\dot{\varepsilon}_q^p$ the increment of the volumetric and deviatoric strain, respectively. The size of the bounding surface $\overline{p}'_c$ can change due to the simultaneous variation of suction and plastic volumetric strain. The following expression is selected for the size of the bounding surface(Loret and Khalili 2002, Moghaddasi et al 2021b)

$$\overline{p}'_c(\varepsilon_p^p, s) = \overline{p}'_{c0} \Psi(s) exp\left(\frac{\upsilon_i \Delta\varepsilon_p^p}{\lambda(s) - \kappa} \right) \tag{9}$$

Where

$$\Psi(s) = exp\left(\frac{N(s) - N(s_i)}{(\lambda(s) - k)} - \frac{(\lambda(s) - \lambda(s_i))}{(\lambda(s) - k)} ln\left(\frac{\overline{p}'_{c0}}{p'_{ref}} \right) \right) \tag{10}$$

In which the slope of unloading-reloading curve on $\upsilon \sim lnp'$ plane is denoted by κ, the increment of the plastic volumetric strain is indicated by $\Delta\varepsilon_p^p$, υ_i and $\overline{p}'_{c0}$ are the initial specific volume and hardening parameter. The suction hardening parameter are also $N(s_i)$ and $\lambda(s_i)$ where they are intercept and the slope of limiting isotropic compression line (LICL) on $\upsilon \sim lnp'$ plane at the suction s. Similar quantities for initial suction are denoted by $N(s_i)$ and $\lambda(s_i)$ respectively. Following the work of Loret and Khalili 2000, two equations for the evolution of the slope and intersect of LICL with matric suction can be introduced

$$N(s) = (N(0) - \kappa)\ln(1 + \frac{k_{m1}}{s_e}(s\chi - s_e)) + N(0) \tag{11}$$

$$\lambda(s) = (\lambda(0) - \kappa)\ln(1 + \frac{k_{m2}}{s_e}(s\chi - s_e)) + \lambda(0) \tag{12}$$

Where k_{m1} and k_{m2} are two material parameters. Also the bulk modulus K, and the shear strain G, are defined to form the elastic stiffness matrix as

$$K = \frac{\upsilon p'}{\kappa}, \quad G = \frac{3(1-2\nu)\upsilon p}{2(1+\nu)\kappa} \tag{13}$$

Where ν is the Poisson's ratio.

3 A NOVEL INCREMENTAL ELASTO-PLASTIC RELATIONSHIP

The explicit integration of the plasticity model is followed. The aim here is to reformulate conventional integration method in such way that the hardening effect of plastic strain and the matric suction are separated . The plastic deformation can be expressed as $\dot{\boldsymbol{\varepsilon}}^{\boldsymbol{p}} = \dot{\Lambda}\boldsymbol{m}$ where $\boldsymbol{m}$ is the direction of the plastic deformation and $\dot{\Lambda}$ is the plastic multiplier. Therefore, the consistency condition on the bounding surface can be written

$$dF = \overline{\boldsymbol{n}}^{\boldsymbol{T}} \dot{\overline{\boldsymbol{\sigma}}}' - \dot{\Lambda} h_b + \frac{\partial F}{\partial \overline{p}_c} \frac{\partial \overline{p}'_c}{\partial s} \dot{s} / \left\| \frac{\partial F}{\partial \overline{\boldsymbol{\sigma}}'} \right\| = 0 \tag{14}$$

In which $\boldsymbol{n}$ is the unit normal vector at the image point and h_b is the hardening module on the bounding surface defined as

$$h_b = -\frac{\partial F}{\partial \overline{p}_c} \frac{\partial \overline{p}_c}{\partial \dot{\varepsilon}_p^p} \boldsymbol{m}_{\boldsymbol{p}} / \left\| \frac{\partial F}{\partial \overline{\boldsymbol{\sigma}}'} \right\| \tag{15}$$

In the bounding surface plasticity, it is assumed that the plastic multiplier on the bounding and loading surface is identical. Therefore the consistency condition on the loading surface can be correlated to the consistency condition on the bounding surface as

$$\dot{\Lambda} = \frac{\overline{\boldsymbol{n}}^{\boldsymbol{T}} \dot{\overline{\boldsymbol{\sigma}}}' + \frac{\partial F}{\partial \overline{p}_C} \frac{\partial \overline{p}_C}{\partial s} \dot{s} / \left\| \frac{\partial F}{\partial \overline{\boldsymbol{\sigma}}'} \right\|}{h_b} = \frac{\boldsymbol{n}^{\boldsymbol{T}} \dot{\boldsymbol{\sigma}}' + \frac{\partial f}{\partial \hat{p}'_c} \frac{\partial \hat{p}'_c}{\partial s} \dot{s} / \left\| \frac{\partial f}{\partial \dot{\boldsymbol{\sigma}}'} \right\|}{h} \tag{16}$$

Where f is the loading surface with the size of $\hat{p}'_c$. Also h is the hardening modules at the current stress point, which can be decomposed into

$$h = h_b + h_f \tag{17}$$

The hardening modulus h_f should fulfill the requirements such that it is zero on the bounding surface and infinity at the projection centre. Considering this requirement, the following equation is selected for h_f

$$h_f = \frac{\partial \bar{p}_c'}{\partial \varepsilon_p^p} \frac{p'}{\bar{p}_c'} \left(\frac{\bar{p}_c'}{\hat{p}_c'} - 1 \right) k_m \left(\eta_p - \eta \right) \tag{18}$$

Where $\eta = q / p$ is the stress ratio, $\eta_p = \left(1 - 2\xi\right) M_{cs}\left(\theta\right)$ is the slope of peak strength line, k_m is material parameter. Decomposing the rate of the strain tensor to elastic and plastic components, the increment of effective stress can be calculated as

$$\dot{\boldsymbol{\sigma}}' = \boldsymbol{D}^e \dot{\boldsymbol{\varepsilon}}^e = \boldsymbol{D}^e (\dot{\boldsymbol{\varepsilon}} - \dot{\Lambda} \boldsymbol{m}) \tag{19}$$

where $\boldsymbol{D}^e$ is the elastic stiffness matrix. Substituting the right part of equation (16) in (19), an incremental elasto-plastic relationship can be achieved

$$\dot{\boldsymbol{\sigma}}' = \left[\boldsymbol{D}^e - \frac{\boldsymbol{D}^e \boldsymbol{m} \boldsymbol{n}^T \boldsymbol{D}^e}{\boldsymbol{n}^T \boldsymbol{D}^e \boldsymbol{m} + h} \right] \dot{\boldsymbol{\varepsilon}} - \frac{\boldsymbol{D}^e \boldsymbol{m} \dfrac{\partial f}{\partial \hat{p}_c'} \dfrac{\partial \hat{p}_c'}{\partial s} \dot{s} \Big/ \left\| \dfrac{\partial f}{\partial \dot{\boldsymbol{\sigma}}'} \right\|}{\boldsymbol{n}^T \boldsymbol{D}^e \boldsymbol{m} + h} \tag{20}$$

4 APPLICATION OF THE MODEL

A wetting test performed in a constant volume oedometer apparatus is simulated. The key feature of this unconventional test is a water retention curve under constant void ratio can be obtained. Similar experimental test has been conducted by Pasha 2016 on the sandy clay soil, which is used here for the validation of the presented model. The initial pore air pressure of 900 kPa was applied to the sample and remained constant throughout the test. Also the pore water pressure was initially set to be 10kPa and decreased in some specific steps from 10 kPa to 900 kPa. Two samples with initial void ratio of 0.974 and 1.31 subjected to wetting test are simulated. The initial vertical net stress of 20 kPa was applied on both samples, while it changed during the test to prevent the volume change. The set of material parameter's used for simulation are $k = 0.007$, $\nu = 0.3$, $N = 2$, $R = 1.6$, $A = 2$, $k_m = 200$, $M_{max} = 1.1$, $\lambda(0)$=0.083, $N(0)$=2.25, k_{m1}= 4.5 and k_{m2}=1.2. The basic hydraulic parameters are then given as $s_{e0} = 4$ kPa, $\lambda_{p0} = 0.22$, $e_0 = 0.974$ and $S_{res} = 0.1$.

Although a piecewise decline of the matric suction with respect to time has been applied to soils samples, smooth functions were used in the numerical calculation to replicate the suction variation with time as shown in the Figure 1a. The predicted and measured WRC for two samples are depicted in the Figure 1b. It is clear that the hydraulic model can predict satisfactorily the variation of degree of saturation at different void ratios. The change of the net normal stress with respect to the time is calculated with the formulated model and compared to the experimental data as shown in the Figure 2. The initial swelling tendency was noted for the sample with low void ratio(i.e. e=0.974), followed to a tendency to collapse after the peak net stress achieved. For the sample with high void ratio, the tendency to collapse was observed throughout the wetting test, leading to significant decline of net stress. Even though the volumetric strain remained unchanged in the test, the model could predict the change of effective stress due to the change of suction since the effect of suction and strain tensor was separated in the novel incremental relationship introduced. Further, as a bounding surface plasticity model was formulated, a smooth transition from an initial elastic response to the plastic behaviour was well captured.

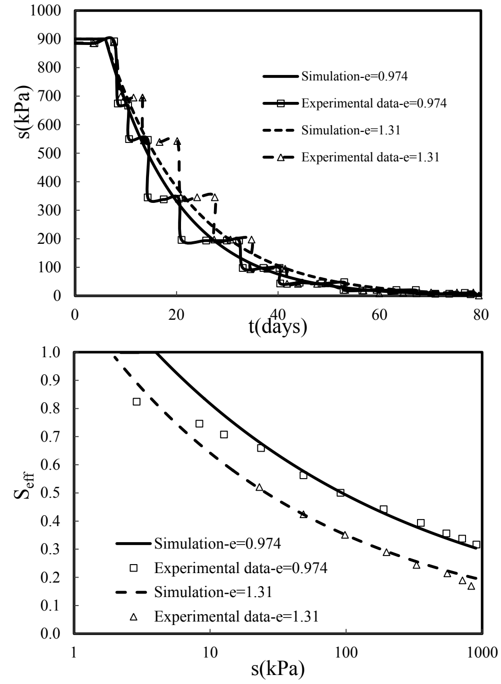

Figure1.a) the change of matric suction with time b)the predicted and measured WRC

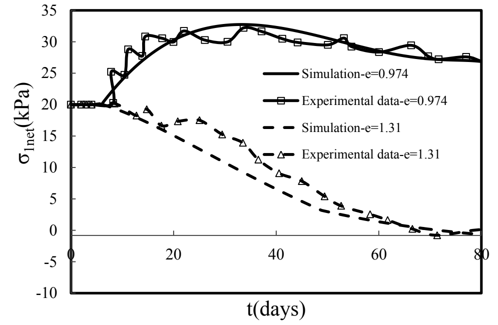

Figure 2. The variation of the net vertical stress with respect to time

5 CONCLUSION

Following the effective stress concept, a bounding surface plasticity formulation has been developed to recover the behaviour unsaturated soils subject to the various hydro-mechanical loading. The mechanical part of the model enjoys a non-associated flow rule and a flexible shape of bounding surface, which can capture the strength parameters of both cohesion and cohesionless unsaturated soils. A novel incremental elasto-plastic relationship was suggested to isolate the influence of state parameters (e.g. matric suction and strain tensor) on the predicted rate of the effective stress. To achieve this feature in the bounding surface plasticity models, the equivalency of the plastic deformation on both loading and bounding surfaces were utilized. The proposed alternative formulations can be used for the explicit integration of the highly nonlinear model. Also for the hydraulic model, a porosity dependent WRC has been adopted requiring no additional material parameters to describe the volume change dependency of WRC. The application of the model recovering the results of a constant volume oedometer test was demonstrated successfully. For this test upon wetting, it was shown that the model can well predict the tendency of samples to the elastic shrinkage or plastic collapse. However, if both hardening variables (e.g. matric suction and strain tensor) were embedded in the elasto-plastic stiffness matrix, the model could only predict the elastic shrinkage, which is solely related to the change of effective stress.

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