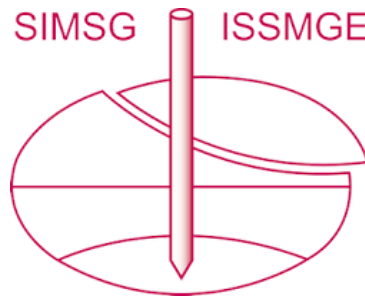


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Development and validation of deformation analysis method for unsaturated soil based on new effective saturation degree considering trapped air

Développement de la méthode d'analyse de déformation des sols insaturés basée sur un nouveau degré de saturation effectif

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ABSTRACT: The soil-water characteristic (SWC) equation describes the relationship between suction and effective saturation degree defined between the maximum and minimum amounts of water. When the degree of saturation is used as the amount of water, the maximum saturation degree is lower than 100% due to the trapped air, and the minimum saturation degree is higher than 0% due to the adsorbed water. In this study, the effective saturation degree was newly defined by dividing pore air into “trapped air” and “continuous air” which exists as a continuous phase, and by dividing pore water into “adsorbed water” and “free water”. Based on this new concept, a finite deformation analysis method for unsaturated soil was developed and it was validated through two analysis examples. It was demonstrated that a mere consideration of transition between trapped air and continuous air in the SWC model allowed us to express the hysteresis of the saturation degree during water absorption-drainage, and it was shown that an unexhausted and undrained shearing test at a suction of 0 kPa could be well-simulated by considering the compressibility of the trapped air.

RÉSUMÉ : L'équation caractéristique sol-eau (SWC) décrit la relation entre l'aspiration et le degré de saturation effectif défini entre les quantités d'eau maximale et minimale. Quand le degré de saturation est utilisé comme quantité d'eau, le degré maximal de saturation est inférieur à 100% à cause de l'air emprisonné, et le degré minimal de saturation est supérieur à 0% à cause de l'eau adsorbée. Dans cette étude, le degré de saturation effectif a été nouvellement défini en divisant l'air interstitiel entre «air emprisonné» et «air continu» qui existe en phase continue, et en divisant l'eau interstitielle en «eau adsorbée» et «eau libre». Sur la base de ce nouveau concept, nous avons développé une méthode d'analyse de déformation des sols insaturés, méthode validée au moyen de deux exemples d'analyses. En d'autres termes, nous avons démontré qu'une simple prise en compte de la transition entre l'air emprisonné et l'air continu dans le modèle SWC nous permettait d'exprimer l'hystérésis du degré de saturation pendant l'absorption et le drainage de l'eau, et montré qu'un essai de cisaillement non épuisé et non-drainé avec une aspiration de 0 kPa pouvait être bien simulé en considérant la compressibilité de l'air emprisonné.

KEYWORDS: deformation analysis method, soil-water characteristic curve, effective saturation degree, trapped air.

1 INTRODUCTION

The soil water characteristic equation describes the relationship between suction and the effective degree of saturation defined between the maximum and minimum amounts of water. When the degree of saturation is used to represent the amount of water, there is trapped air in pore water under low suction, so the maximum degree of saturation is lower than 100%. Also, the minimum degree of saturation under high suction is higher than 0% due to the presence of adsorbed water on the soil particle surfaces. In this study, the effective degree of saturation was newly defined by dividing pore air into “trapped air” and “continuous air” that exists as a continuous phase and by dividing pore water into “adsorbed water” and “free water.” In addition, a finite deformation analysis method for unsaturated soil was proposed based on the new effective degree of saturation. Also, by comparing numerical simulation results obtained by this method with experimental results, we demonstrated that it is possible to express the behavior of unsaturated soil caused by the presence of trapped air, which cannot be expressed with previous seepage/deformation analysis methods for unsaturated soil.

2 DEFINITION OF THE NEW EFFECTIVE DEGREE OF SATURATION AND VARIOUS QUANTITIES

Figure 1 shows the phases in a soil. The soil particles, adsorbed water, free water, trapped air, and continuous air are treated separately. The soil particles and the adsorbed water form the “soil skeleton,” and the other parts are considered to be the

“effective void” that is effective in the volumetric change of the soil skeleton. The ratio of effective void to the total soil is defined as the effective porosity (n_e). The trapped air is trapped by free water, so the ratio of free water + trapped air to the effective void is newly defined as the effective degree of saturation (S_e^l). Also, the volume ratio of the free water to free water + trapped air is defined as the free water degree of saturation (S_1^w). The quantity of adsorbed water is defined by the water content (w_t), and in this research, it is considered to be a material constant (the adsorbed water is assumed to be incompressible). Note that G_s in the figure is the specific gravity of the soil particle, and when the volume of the soil particles is assumed to be 1, the volume of the adsorbed water is $w_t G_s$.

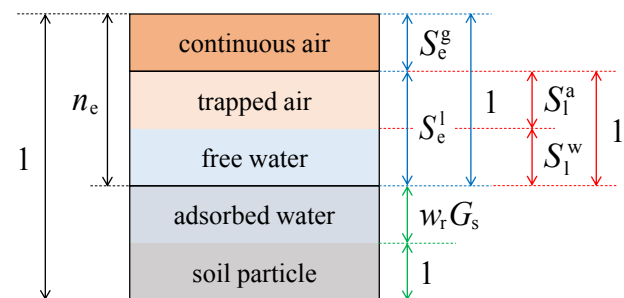


Figure 1. Phases in a soil.

3 SOIL SKELETON-FREE WATER-TRAPPED AIR-CONTINUOUS AIR COUPLED FINITE DEFORMATION ANALYSIS METHOD

As stated in the previous section, it is considered that unsaturated soil is composed of four phases, the soil skeleton, free water, trapped air, and continuous air, and unlike previous three-phase analyses, the equation of motion and mass conservation equation are applied to four phases. In this paper, the detailed deformation equations are omitted because of limitations of space. The equations were derived based on the \mathbf{u} - p formulation, following de Boer (1998), Nishimura (1999), Noda et al. (2008), and Noda and Yoshikawa (2015). As a result, the following equation of motion, soil skeleton-free water coupled equation, soil skeleton-trapped air coupled equation, and soil skeleton-continuous air coupled equation were obtained.

$$\rho \dot{\mathbf{v}}_s = \text{div} \mathbf{T} + \rho \mathbf{b} \quad (1)$$

$$S_c^l S_1^w \text{div} \mathbf{v}_s + \frac{1}{\rho^{lw}} \text{div} \{ \rho_{lw} (\mathbf{v}_{lw} - \mathbf{v}_s) \} + n_c S_1^w \dot{S}_c^l + n_c S_c^l \dot{S}_1^w + \frac{n_c S_c^l S_1^w}{K_w} \dot{p}^l = 0 \quad (2)$$

$$S_c^l S_1^a \text{div} \mathbf{v}_s + \frac{1}{\rho^{la}} \text{div} \{ \rho_{la} (\mathbf{v}_{la} - \mathbf{v}_s) \} + n_c S_1^a \dot{S}_c^l + n_c S_c^l \dot{S}_1^a + \frac{n_c S_c^l S_1^a}{K_a} \dot{p}^l = \frac{m_{la}}{\rho^{la}} \quad (3)$$

$$S_c^g \text{div} \mathbf{v}_s + \frac{1}{\rho^{ga}} \text{div} \{ \rho_{ga} (\mathbf{v}_{ga} - \mathbf{v}_s) \} + n_c \dot{S}_c^g + \frac{n_c S_c^g}{K_a} \dot{p}^g = \frac{m_{ga}}{\rho^{ga}} \quad (4)$$

Here, the overdots indicate material time derivatives viewed from the soil skeleton. \mathbf{v}_s is the soil skeleton velocity vector, $\dot{\mathbf{v}}_s$ is the soil skeleton acceleration vector, \mathbf{T} is the Cauchy total stress tensor (tension is positive), and \mathbf{b} is the body force per unit mass. p^l is the pressure of the free water, and p^g is the pressure of the continuous air; it is assumed that the pressure of the trapped air is equal to the pressure of the free water (compression is positive for p^l and p^g). ρ is the soil density as a mixture; ρ^{lw} , ρ^{la} , and ρ^{ga} are the true densities of the free water, trapped air, and continuous air as single bodies, respectively; and, ρ_{lw} , ρ_{la} , and ρ_{ga} are the densities of the free water, trapped air, and continuous air as component elements of the mixture. \mathbf{v}_{lw} , \mathbf{v}_{la} , and \mathbf{v}_{ga} are the velocity vectors of the free water, trapped air, and continuous air, respectively. K_w and K_a are the bulk moduli of water and air, respectively. Also, $S_c^g = 1 - S_c^l$, and $S_1^a = 1 - S_1^w$.

m_{la} and m_{ga} are the mass changes of the trapped air and continuous air per unit time per unit volume, respectively. It is considered that some continuous air changes into trapped air when water is absorbed and that some trapped air changes into continuous air when water is discharged. Therefore, in this paper the constitutive models for m_{la} and m_{ga} as described in the following (i) and (ii) are considered. Note that in this paper, only the mass conversion between trapped air and continuous air is considered, so $m_{la} + m_{ga} = 0$.

(i) Transition of continuous air into trapped air (for $\dot{S}_c^l \geq 0$)

Consider that transition occurs when the material-time derivative viewed from the soil skeleton for the effective degree of saturation (S_c^l) is $\dot{S}_c^l \geq 0$. When there is no continuous air at $S_c^g = 0$ and no free water at $S_1^g = 1$, there is no transition into trapped air, so the simple model indicated by the following equation was considered.

$$\frac{m_{la}}{\rho^{la}} = n_c \alpha^* S_c^g (1 - S_c^g) \dot{S}_c^l \quad (5)$$

Here, α^* is a material constant, and the larger the value the greater the amount of transition.

(ii) Transition of trapped air into continuous air (for $\dot{S}_c^l < 0$)

Consider that transition occurs when $\dot{S}_c^l < 0$. When there is no trapped air at $S_1^a = 0$, transition does not occur. There is no free water when $S_1^a = 1$, which means that the effective void is all continuous air, so this state cannot be defined. Therefore, the simple model indicated by the following equation was considered.

$$\frac{m_{ga}}{\rho^{ga}} = n_c \mu^* \ln(1 - S_1^a) \dot{S}_c^l \quad (6)$$

Here, μ^* is a material constant, and the larger the value the greater the amount of transition.

The stress equation is described by the following equation using the newly defined effective degree of saturation (S_c^l), based on the concept of skeleton stress (Jommi, 2000).

$$-\mathbf{T}' = -\mathbf{T} - (S_c^l p^l + S_c^g p^g) \mathbf{I} \quad (7)$$

Here, \mathbf{T}' is the skeleton stress tensor (tension is positive), and \mathbf{I} is the identity tensor.

By solving the initial and boundary value problem for the 6 equations obtained from the three equations of Eq. 1 and Eqs. 2-4, six unknowns (i.e. the three components of \mathbf{v}_s , p^l , p^g , and S_1^w) are obtained. Note that the effective degree of saturation (S_c^l) is calculated from the soil water characteristic model. Also, to take into consideration geometric nonlinearity and material nonlinearity, the rate type equation of motion obtained by the material time derivative viewed from the soil skeleton is used (Noda et al. 2008).

4 CHARACTERISTICS OF THE SOLUTION METHOD

As stated in Section 2, the effective degree of saturation (S_c^l) is defined as the ratio of the volume of the free water + trapped air to the effective void. Here, the following equations are used to represent the effective degree of saturation (S_c^l) and the degree of saturation (S_r).

$$S_c^l = \frac{V_{Wf} + V_{At}}{V_{Wf} + V_{At} + V_{Ac}} \quad (8)$$

$$S_r = \frac{V_{Wa} + V_{Wf}}{V_{Wa} + V_{Wf} + V_{At} + V_{Ac}} \quad (9)$$

Note that V_{Wa} , V_{Wf} , V_{At} and V_{Ac} are the volume of adsorbed water, free water, trapped air, and continuous air respectively.

(i) When there is no continuous air under low suction

When 0 is substituted into V_{Ac} , S_c^l and S_r are expressed by the following equations.

$$S_c^l = \frac{V_{Wf} + V_{At}}{V_{Wf} + V_{At}} = 1 \quad (10)$$

$$S_r = \frac{V_{Wa} + V_{Wf}}{V_{Wa} + V_{Wf} + V_{At}} \quad (11)$$

Based on Eq. 10, S_e^1 is 1; however, based on Eq. 11, if the trapped air compresses/expands, namely, V_{At} changes, it is possible for S_r to change. In this case, the free water degree of saturation (S_1^w : the ratio of the volume of the free water to free water + trapped air) changes. In previous soil water characteristic models, if the maximum degree of saturation is defined, a rise in the degree of saturation to greater than the maximum cannot be expressed, so the compressibility of the trapped air also cannot be evaluated.

(ii) When there is no free water under high suction

Since there is no free water, there is no trapped air in free water. Therefore, when 0 is substituted into V_{Wf} and V_{At} , S_e^1 and S_r are expressed by the following equations.

$$S_e^1 = \frac{0}{V_{Ac}} = 0 \quad (12)$$

$$S_r = \frac{V_{Wa}}{V_{Wa} + V_{Ac}} \quad (13)$$

Based on Eq. 12, S_e^1 is 0. Since pore water exists only as adsorbed water on the surface of soil particles and is defined by the water content (w_r), V_{Wa} is a constant. Therefore, based on Eq. 13, if the continuous air is compressed/extended, namely, V_{Ac} changes, S_r will naturally change. In previous soil water characteristic models, if the minimum degree of saturation is defined, it is necessary to change the minimum degree of saturation depending on the void ratio in order to express the water content of the adsorbed water as constant.

5 VALIDATION OF THE ANALYSIS METHOD

In this section, the effectiveness of taking into consideration the trapped air is demonstrated through two analysis examples. In Section 5.1, the effectiveness of the model for transition between trapped air and continuous air with water absorption and drainage as described in Section 3 is demonstrated by simulation of a soil water retention test. In Section 5.2, the effectiveness of expressing the compressibility of trapped air as described in Section 4 is demonstrated by simulation of an unexhausted and undrained shear test with suction of 0 kPa.

Note that in the simulations, a uniform deformation field was assumed, and the responses of the constitutive model for the soil skeleton and soil water characteristic model were output. An elasto-plastic constitutive model, the SYS Cam-clay model (Asaoka et al. 2002), was used as the constitutive model for the soil skeleton, and the van Genuchten (1980) model was used as the soil water characteristic model. The new effective degree of saturation (S_e^1) was related to suction ($p^s = p^g - p^l$) using the van Genuchten (1980) soil water characteristic model.

5.1 Soil water retention test and numerical simulation

First, an overview of the soil water retention test is described. The soil specimen was non-plastic silt (DL clay). Suction was controlled by a microporous membrane (Nishimura, 2012). The experimental procedure was as follows. (i) A soil sample adjusted to a water content of 20% was compacted within a mold, and a cylindrical specimen (diameter 50 mm, height 100 mm) was produced with a void ratio of 1.18 and a degree of saturation of 46%. (ii) The specimen was placed in a triaxial test apparatus, and the cell pressure was increased to 20 kPa under exhausted and undrained conditions. (iii) The cell pressure and air pressure were increased simultaneously to 60 kPa and 40 kPa, respectively (at this time, the water pressure was about 20 kPa, and suction was about 20 kPa). (iv) After undrained condition was changed into drained condition without changing suction of the specimen, the cell pressure was increased to 240 kPa and

isotropic consolidation was carried out at a net stress of 200 kPa. (v) The water pressure was increased to 40 kPa, which reduced the suction from 20 kPa to 0 kPa (this process is referred to as the 0th time). (vi) The cell pressure, air pressure, and water pressure were increased simultaneously to 450 kPa, 250 kPa, and 250 kPa, respectively. This process corresponds to the back pressure rising process in triaxial tests of saturated soil, and it increases the degree of saturation. (vii) Under constant net stress, a test was performed in which the suction was increased in stages from 0 kPa in the sequence of 0→20→0→25→0→30→0 kPa under drained conditions, thereby repeating water absorption and drainage three times.

Figure 2 shows the experimental results for the relationship between the degree of saturation (S_r), void ratio (e), and suction (p^s) under procedures (v) to (vii) above. Note that the points for the experimental data were connected by a straight line. First, if the process of water absorption and drainage is repeated under the condition that suction is increased in each drainage stage after the back pressure rise, the degree of saturation at suction of 0 kPa gradually decreases, and the amount of trapped air increases. As a result, the hysteresis during water absorption and drainage can be observed. Next, looking at the relationship between the void ratio (e) and suction (p^s), large volumetric compression was seen due to wetting-induced collapse when the 0th time suction was changed from 20 kPa to 0 kPa, but thereafter the volumetric change was small.

Figure 3 shows the numerical simulation results. Since the volumetric change was small after the 0th time water absorption process, numerical simulation was performed under no deformation condition in order to validate only the soil water characteristic model. In other words, numerical simulation was performed with the start of the back pressure rising process after the 0th time water absorption process under no deformation condition. First, in the back pressure rising process, the increase in the degree of saturation (S_r) at $p^s = 0$ kPa can be expressed by the trapped air compression (increase of S_1^w). Next, when the process of water absorption and drainage is repeated while suction is increased in each drainage stage, it is possible to reproduce the decrease in the degree of saturation at $p^s = 0$ kPa, which means the amount of trapped air increases (reduction in S_1^w). Thus, the hysteresis in the relationship between the degree of saturation (S_r) and suction (p^s) can be expressed without introducing hysteresis into the relationship between the effective degree of saturation (S_e^1) and suction (p^s), by taking into consideration the transition between trapped air and continuous air.

In previous methods, the maximum degree of saturation of the soil water characteristic curve is given as a material constant at $p^s = 0$ kPa. Therefore, this variation in the degree of saturation cannot be expressed. Here, the amount of trapped air in the first water absorption and drainage process was smaller in the numerical simulation than that in the experiment. There is room for improvement of the proposed analysis method in the future, mainly in the simple transition models indicated in Eqs. 5 and 6.

5.2 Numerical simulation of an unexhausted and undrained shear test (Kodaka et al. 2006)

The unsaturated triaxial compression tests of Kodaka et al. (2006) were referred. Using non-plastic silt (DL clay), cylindrical triaxial test specimens were produced with an initial void ratio of 1.14, water content of 20%, and degree of saturation of 46.5% (at this time, suction was 20 kPa). After application of the predetermined suction and net stress of 200 kPa, shear tests were performed with a constant cell pressure. In this section, the unexhausted and undrained shear test with suction of 0 kPa was simulated.

Figure 4 shows a comparison of the experimental result and the numerical simulation results. First, in previous three-phase analysis methods, when suction is 0 kPa, the degree of saturation

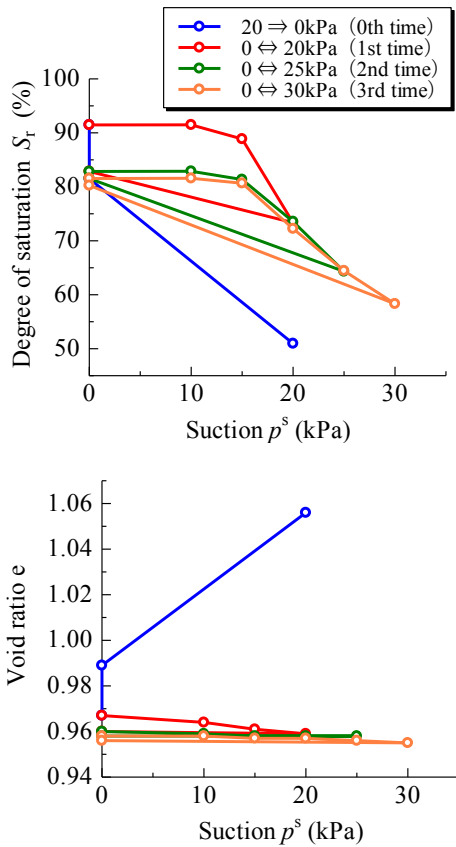


Figure 2. Experimental result of soil water retention test.

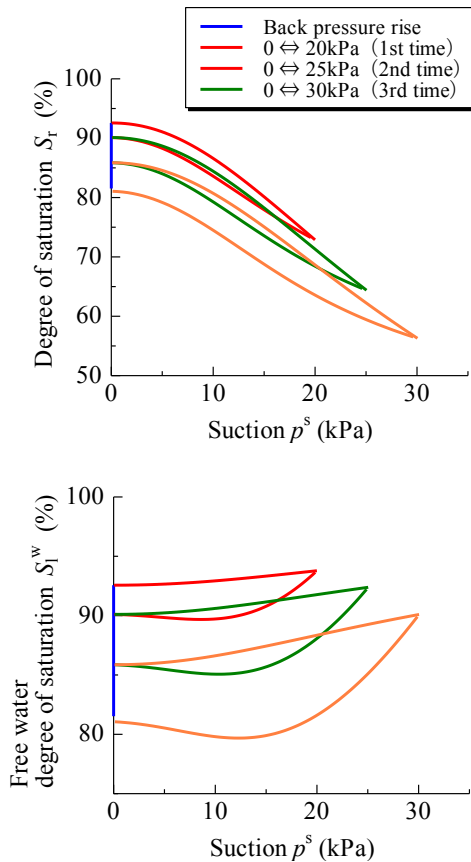


Figure 3. Simulation result of soil water retention test (expression of the hysteresis in the degree of saturation caused by the trapped air).

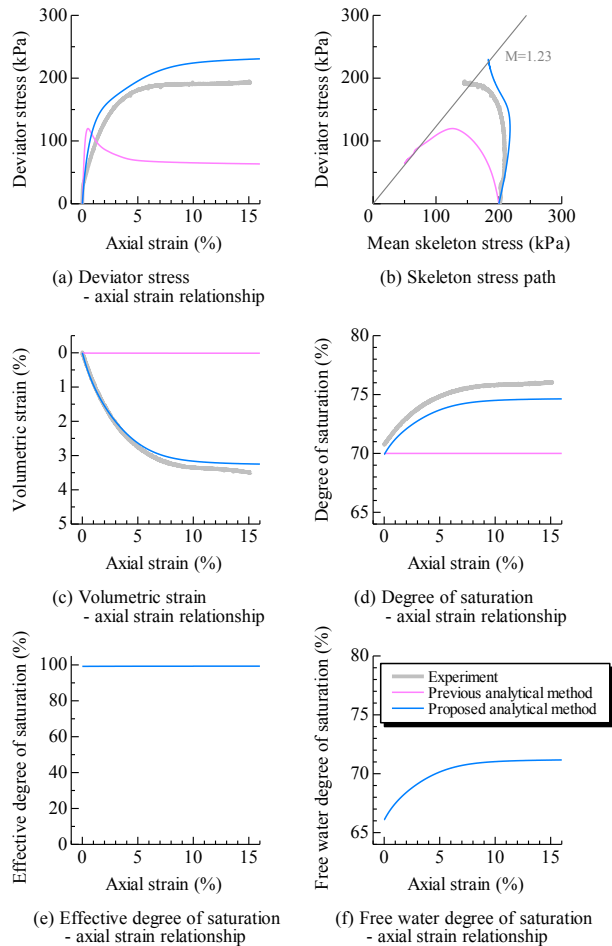


Figure 4. Comparison of unexhausted and undrained shear experimental results (Kodaka et al., 2006) and simulation results (expression of increase in the degree of saturation at suction of 0 kPa).

is the maximum degree of saturation of the soil water characteristic curve (here it is 70%), so it is not possible to represent an increase of the degree of saturation greater than this value. Therefore, there is no volumetric compression under exhausted and undrained conditions, resulting in constant volume shearing. On the other hand, when using proposed analysis method, it was possible to express volumetric compression behavior and increases in the degree of saturation similar to the experimental results. Since suction was 0 at this time, the effective degree of saturation did not change from 1, however the trapped air in the free water was compressed; therefore, it can be seen that it was possible to express the increase in the degree of saturation. Also, the deviator stress-axial strain relationship and skeleton stress path were reproduced well, and the deviator stresses differed by as much as a factor of two.

6 CONCLUSIONS

In this research, a new effective degree of saturation was defined by focusing on trapped air, and a soil skeleton-free water-trapped air-continuous air coupled finite deformation analysis method was proposed based on this effective degree of saturation. Its major characteristics are that pore air is clearly divided into trapped air and continuous air and pore water is divided into adsorbed water and free water. It was emphasized that at low suction it is possible to express the variation in the degree of saturation due to compression/expansion of the trapped air and that at high suction a complex model is not required because it is possible to define the amount of adsorbed water by the water

content. Also, a model is proposed for the transition of trapped air and continuous air. By comparing the results of a soil water retention test and the simulation result, it was found that the hysteresis in the degree of saturation (S_r)-suction (p^s) relationship due to the presence of trapped air could be expressed without introducing hysteresis into the effective degree of saturation (S_e^l)-suction (p^s) relationship. In addition, in the simulation of the unexhausted and undrained shear test at suction of 0 kPa, it was possible to reproduce well the experimental results due to the compression of trapped air. This indicates that in previous methods that could not take into consideration the compressibility of trapped air, shear strength was under-evaluated by about half. In this paper, simulation results were presented with a focus on the change in the degree of saturation at zero suction and the authors would like to emphasize again that previous methods that prescribed the maximum degree of saturation as a material constant cannot express this type of behavior.

7 ACKNOWLEDGEMENTS

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