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Method to estimate excess pore pressures from total stress seismic site response analyses

Méthode d'estimation des pressions interstitielles excédentaires à partir des analyses de la réponse du site sismique sous contrainte totale

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ABSTRACT: Equivalent linear seismic site response analyses are easier to use than nonlinear methods and accurate up to shear strains of about 0.4-1%, however, they cannot estimate excess pore pressures necessary for liquefaction analyses. The objective of this research is to develop and test a method to estimate excess pore pressures from the results of an equivalent linear seismic site response analysis. The proposed method is as follows: 1) perform an equivalent linear analysis to estimate the shear stress time series at each soil layer; 2) transform the irregular shear stress time series into a series of uniform cycle parcels with varying amplitudes using the rainflow method; 3) use the pore pressure accumulation method of Andersen et al. (1994) to estimate total excess pore pressures generated due to the uniform cycle parcels; 4) update the initial small strain shear stiffness in each layer to account for the reduction in the effective stress due to the estimated excess pore pressures; 5) iterate over steps 1 through 4 until the final estimated excess pore pressures from two consecutive analyses are within a set tolerance level. The new method in general predicts similar excess pore pressure ratios (r_u) as commonly used nonlinear effective stress analyses, however, more work is required to validate it against case histories.

RÉSUMÉ : Les analyses de réponse de site sismique linéaire équivalente sont plus faciles à utiliser que les méthodes non linéaires et précises jusqu'à des déformations de cisaillement d'environ 0,4 à 1%, cependant, elles ne peuvent pas estimer les pressions interstitielles excessives. L'objectif de cette recherche est de développer et de tester une méthode pour estimer les pressions interstitielles excédentaires à partir des résultats d'une analyse de réponse de site sismique linéaire équivalente. La méthode proposée est la suivante: 1) effectuer une analyse linéaire équivalente pour estimer la série chronologique des contraintes de cisaillement à chaque couche de sol; 2) transformer la série chronologique des contraintes de cisaillement irrégulières en une série de parcelles à cycle uniforme avec des amplitudes variables en utilisant la méthode de l'écoulement de la pluie; 3) utiliser la méthode d'accumulation de pression interstitielle d'Andersen et al. (1994) pour estimer les surpressions interstitielles totales générées par les parcelles à cycle uniforme; 4) mettre à jour la rigidité de cisaillement à petite déformation initiale dans chaque couche pour tenir compte de la réduction de la contrainte effective due aux surpressions interstitielles estimées; 5) répétez les étapes 1 à 4 jusqu'à ce que les pressions interstitielles en excès estimées finales de deux analyses consécutives se situent dans un niveau de tolérance défini. La nouvelle méthode prédit en général des rapports de pression interstitielle (r_u) similaires à ceux des analyses de contraintes efficaces non linéaires couramment utilisées, cependant, plus de travail est nécessaire pour la valider par rapport aux antécédents de cas.

KEYWORDS: Seismic site response; rainflow counting; cyclic contour diagrams; liquefaction; cyclic softening.

1 INTRODUCTION

Seismic site response analyses estimate the effect that near surface soils have on the intensity, duration, and frequency content of earthquake ground motions. They are important to provide accurate input parameters to assess the seismic performance of foundations, tunnels, retaining walls, slopes and other infrastructure.

The most frequent approaches to seismic site response analysis are one dimensional (1D) equivalent linear analyses (ELA) solved in the frequency domain and 1D nonlinear analyses (NLA) solved in the time domain. Equivalent linear analyses are performed more often than nonlinear analyses (Matasovic and Hashash 2012) because of their ease of use, robustness, low computational requirements and accuracy up to shear strains of 0.04% to 1% (Carlton and Tokimatsu, 2016). However, one of the main limitations of equivalent linear analyses is that they are based on a total stress representation of soil behavior and cannot predict excess pore pressure generation. Cyclic shearing of fully saturated soils causes plastic deformations due to the progressive collapse of the soil skeleton. As the soil skeleton collapses residual excess pore water pressures are generated, which decrease the effective stress. Because the stiffness and strength of soils are dependent on the effective confining pressure, as the effective stress decreases the stiffness and strength also decrease. As a result, the generation and redistribution of excess pore water pressure within a soil deposit can significantly affect the seismic response of a site (Matasovic and Vucetic, 1993).

The objective of this study was to develop a method to estimate excess pore pressures from total stress equivalent linear analyses. The following section describes the methodology. We then compare the excess pore pressures estimated from the proposed methodology with those estimated using traditional 1D nonlinear effective stress analyses. We finish the paper with a discussion of the potential applications and limitations of the methodology.

2 PROPOSED NEW METHODOLOGY

2.1 *Overview*

We developed the following work flow in the Python programming language to estimate excess pore pressures from equivalent linear seismic site response analyses:

1. Perform an equivalent linear total stress analysis in the frequency domain to estimate the shear stress time series at each soil layer.
2. Transform the irregular shear stress time series at each layer into a series of uniform cycles (load parcels) with

varying amplitudes using the rainflow method (Matsuishi and Endo, 1968).

3. Calculate the excess pore pressures in each layer at the end of shaking from the uniform shear stress cycles found in step 2 using the pore pressure accumulation procedure of Andersen et al. (1994).
4. Update the initial small strain shear stiffness in each layer to account for the reduction in the effective stress due to the estimated final excess pore pressures.
5. Iterate over steps 1-4 until the final estimated pore pressures from two consecutive analyses are within a set tolerance level in each layer.

The following sections describe each of the steps above in more detail.

2.2 *Equivalent linear analyses*

The first step in the proposed methodology is to perform equivalent linear analyses in the frequency domain. To conduct the equivalent linear analyses, we used a modified version of the program SHAKE (Schnabel et al., 1972).

Seismic site response analyses calculate the shear stress, strain, and acceleration time series at each soil layer for a given input acceleration time series and soil profile. Linear models computed in the frequency domain take an acceleration time series in the time domain and convert it to the frequency domain using a Fast Fourier Transform (FFT). The Fourier series is then multiplied by a transfer function that determines how each frequency in the input motion is either amplified or deamplified to produce the Fourier series of the output motion. The Fourier series of the output motion is then transformed back to the time domain using the inverse FFT. Transfer functions are solutions to the 1-D wave equation of a vertically propagating horizontal shear wave. They are dependent on frequency and the stiffness, damping, and density properties of the soil profile (Kramer 1996).

The equivalent linear method is essentially a series of linear analyses where the stiffness and damping are adjusted after each computation. The response of the soil profile is first calculated using the small strain stiffness and damping in a linear analysis. From this initial estimate, shear strain time series for each layer are computed. Then, for each layer, the effective shear strain is calculated as some fraction of the maximum shear strain, usually 0.65 (Carlton, 2016). The values of stiffness and damping at the effective shear strain are then determined from shear modulus reduction and damping curves (e.g. Darendeli, 2001). Shear modulus reduction and damping curves are approximations of the hysteretic stress-strain behavior of soils. They model the strain dependence of the shear modulus and damping. The shear modulus is approximated using the secant shear modulus, and the damping ratio as the equivalent damping from one hysteresis cycle. The process is repeated until the difference between the stiffness and damping properties in two consecutive iterations falls below a set tolerance level.

2.3 *Rainflow counting*

Once the equivalent linear analysis has converged, we use the rainflow counting method (Matsuishi and Endo, 1968) to convert the stress time series at each layer into a series of uniform cycle parcels.

The rainflow counting method creates uniform cycle parcels by considering successive ranges in a time series. If the absolute value of the next range in the time series is larger than the current range, the current range is counted as one cycle with the amplitude equal to the range. The two points comprising the current range are then removed and the process starts over at the beginning of the time series. If the current range includes the earliest time point remaining in the time series, it is counted as one half cycle and only the earliest time point is removed. If the end of the time series is reached, all the remaining ranges are counted as half cycles (ASTM E 1049-85, 1997). In this way, the rainflow method extracts successively larger and larger cycles.

The rainflow method is a common method to convert broad-banded earthquake motions into a series of uniform cycle parcels because it counts both low and high frequency cycles. However, the methodology described in this paper is not dependent on the rainflow counting method, and other cycle counting methods such as mean-crossing, level-crossing and peak counting could be used. Stelzer et al. (2020) show that the rainflow and mean-crossing methods generally give similar results, but that the level-crossing and peak counting methods predict a larger equivalent number of cycles. Therefore, it is expected that the cycle counting method used could have an effect on the excess pore pressures predicted. However, the effect of cycle counting method was not investigated in this study.

2.4 *Pore pressure accumulation*

After the stress time series in each layer is converted to a series of uniform cycle parcels, we applied the pore pressure accumulation procedure of Andersen et al. (1994) to estimate the excess pore pressure in each layer at the end of shaking.

An essential component of the pore pressure accumulation procedure is the cyclic contour diagram (Andersen et al., 1988). Cyclic contour diagrams are visual representations of the relation between average stress and cyclic stress with number of cycles, shear strain, or pore pressure. They are derived by interpolating and extrapolating data from cyclic laboratory tests with different combinations of average and cyclic stress to provide information at other stress conditions than those tested. The cyclic contour diagram approach has been applied to offshore foundation design to account for cyclic loading due to wind and waves for many years (e.g. Andersen et al., 1988). However, it has been less used for earthquake design. In the present method we assume zero average stress, which is consistent for earthquake loading on level ground.

Figure 1 shows cyclic contour diagrams for permanent excess pore pressure ratio (r_u) based on DSS tests with zero average stress. The pore pressure accumulation procedure developed by Andersen et al. (1994) first sorts the cycle parcels from smallest cyclic shear stress to largest. Then, the cyclic contour diagram for the given soil is used to estimate the excess pore pressure generated due to the number of cycles and cyclic shear stress applied from the first parcel. Next, the pore pressure contour reached at the end of the first cycle parcel is followed up to the cyclic stress of the next cycle parcel. Starting from this pore pressure contour, the next cycle parcel is applied. This procedure is repeated until all cycle parcels have been applied. Figure 1 shows an example calculation with 70, 18, and 7 cycles at normalized cyclic shear stress values of 0.05, 0.085 and 0.15, respectively. The pore pressure accumulation procedure predicts that this sequence of cycle parcels will result in $r_u = 0.25$.

An important assumption of the pore pressure accumulation procedure is that the excess pore pressures at the start of one cycle parcel are the same as at the end of the previous cycle parcel. This assumption may not be valid for very dense or very loose sands because when the shear stress is increased from one cycle parcel to the next they will tend to dilate or contract, respectively, resulting in a change of pore pressure with zero additional cycles (Jostad et al., 1997). This represents a limitation of the current methodology.

Two other important assumptions of the pore pressure accumulation procedure are that each cycle parcel is fully undrained and applied at a constant rate (Andersen, 2015). The fully undrained assumption is appropriate for earthquake analyses. On the other hand, cycle counting methods such as the rainflow method lump cycles according to amplitude and ignore frequency. Therefore, one cycle parcel could represent cycles with many different frequencies. However, high frequency cycles generally induce low shear strain and low frequency cycles high strain (Assimaki and Kausel, 2004). Therefore, this limitation most likely has only a small effect on the predicted pore pressures.

2.5 *Effect of pore pressure on small strain stiffness*

Initial analyses showed that the pore pressure accumulation methodology consistently over predicted the excess pore pressures compared with traditional nonlinear effective stress site response analyses. Therefore, we implemented an equivalent linear iterative scheme for the excess pore pressure, similar to the equivalent linear approach for shear strain. After the equivalent linear analysis had converged, we updated the initial small strain shear modulus in each layer to account for the reduction in the effective stress due to the estimated excess pore pressures at the end of shaking. We then repeated the analyses until pore pressures from two consecutive runs were within a set tolerance level in each layer.

To update the small strain shear modulus (G_{max}), we modified the approach adopted by Finn et al. (1977), Matasovic and Vucetic (1993) and implemented in the 1D site response analysis code DeepSoil (Hashash et al., 2020) to:

$$G_{max,i+1} = G_{max,1} * \sqrt{1 - r_{u,i} * 0.65} \quad (1)$$

where $G_{max,i+1}$ is the G_{max} value for the next iteration, $G_{max,1}$ is the initial G_{max} value, and $r_{u,i}$ is the estimated excess pore pressure ratio for the current analysis. This approach is based on the finding that the small strain shear modulus is dependent on the square root of the mean effective stress (e.g. Hardin and Drnevich, 1972; Carlton and Pestana, 2016). Our modification is the 0.65 factor on $r_{u,i}$. This is because unlike effective stress analyses where the stiffness is updated at each time step, the methodology proposed in this paper uses constant stiffness values for the entire acceleration time series. Therefore, similar to the effective strain concept in equivalent linear analyses, we use an effective pore pressure value that balances the influence of pore pressure build up over the entire acceleration time series. We chose 0.65 to estimate the effective pore pressure value because this is a common value used for equivalent linear analyses (Carlton, 2014).

3 COMPARISON WITH TRADITIONAL EFFECTIVE STRESS ANALYSES

3.1 *Model setup*

To test the methodology described above, we performed analyses for 16 acceleration time series and one soil profile. We then compared the results from the method proposed in this paper to traditional nonlinear (NLA) effective stress analyses in the program DeepSoil (Hashash et al., 2020). The following sections describe the selected acceleration time series, soil profiles and setup of the site response analyses.

3.1.1 *Acceleration time series*

We selected both horizontal components of eight ground motions from the PEER NGA West 2 database (Ancheta et al., 2014) for a total of 16 acceleration time series. Table 1 lists the peak ground acceleration (PGA), peak ground velocity (PGV), mean period (Tm), duration between when 5% and 95% ($D_{5\text{-}95}$) of the Arias intensity is reached, and the Arias intensity (Ia). The ID consists of the RSN number given in the PEER NGA West 2 database followed by an X and the azimuth of the recording in degrees to differentiate between the two horizontal components. We

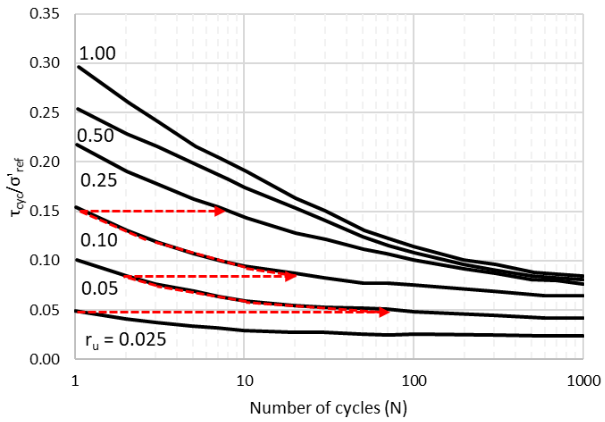

Figure 1 Permanent excess pore pressure ratio (r_u) predicted from DSS tests as a function of normalized cyclic shear stress and number of cycles with zero average stress (level ground conditions).

Table 1. Ground motion parameters for selected acceleration time series

ID	PGA (g)	PGV (cm/s)	Tm(s)	$D_{5\text{-}95}$ (s)	Ia (m/s)
0028X050	0.060	6.618	0.74	29.0	0.056
0028X320	0.063	7.344	0.87	26.7	0.063
0070X021	0.151	18.188	0.74	15.7	0.261
0070X111	0.114	14.796	0.60	18.4	0.200
0290X000	0.108	7.136	0.49	26.7	0.279
0290X270	0.140	11.834	0.48	25.1	0.468
0335X000	0.074	16.324	1.18	21.6	0.178
0335X090	0.132	16.343	0.99	16.6	0.219
0405X270	0.255	15.490	0.43	9.7	0.697
0405X360	0.336	17.453	0.36	6.2	0.898
0694X092	0.215	11.945	0.38	10.5	0.353
0694X182	0.236	12.238	0.36	8.3	0.456
0811X000	0.373	27.407	0.28	10.5	3.704
0811X090	0.654	38.162	0.27	11.0	6.269
1035X000	0.156	18.897	0.61	20.4	0.209
1035X090	0.123	11.289	0.60	18.9	0.184

selected acceleration time series with a range of Arias intensity and duration to test the robustness of the proposed methodology. We also selected acceleration time series with no directivity pulse and with Vs_{30} values between 350 m/s to 450 m/s. We used the input motions as recorded with no deconvolution and an elastic half-space underlying the soil profile with Vs = 400 m/s, unit weight of 20 kN/m^3, and damping ratio of 1%. The moment magnitudes and rupture distances of the selected acceleration time series vary between 5.77 and 6.93, and 11.5 and 40 km, respectively.

3.1.2 *Soil profile*

We modelled one sand profile 50 meters thick with a uniform unit weight of 19.1 kN/m^3, relative density Dr = 60%, fines content FC = 5-10%, and shear wave velocity profile according to Carlton and Tokimatsu (2014) for a generic NEHRP E site (Figure 2). According to Anderson (2015), a sand with Dr = 60% and FC = 5-10% has a DSS cyclic shear strength ratio equal to about τ_f/σ'_{ref} = 0.19, where τ_f is the cyclic shear strength, $\sigma'_{ref} = p_a*(\sigma'_v/p_a)^{0.9}$, σ'_v is the vertical effective stress and p_a is atmospheric pressure. Based on this, we calculated the cyclic strength profile as shown in Figure 2. We then used the pore pressure ratio contour diagram in Andersen (2015) for sand with τ_f/σ'_{ref} = 0.19 (Figure 1) for the proposed new methodology outlined in section 2. For the shear modulus reduction and damping curves, we used the model of Darendeli (2001) with plasticity index = 0. We set the tolerance level for the equivalent linear analyses and the pore pressure iterations as five percent.

For the nonlinear analyses in DeepSoil, we used the model of Groholoski et al. (2016) to define the backbone curve and ensure that the shear stress at large shear strains matched the selected cyclic shear strength of the soil, and we used the frequency independent damping formulation proposed by Phillips and Hashash (2009). We matched the model of Groholoski et al. (2016) to the shear modulus reduction and damping curves given by Darendeli (2001) for plasticity index = 0 to be consistent with the equivalent linear analyses. We adjusted the thickness of the soil layers so that the maximum frequency propagated through the site was 25 Hz. We used the same profile layering for both the nonlinear and equivalent linear analyses.

To model pore pressure generation in the nonlinear analyses we used the effective stress models of Green et al. (2000) and Park et al. (2015). The Green et al. (2000) model (GMP) relates the residual excess pore pressure to the amount of energy dissipated per unit volume of soil. The Park et al. (2015) model (PEA) relates the residual excess pore pressure to the cyclic shear stress through the concept of an incremental damage parameter. We calibrated both models using the methodologies described in their respective papers with the same cyclic DSS tests used to develop the cyclic contour diagram shown in Figure 1. The selected input values for the Green et al. (2000) model in DeepSoil are PEC = 3.58 and α = 1, where PEC is the pseudo-energy capacity and α is a scale factor introduce by DeepSoil. The value of PEC estimated from the DSS tests matches well with the value calculated from the correlations with Dr and FC proposed by Polito et al. (2008). The selected input values for the Park et al. (2015) model in DeepSoil are $CSR_t = 0$, $D_{ru=1} = 0.007$, $\alpha = 5.191$ and $\beta = 0.75$, where CSR_t is the minimum shear stress ratio that generates pore pressures, $D_{ru=1}$ is the value of the damage parameter when $r_u = 1$, and α and β are curve fitting parameters. For both models, we set the degradation parameter in DeepSoil to ν = 1. The degradation parameter controls the rate of reduction of the shear strength due to the generated excess pore pressure.

3.2 *Comparison*

Figure 3 shows the peak ground acceleration (PGA), maximum shear strain (γ), maximum shear stress ratio (τ/σ'_v) and r_u with depth for acceleration time series 0070X111 for all three analysis methods. All three methods predict very similar values of r_u below 10 meters. However, at shallower depths the Park et al. (2015) model (PEA) starts to deviate from the other two, predicting a smooth increase in r_u as the depth decreases, whereas the new method (NEW) and the Green et al. (2000) model (GMP)

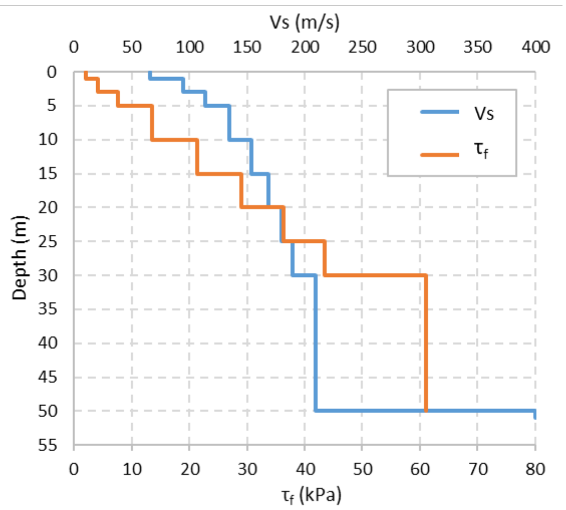

Figure 2 Shear wave velocity and shear strength profile with depth

predict spikes in r_u at depths of 10, 5, 3 and 1 meter. The new method and PEA model predict the largest r_u at one meter depth, whereas the GMP model predicts the largest r_u at three meters depth.

The PEA model predicts a smooth increase in r_u as depth decreases because it is based on the shear stress ratio, which increases smoothly as depth decreases. The GMP model, on the other hand, is based on stress and strain (energy), and therefore predicts spikes in r_u where there are spikes in the shear strain. These spikes occur at layers above velocity changes (see Figure 2), where the impedance contrast causes amplification of the ground motion. For this reason, the GMP model predicts the maximum r_u at three meters depth, where the shear strain is largest, and the PEA model at one meter depth where the shear stress ratio is largest. The maximum shear stress and strain profiles for both models are almost exactly the same, which shows that the difference in the predicted r_u is based on the model formulation. Surprisingly, the new method predicts values of r_u closer to the GMP model than the PEA model. This is surprising because the new method is also stress based, similar to the PEA model.

Figure 4 and Figure 5 compare the predicted r_u with depth between the new proposed method and the GMP and PEA models for all 16 acceleration time series. Similar trends as seen in Figure 3 for ground motion 0070X111 are seen for the other ground motions as well.

Figure 6 shows the difference between r_u with depth for each of the three models averaged over all 16 ground motions. The average differences in predicted r_u between the new method and the GMP model (NEW – GMP) and the new method and the PEA model (NEW – PEA) are similar to the differences between the GMP and PEA models (GMP- PEA). This indicates that the new method predicts r_u within the range of uncertainty of the other two methods. The average differences in r_u between the new method and the GMP model are generally between ± 0.10 and

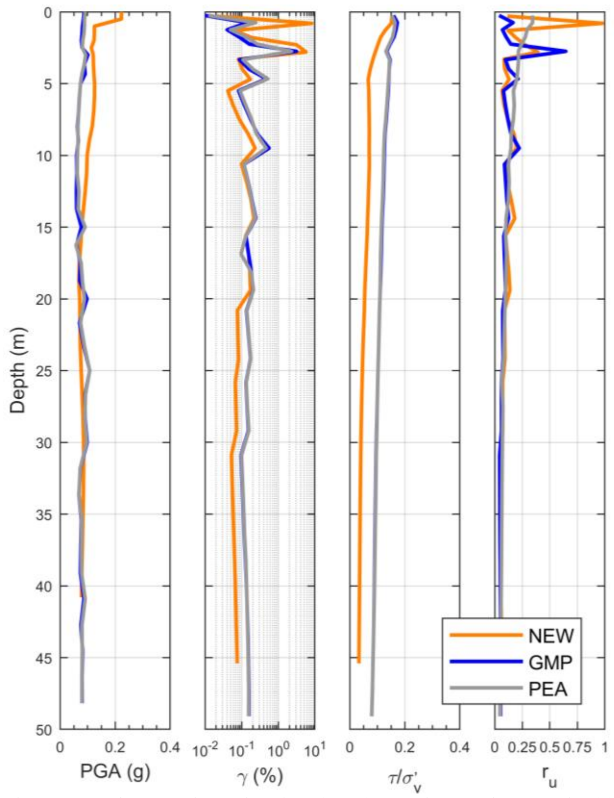

Figure 3 Peak ground acceleration (PGA), maximum shear strain (γ), maximum shear stress ratio (τ/σ'_v), and excess pore pressure ratio (r_u) with depth for 0070X111.

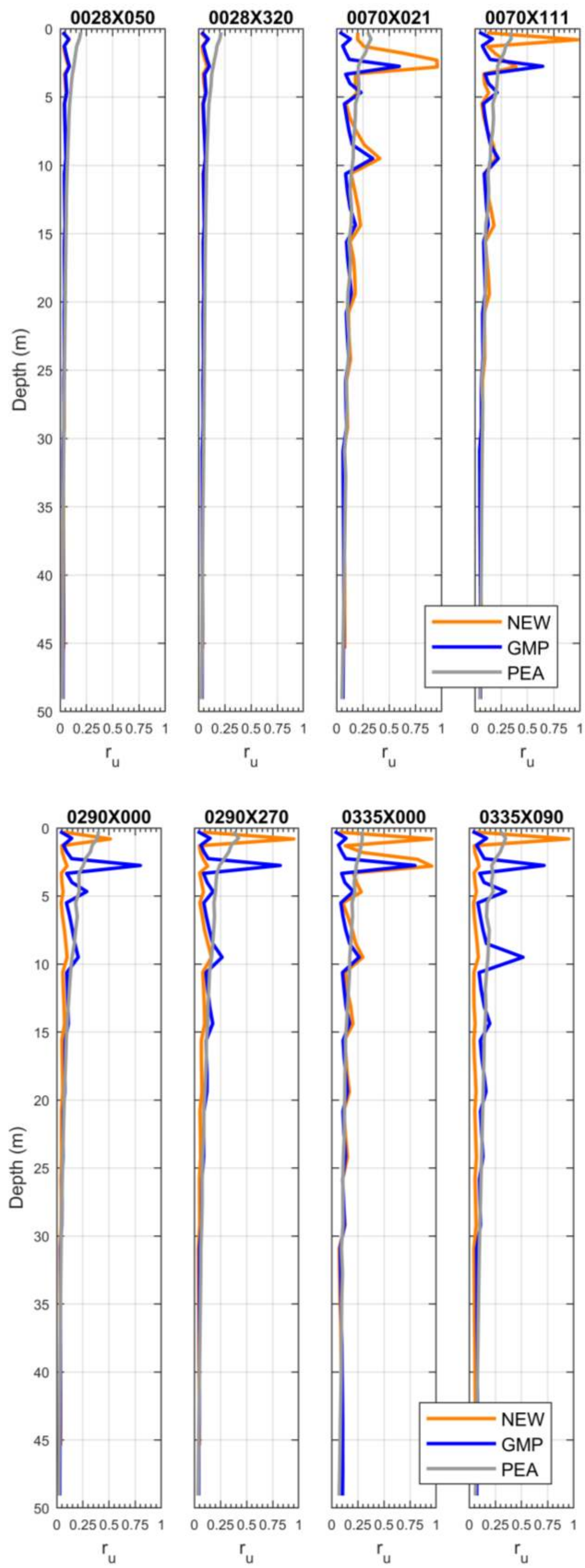

Figure 4. Comparison of the predicted final r_u with depth for the new proposed method (NEW), and the Green et al. (2000) (GMP) and Park et al. (2015) (PEA) methods in DeepSoil for the first 8 ground motions

centered around zero, which shows no bias in the predicted r_u values from the new method compared with the GMP model. The average differences between the new method and the PEA model are also centered around zero for depths deeper than seven meters.

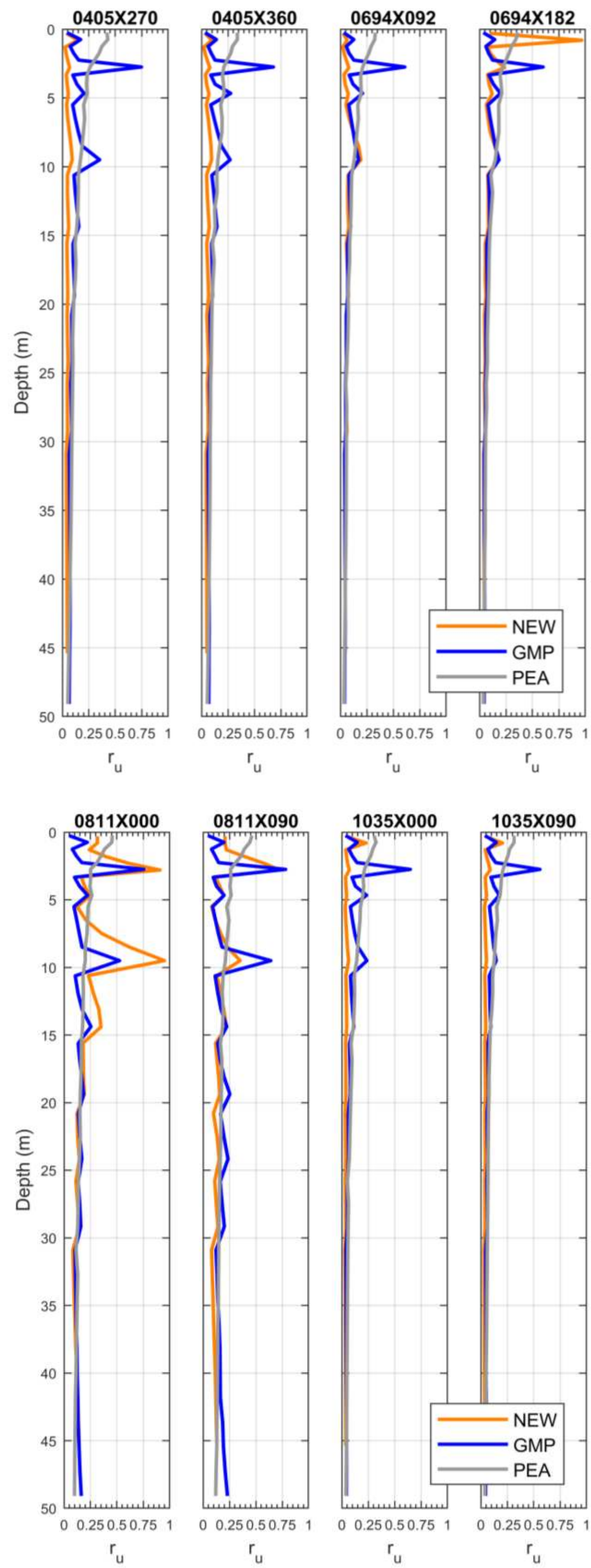

Figure 5. Comparison of the predicted final r_u with depth for the new proposed method (NEW), and the Green et al. (2000) (GMP) and Park et al. (2015) (PEA) methods in DeepSoil for the second 8 ground motions

For depths between zero to seven meters, the PEA model predicts larger values of r_u than both the new method and the GMP model, except at depths of one and three meters, where the new method and GMP model predict spikes in the value of r_u.

4 DISCUSSION

The results of the comparison show that the proposed method in general predicts similar excess pore pressure ratios as traditional effective stress analyses. However, the results are based on only 16 ground motions and one uniform sand site with increasing velocity with depth. Sites with velocity inversions or interbedded silt and clay layers could predict different results.

The computational time for the proposed method is not significantly faster than performing a 1D effective stress NLA in DeepSoil. Therefore, we see the main benefit of the proposed method for large 2D and 3D analyses, where time domain analyses can be much more computationally expensive than frequency domain equivalent linear analyses.

The main application of the proposed methodology is liquefaction analysis, however, the methodology could be extended to estimate cyclic degradation of fine grained soils due to earthquake shaking using the same principles.

5 CONCLUSION

We propose a novel new method to estimate excess pore pressures from equivalent linear site response analyses. The method takes advantage of knowledge and techniques commonly used in design of offshore foundations due to wind and wave loads. We describe the new methodology, compare it to results from traditional effective stress analyses for 16 acceleration time series and one hypothetical sand site, and discuss its potential applications and limitations. The results show that the new method is able to capture similar trends in r_u with depth as effective stress analyses, however, more work is required to validate it against case histories.

6 ACKNOWLEDGEMENTS

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7 REFERENCES

Ancheta, T. D., Darragh, R. B., Stewart, J. P., Seyhan, E., Silva,W. J., Chiou, B. S. J., Wooddell, K. E., Graves, R. W., Kottke, A. R., Boore, D. M., Kishida, T., and Donahue, J.L., 2014. NGA-West2 database, *Earthquake Spectra* 30, 989–1005.

Andersen, K.H. 2015. The Third ISSMGE McClelland Lecture: Cyclic soil parameters for offshore foundation design. *Frontiers in Offshore Geotechnics III* – Meyer V. (Ed.). Taylor & Francis.

Andersen, K.H., Allard, M.A. & Hermstad, J. 1994. Centrifuge model tests of a gravity platform on very dense sand; II: Interpretation. 7th *BOSS*. Cambridge, Mass. Proc. (1): 255–282.

Andersen, K.H., Kleven, A. & Heien, D. 1988. Cyclic soil data for design of gravity structures. *ASCE, J. of Geotech. Engrg*, 114 (5): 517–539.

Assimaki, D., Kausel, E. 2004. An equivalent linear algorithm with frequency- and pressure-dependent moduli and damping for the seismic analysis of deep sites. *Soil Dyn. Earthq. Eng.,* 22, 959-965.

ASTM 1997. Standard practices for cycle counting in fatigue analysis. Standard E 1049 – 85 (Reapproved 1997), *American Society for Testing and Materials*, West Conshohocken, PA, USA.

Carlton, B.D. 2016. Ground Motion Parameter Sensitivity to Shear Strain Ratio in Equivalent Linear Analyses. *Eqk. Spec.*, 32(3), 1889-1901.

Carlton, B.D., and Pestana, J. 2016. A unified model for estimating the in-situ small strain shear modulus of clays, silts, sands, and gravels. *Soil Dynamics and Earthquake Engineering*, 88, p. 345-355.

Carlton, B.D., and Tokimatsu, K., 2016. Comparison of Equivalent Linear and Nonlinear Site Response Analysis Results and Model to Estimate Maximum Shear Strain. *Eqk. Spectra*, 32(3), 1867-1887.

Darendeli, M. B., 2001. Development of a New Family of Normalized Modulus Reduction and Material Damping Curves, Ph.D. dissertation, University of Texas, Austin.

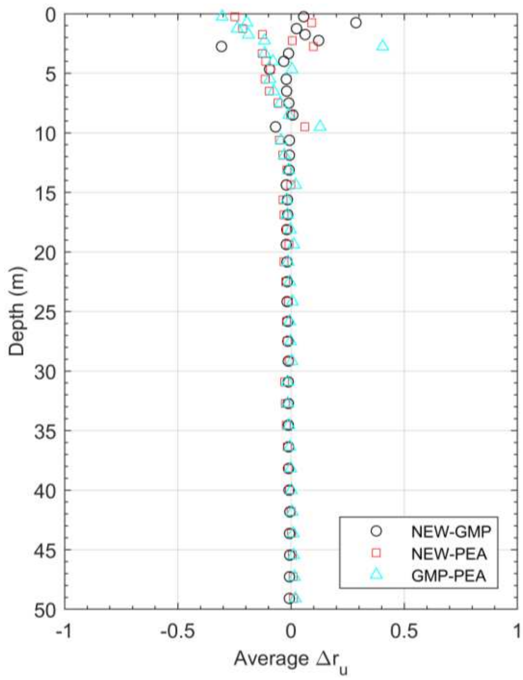

Figure 6. Average difference in r_u between each of the three models

Finn, W. D. L., Lee, K. L, and Martin, G. R. 1977. An effective stress model for liquefaction. *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 103, No. GT6, 517-533.

Green, R.A., Mitchell, J.K. and Polito, C.P. 2000. An Energy-Based Pore Pressure Generation Model for Cohesionless Soils. Proceedings: *John Booker Memorial Symposium*, Melbourne, Australia, November 16-17.

Groholski, D., Hashash, Y., Kim, B., Musgrove, M., Harmon, J., and Stewart, J. 2016. Simplified Model for Small-Strain Nonlinearity and Strength in 1D Seismic Site Response Analysis. *J. Geotech. Geoenviron. Eng.*, 142(9).

Hardin, B. O. and Drnevich, V. P. 1972. Shear modulus and damping in soils: Design equations and curves. *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 98, No. SM7, 667-692.

Hashash, Y.M.A., Musgrove, M.I., Harmon, J.A., Ilhan, O., Xing, G., Groholski, D.R., Phillips, C.A., and Park, D. 2020. DEEPSOIL 7.0, User Manual. Urbana, IL, Board of Trustees of UIUC.

Jostad, H.P., Andersen, K.H. & Tjelta, T.I. 1997. Analyses of skirted foundations and anchors in sand subjected to cyclic loading. Proc., *BOSS*, 8. Delft, The Netherlands, (1): 49–162.

Kramer, Steven L. 1996. Geotechnical Earthquake Engineering. Prentice Hall; New Jersey.

Matasovic, N., and Hashash, Y., 2012. NCHRP428: Practices and Procedures for Site-Specific Evaluations of Earthquake Ground Motions (A Synthesis of Highway Practice), *National Cooperative Highway Research Program*, *TRB,* Washington, D.C., 78 pp.

Matasovic, Neven, and M. Vucetic 1993. Cyclic Characterization of Liquefiable Sands. *Journal of Geo. and Geoenvironmental Engineering, ASCE*, 119(11), 1805-1822.

Matsuishi, M., and Endo, T. 1968. Fatigue of Metals Subjected to Varying Stress. Paper presented to Japan Society of Mechanical Engineers, Fukuoka, Japan, March (in Japanese).

Park, T., Park, D., Ahn, J.K. 2015. Pore pressure model based on accumulated stress. *Bull Earthquake Eng*., 13: 1913–1926.

Phillips C, Hashash YMA 2009. Damping formulation for nonlinear 1D site response analyses. *Soil Dyn. Earthq. Eng.*, 29, 1143–58.

Polito, C. P., Green, R. A., and Lee, J. H. 2008. Pore pressure generation models for sands and silty soils subjected to cyclic loading. *Journal of Geo. and Geoenvironmental Engineering*, 134(10), 1490-1500.

Schnabel, P.B, J. Lysmer and H.B. Seed 1972. SHAKE: A computer program for earthquake response analysis of horizontally layered site. Report no. UCB/EERC-72/12, Earthquake Engrg. Research Center, Univ. of Calif., Berkeley, California, 102p

Stelzer, R., Carlton, B.D., Mazzoni, S. 2020. Comparison of cycle counting methods for potential liquefaction or structural fatigue assessment. Proceedings 17th *WCEE*, Sendai, Japan, 13-18.