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Calibration of a cyclic constitutive model to Bay of Campeche carbonate sand

Calibration d'un modèle constitutif cyclique sur le sable carbonaté de la Baie de Campeche

Ernesto Vega Fernández & Francisco A. Flores López
Research Department, IGM Ingenieros Geotecnistas Mexicanos, Mexico, ernesto.veg@igmmexico.com

Victor M. Taboada
Marine Geotechnical Manager Americas, Fugro, USA

Diego Cruz Roque & Prócoro Barrera Nabor
Engineering Department, Instituto Mexicano del Petróleo, Mexico

ABSTRACT: A nonlinear constitutive model identified as MRDF-UIUC of cyclic stress-strain behavior available in the DEEPSOIL software (Hashash, et al., 2016) developed at the University of Illinois at Urbana-Champaign (UIUC) by Phillips and Hashash (2009) was calibrated with results from 53 isotropically consolidated resonant column tests and 27 strain-controlled cyclic direct simple shear tests of siliceous carbonate sand (50 % to 90 %) and carbonate sand (90 % to 100 %) specimens obtained from the Bay of Campeche. The computed normalized shear modulus (G/G_{max}) values larger than 0.2 and the material damping ratio values larger than 5 % fall within ± 30 % of the laboratory measurements. Overprediction was observed when the values of normalized shear modulus were smaller than 0.2 and the material damping ratio values were smaller than 5 %. The calibrated model is limited to effective confining stress between 15 kPa and 900 kPa and can be used to perform seismic site response analyses with the DEEPSOIL software and predict normalized shear modulus curves and material damping ratio curves of Bay of Campeche carbonate sand when laboratory measurements are not available.

RÉSUMÉ : Un modèle constitutif non linéaire identifié comme MRDF-UIUC du comportement cyclique contrainte-déformation disponible dans le logiciel DEEPSOIL (Hashash, et al, 2016) développé à l'Université de l'Illinois à Urbana-Champaign (UIUC) par Phillips et Hashash (2009) a été calibré avec les résultats de 53 essais sur colonne résonante isotropiquement consolidée et 27 essais de cisaillement simple direct cyclique à contrainte contrôlée sur des échantillons de sable carbonaté siliceux (50% à 90%) et de sable carbonaté (90% à 100%) obtenus dans la baie de Campeche. Les valeurs calculées du module de cisaillement normalisé (G/G_{max}) supérieures à 0,2 et les valeurs du rapport d'amortissement du matériau supérieures à 5% se situent à ± 30% des mesures de laboratoire. Une surestimation a été observée lorsque les valeurs du module de cisaillement normalisé étaient inférieures à 0,2 et que les valeurs du taux d'amortissement du matériau étaient inférieures à 5%. Le modèle calibré est limité à une contrainte de confinement efficace entre 15 kPa et 900 kPa, et peut être utilisé pour effectuer des analyses de réponse des sites sismiques avec le logiciel DEEPSOIL et prédire les courbes de module de cisaillement normalisé et les courbes de rapport d'amortissement des matériaux du sable carbonaté de la baie de Campeche lorsque les mesures de laboratoire ne sont pas disponibles.

KEYWORDS: Cyclic constitutive model, static parameters, stress-strain behavior, carbonate sand, DEEPSOIL software

1 INTRODUCTION AND OBJECTIVE

It is estimated (Lee, 1982) that approximately 48 % of the world's seabed is covered by calcareous ooze (containing more than 30 % of carbonate content). Since calcareous materials are predominately produced by living organisms, primary deposition can occur only in locations where water conditions favor calcium-producing marine organisms. These conditions are determined mainly by salinity and temperature. Carbonate deposits in marine environment are formed by the settlement of calcium-rich skeletons of marine organisms. Present-day deposition occurs predominantly in waters that are warmer than 18 °C throughout the year. This zone generally lies between 30° N and 30° S latitude. However, this zone cannot be considered a precise determinant for the occurrence of carbonate soils. Because the conditions of temperature, sea level, and salinity have changed through the geologic time, old deposits of carbonate soils can be found buried under more recent soils that are outside zones of probable current active deposition. Carbonate sand has been detected in most of the Bay of Campeche, mainly in the areas that make up the Cantarell and Kumaza fields.

The carbonate sands have high shear strength and may also have high compressibility that are attributed to the low resistance to grain crushing, which consequently can lead to further degradation of stiffness. There are few available studies focused on the measurement in the laboratory of the dynamic behavior of marine carbonate sands. Carraro and Bortolotto (2015) as well as Senetakis and Ranjith (2017) show the behavior of carbonate sands from Australia based on a few resonant column tests at low shear strains.

To cover this need, a database was established and tailored for a nonlinear characterization of the cyclic response of sandy soil units in the Bay of Campeche. The data were collected from Flores López, et al., (2018), which include results of 84 resonant column tests and 252 cyclic direct simple shear tests performed in sand specimens obtained from the Bay of Campeche between 1993 and 2015. The curves of normalized modulus reduction and damping ratio were organized in three groups according to the percentage of carbonate content: 1) calcareous sands (10 % to 50 %), 2) siliceous carbonate sand (50 % to 90 %), and 3) carbonate sands (90 % to 100 %).

Figure 1 presents the study area of the Bay of Campeche and Tabasco Coastline, which are in the southern portion of the Gulf of Mexico. The dynamic structural analyses of the oil platforms require acceleration time histories or acceleration spectra that already include the soil amplification of the earthquake motions. Two of the most important dynamic soil properties required to conduct an equivalent-linear seismic site-response analysis to evaluate the soil amplification are:

i. A curve of G/G_{max} versus cyclic strain, γ, also called modulus reduction curve, where G is the shear modulus and G_{max} is the maximum shear modulus at very low shear strains of the order of 10^{-4} %;
ii. A curve of material damping ratio D versus γ, where D is defined from the measured area inside a complete hysteretic loop, W_D, which corresponds to the energy dissipated in one cycle, and the maximum strain energy stored during one cycle, W_S, (see Figure 2) through the basic expression shown in Equation 1.

$$D = \frac{1}{2\pi}\frac{W_D}{G\gamma_c^2} = \frac{1}{4\pi}\frac{W_D}{W_s} \quad (1)$$

Figure 1. Location of the area of study in the Bay of Campeche and Tabasco Coastline

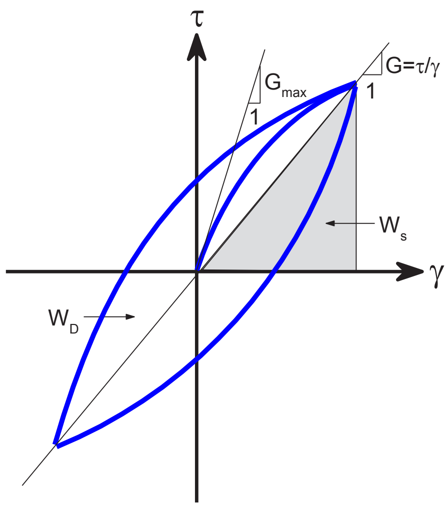

Figure 2. Hysteresis loop for one cycle of loading showing G_{max}, G, and D

The objective of this research is to use the database of Bay of Campeche siliceous carbonate sand and carbonate sand to calibrate the cyclic soil model MRDF-UIUC (Phillips and Hashash, 2009) available in the DEEPSOIL software (Hashash, et al., 2016) to perform seismic site response analysis and obtain the modulus reduction curves and the material damping ratio curves and to aid geotechnical engineers in obtaining these curves when there is no available dynamic laboratory testing data.

2 DATABASE OF DYNAMIC PROPERTIES OF BAY OF CAMPECHE CARBONATE SAND

The data used in this research correspond to siliceous carbonate sand and carbonate sand based on the carbonate soil classification system proposed by Clark and Walker (1977) as shown in Table 1.

Table 1. Carbonate soil classification proposed by Clark and Walker in 1977 (well cemented soils are not included)

Grain Size, mm 0.074 to 4.76	Carbonate Content, %
Carbonate Sand	90 – 100
Siliceous Carbonate Sand	50 – 90
Calcareous Sand	10 – 50
Silica Sand	0 – 10

The database with a total of 2779 pairs of values of normalized shear modulus G/G_{max} and material damping ratio D versus cyclic shear strain used in this study was compiled by Taboada, et al., (2016) and Flores López, et al., (2018) and includes isotropically consolidated resonant column and strain-controlled cyclic direct simple shear test results of 216 specimens of sand from the Bay of Campeche and Tabasco Coastline. The values from 54 specimens containing more than 50 % of carbonate content were selected for this research.

Histograms of the 54 specimens with respect to the confining stress, carbonate content, and fines content are presented in Figures 3(a) to 3(c). The effective confining stresses presented in Figure 3(a) are between 15 kPa and 1300 kPa, with an average of 300 kPa. According to the carbonate content shown in Figure 3(b), about 70 % of the soil specimens are siliceous carbonate sand with carbonate content between 50 % and 90 %, and the rest of the specimens are carbonate sand with carbonate content greater than 90 %. About 25 % of the specimens had a fines content in the range of 5 % to 10 % (Figure 3 (c)), classified as sand with silt; meanwhile, the rest had fines content higher than 12 % but less than 41 %, classified as silty sand.

The rest of the specimens containing carbonates content between 0 % and 50 % were calibrated by Flores López, et al., (2020) taking into account the fact that when the carbonate content is smaller than 50 %, there is a negligible effect on the carbonate content on the curves of G/G_{max} and D (Flores López, et al., 2018).

3 MODELING NON-LINEAR STRESS-STRAIN BEHAVIOUR USING THE MRDF-UIUC MODEL

In order to describe the hysteretic behavior of a soil during unload and reload conditions, many models are based on the Masing rules (Masing, 1926). However, Kwok, et al., (2007) found that an overestimation of damping at large strain can result when using the unload-reload stress-strain loops obtained by adhering to the Masing rules. Therefore, several researchers have developed reduction factors or procedures that modify the Masing rules (Pyke, 1979; Muravskii, 2005; Phillips and Hashash, 2009).

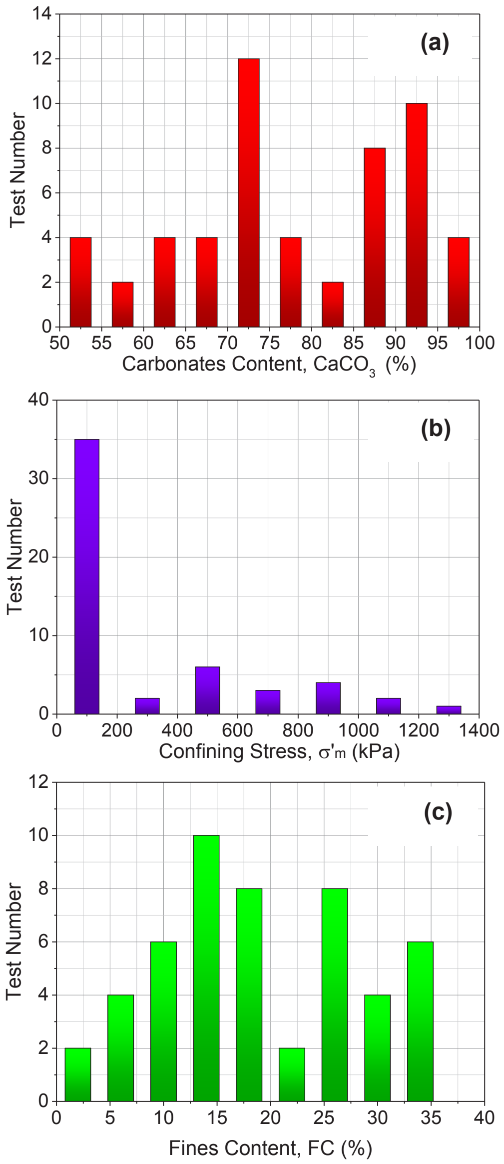

Figure 3. Histograms of Bay of Campeche carbonate sand database

The soil model adopted in this research is the non-Masing MRDF pressure-dependent hyperbolic (Phillips and Hashash, 2009) model included in the DEEPSOIL software (Hashash, et al., 2016) that introduces the following reduction factor into the hyperbolic model:

$$F(\gamma_m) = p_1 - p_2\left(1 - \frac{G(\gamma_m)}{G_0}\right)^{p_3} \tag{2}$$

Where γ_m is the maximum shear strain experienced at any given time, $G(\gamma_m)$ is the shear modulus at γ_m, G_o is the initial shear modulus (G_{max}) and p_1, p_2, and p_3 are fitting parameters.

By setting p_1 as 1 and p_2 as 0, the reduction factor is equal to 1 (regardless of the value of p_3) and the model is reduced to the extended Masing criteria.

The model incorporates the modified hyperbolic model developed by (Matasovic, 1992) and based on the hyperbolic model by (Konder and Zelasko, 1963) but adds two additional parameters (Beta (β) and s) that adjust the shape of the backbone curve:

$$\tau = \frac{\gamma G_0}{1 + \beta\left(\frac{\gamma}{\gamma_r}\right)^s} \tag{3}$$

Where G_o is the initial shear modulus (G_{max}), τ is the shear strength, γ is the shear strain, and γ_r is the reference shear strain (Hardin and Drnevich, 1972) and is considered a material constant; β and s are model parameters; and there is no coupling between the confining pressure and shear stress.

DEEPSOIL extends the model to allow coupling between the confining pressure and the shear stress by making the reference strain γ_r-confining pressure dependent as follows (Hashash and Park, 2002):

$$\gamma_r = a\left(\frac{\sigma_v'}{\sigma_{\text{ref}}}\right)^b \tag{4}$$

Where σ_v' is the effective vertical stress. Reference stress is the vertical effective stress at which $a = \sigma_{ref}$ and b = 0. This is the pressure-dependent hyperbolic model.

The pressure-dependent modified hyperbolic model is almost linear at small strains and results in zero hysteretic damping at small strains. Small-strain damping must be added separately to simulate actual soil behavior that exhibits damping even at very small strains (Hashash and Park, 2002). The small-strain damping ratio is defined as:

$$D_{min} = \left(\frac{1}{\sigma'_v}\right)^d \tag{5}$$

where d can be set to 0 in case pressure-independent small-strain damping is desired.

The hyperbolic/pressure-dependent hyperbolic unload–reload equation is modified with the reduction factor, $F(\gamma_m)$, as follows:

$$\tau = F(\gamma_m)\left[\frac{2G_0\left(\frac{\gamma - \gamma_{rev}}{2}\right)}{1 + \beta\left(\frac{\gamma - \gamma_{rev}}{2\gamma_r}\right)^s} - \frac{G_0(\gamma - \gamma_{rev})}{1 + \beta\left(\frac{\gamma_m}{\gamma_r}\right)^s}\right] + \frac{G_0(\gamma - \gamma_{rev})}{1 + \beta(\gamma_m - \gamma_r)^s} + \tau_{rev} \tag{6}$$

where, γ is the given shear strain, γ_r is the reference shear strain, β is the dimensionless factor, s is the dimensionless exponent, γ_{rev} is the reversal shear strain, τ_{rev} is the reversal shear stress, γ_m is the maximum shear strain, $F(\gamma_m)$ is the reduction factor given in Equation 2, and G_o is the initial shear modulus (G_{max}).

In summary, the 10 parameters to be defined of the non-Masing MRDF-UIUC pressure-dependent hyperbolic model are presented in Table 2.

Table 2. Parameters of the non-Masing MRDF-UIUC pressure-dependent hyperbolic model

Parameter	Symbol
Small-strain damping ratio	D_{min}
Reference strain	γ_{ref}
Reference stress	σ_{ref}
Stress-strain curve parameter	β
Stress-strain curve parameter	s
Pressure dependent (reference strain) parameter	b
Pressure dependent (damping curve) parameter	d
Fitting parameter	p_1
Fitting parameter	p_2
Fitting parameter	p_3

4 CALIBRATION OF THE MRDF-UIUC MODEL TO THE BAY OF CAMPECHE CARBONATE SAND

The MRDF-UIUC model (Phillips and Hashash, 2009) was calibrated with the curves of normalized shear modulus (G/G_{max}) and material damping ratio of the 54 selected specimens containing more than 50 % of carbonate content of the database (Taboada, et al., 2016; Flores López, et al., 2018) of sand from the Bay of Campeche. The database of normalized shear moduli versus shear strain of the 54 selected tests is plotted in Figure 4(a), and the database of material damping ratios versus shear strain is shown in Figure 4(b).

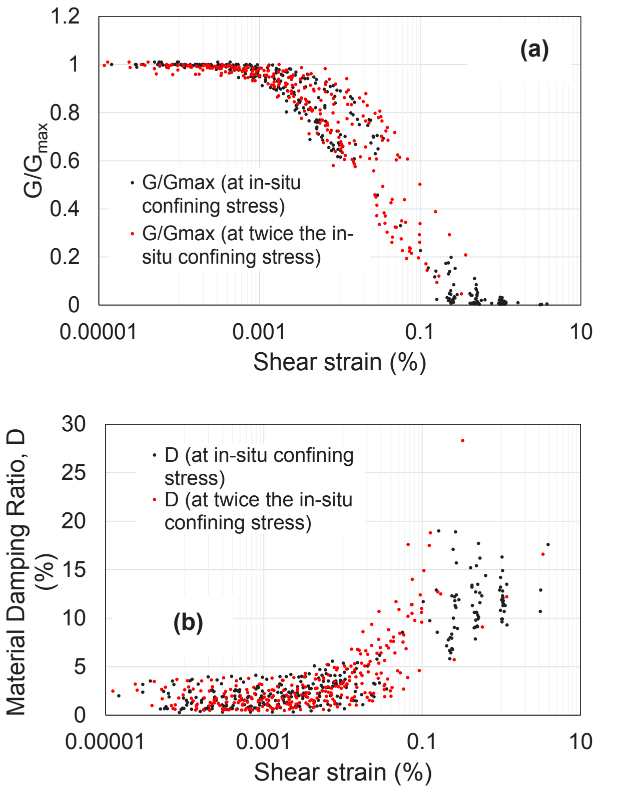

Figure 4. Database of Bay of Campeche carbonate sand presenting: (a) normalized shear modulus versus shear strain and (b) Material damping ratio versus shear strain

A regression analysis for each of the 10 parameters of the non-Masing MRDF-UIUC pressure-dependent hyperbolic model was performed using the database of Bay of Campeche siliceous carbonate sand and carbonate sand presented in Figure 4. The results of the regression analysis are presented in Figure 5.

Semilogarithmic plots of 8 out of the 10 parameters of the model against σ'_m/P_a are shown in Figure 5, where σ'_m is the effective confining stress and P_a is a reference pressure of 100 kPa (effectively, the atmospheric pressure). The parameters b and d are not plotted because those values resulted in 0 in the fitting processes.

The effective confining stress σ'_m is calculated by:

$$\sigma'_m = \frac{\sigma'_v + 2\sigma'_h}{3} = \frac{\sigma'_v(1 + 2K_0)}{3} \tag{7}$$

where, σ'_v is the effective vertical stress, σ'_h is the effective horizontal stress, and K_0 is the coefficient of earth pressure at rest.

The results of the regression analysis performed to determine the 10 parameters of the non-Masing MRDF-UIUC pressure-dependent hyperbolic model are presented below in Equations 8 to 17.

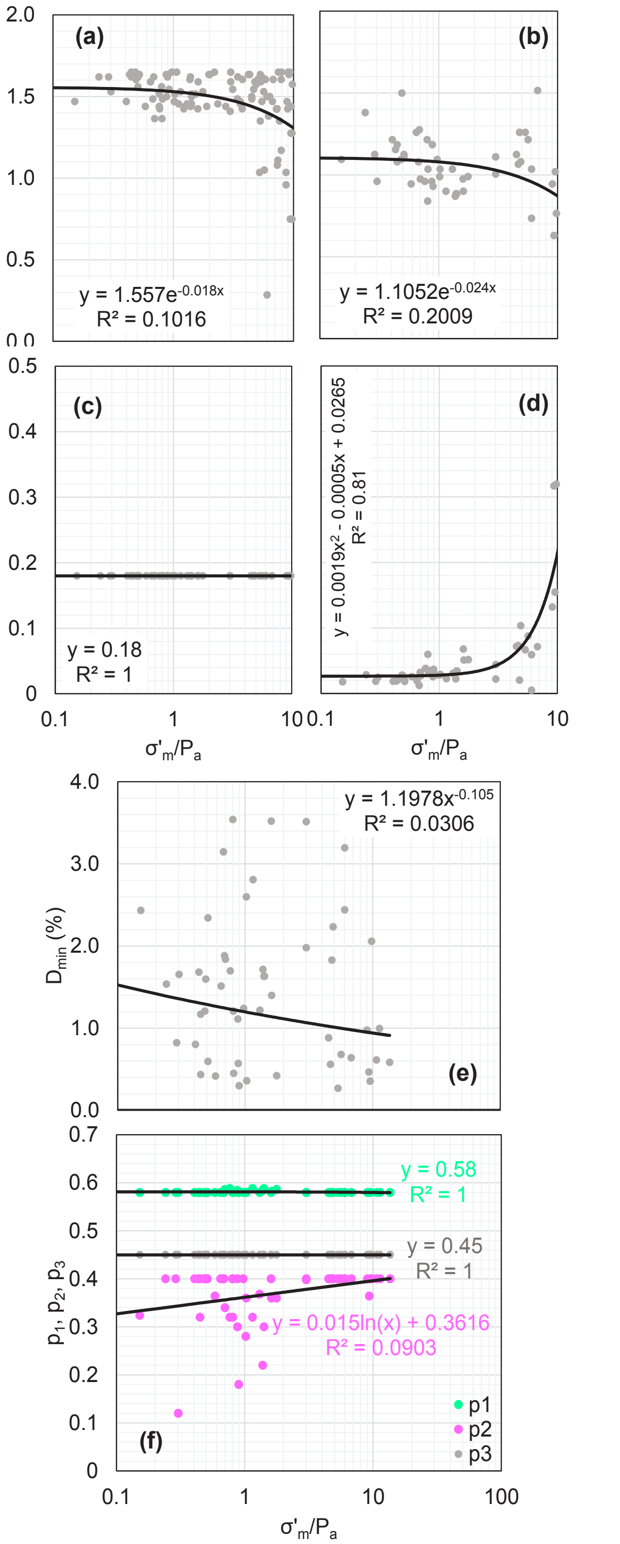

Figure 5. MRDF-UIUC model parameters versus normalized effective confining stress: (a) β; (b) s; (c) Reference stress, σ_{ref}; (d) Reference strain, γ_{ref}; (e) Small-strain damping, D_{min}; and (f) p_1, p_2, and p_3

For the sake of simplicity, the parameters σ_{ref}, b, d, p_1, and p_3 are considered constant with the values of 0.18, 0.00, 0.00, 0.58, and 0.45, respectively.

Plots of normalized shear modulus and material damping ratio versus cyclic shear strain predicted using the MRDF-UIUC model with the calibrated parameters defined in Equations 8 to 17 are showed in Figure 6 for confining stresses in the range of 100 kPa to 1200 kPa.

$$D_{min} = 1.1978 \frac{\sigma'_m}{P_a}^{-0.105} \tag{8}$$

$$\gamma_{ref} = 0.0019 \frac{\sigma'_m}{P_a}^{2} - 0.0005 \frac{\sigma'_m}{P_a} + 0.0265 \tag{9}$$

$$\sigma_{ref} \approx 0.18 \tag{10}$$

$$\beta = 1.557 e^{-0.018 \frac{\sigma'_m}{P_a}} \tag{11}$$

$$s = 1.1052 e^{-0.024 \frac{\sigma'_m}{P_a}} \tag{12}$$

$$b = 0 \tag{13}$$

$$d = 0 \tag{14}$$

$$p_1 \approx 0.58 \tag{15}$$

$$p_2 = 0.015 \ln \frac{\sigma'_m}{P_a} + 0.3616 \tag{16}$$

$$p_3 \approx 0.45 \tag{17}$$

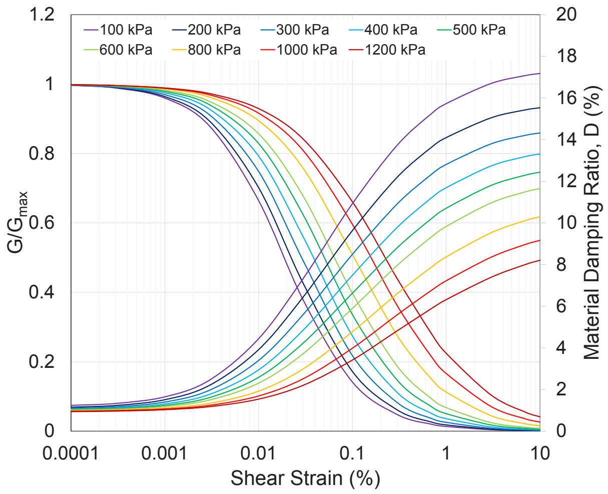

Figure 6. Variation with confining stress of the predicted curves of Normalized shear modulus and Material damping ratio for Bay of Campeche carbonate sand

5 VALIDATION OF CALIBRATED MRDF-UIUC MODEL FOR BAY OF CAMPECHE CARBONATE SAND

Comparison of G/G_{max} between predictions and measurements can be assessed best by plotting predicted against measured values for the 593 data points accumulated from all the tests. This is presented in Figure 7, which shows that the predicted normalized shear modulus (G/G_{max}) values larger than 0.2 fall within ± 30 % of the laboratory measurements while overprediction is observed when the values of normalized shear modulus are smaller than 0.2.

Figure 8 presents a comparison of the predicted material damping ratio D and the 593 measured data points. The predicted material damping ratio values larger than 5 % fall within ± 30 % of the laboratory measurements. Overprediction is observed when the predicted values of material damping ratio are smaller than 5 %.

There are discrepancies between the measured data and the predicted G/G_{max} and material damping ratio (D) values, as can be seen in Figures 7 and 8.

However, the differences are clearer for material damping ratio (D) values below 5 %, corresponding to low shear strains and G/G_{max} values below 0.2 and to large shear strains. Furthermore, Figure 6 shows the material damping ratio (D) values at large strains tend to increase, even when the reduction factor proposed by Phillips and Hashash (2009) was applied.

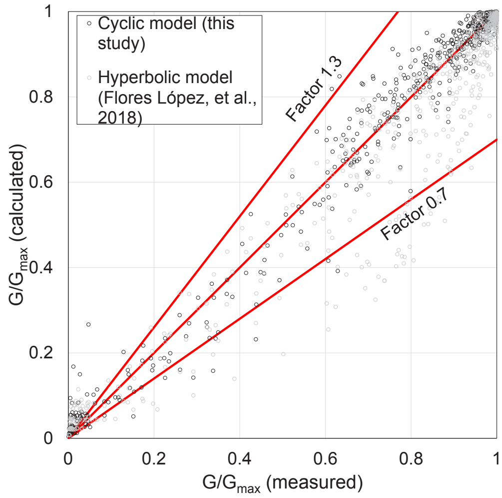

Figure 7. Comparison of measured and calculated G/G_{max} values (593 data points)

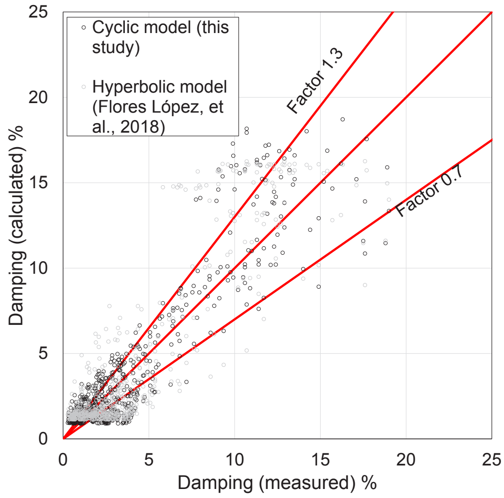

Figure 8. Comparison of measured and calculated material damping ratio (D) values (593 data points)

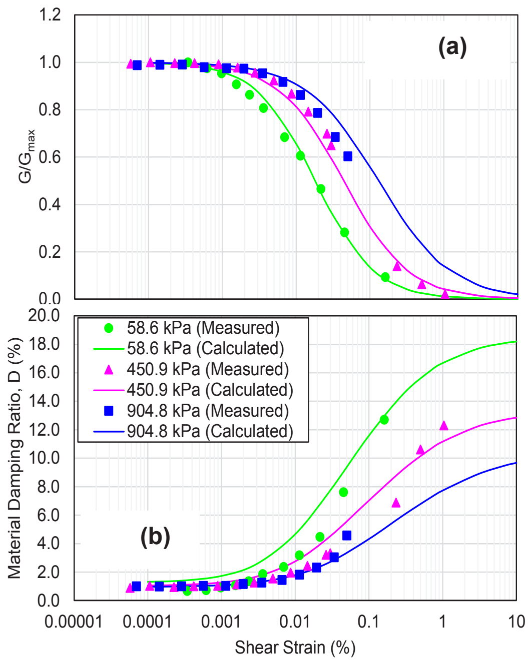

Figure 9. Measured and Predicted G/G_{max}-γ (a) and D-γ (b) curves for σ'_m of 58.6 kPa, 450.9 kPa, and 904.8 kPa

Nevertheless, predicted G/G_{max} and D values resulted in a good agreement with the measured values, as can be seen in Figure 9, where predicted and measured data for three confining stresses are plotted.

5.1 *Comparison with hyperbolic model*

Flores López, et al., (2018) presented predictive equations for estimating normalized shear modulus and material damping of calcareous sand, siliceous carbonate sand, and carbonate sand.

The equations are based on a modified hyperbolic model and were used to calculate values for the selected 593 measured points of siliceous carbonate sand and carbonate sand of this study. A comparison between the predicted values and the measured values are presented in Figures 7 and 8.

In the case of the normalized shear modulus (Figure 7), a good agreement was found between the comparisons made for the modified hyperbolic model equations and the cyclic model equations of this study; however, a smaller number of values fell within ± 30 % of the laboratory measurements in the case of the modified hyperbolic model. For material damping ratio (Figure 8), the number of values within ± 30 % of the laboratory measurements is very similar for both models, and overprediction is found with material damping ratio values smaller than 5%.

6 CONCLUSIONS AND RECOMMENDATIONS

The database of normalized shear modulus and material damping ratios obtained from resonant column tests and strain-controlled cyclic direct simple shear tests (Taboada, et al., 2016; Flores López, et al., 2018) allowed the calibration of the MRDF-UIUC model (Phillips and Hashash, 2009) introduced in DEEPSOIL to predict the non-linear behavior of Bay of Campeche carbonate sand.

The MRDF-UIUC model is defined with 10 parameters reported in Table 2. The parameters b and d were set equal to 0, and the other eight parameters were determined from a regression analysis. This analysis resulted in 3 parameters with constant values and 5 parameters as a function of normalized effective confining stress σ'_m/P_a, where σ'_m is the effective confining stress and P_a is a reference pressure of 100 kPa. The relationships as a function of σ'_m/P_a and the constant values that define the 10 parameters are summarized in Equations 8 to 17.

Comparisons between the predicted values developed in this study with the MRDF-UIUC model and measured values were performed, showing that the predicted normalized shear modulus (G/G_{max}) values larger than 0.2 and material damping ratio values larger than 5 % fall within ± 30 % of the laboratory measurements while overprediction is observed when the values of normalized shear modulus are smaller than 0.2 and material damping ratio values smaller than 5 %. The same comparison was performed using predicted values of a modified hyperbolic model, founding that the MRDF-UIUC model values have a better adjustment within ± 30 % of the laboratory measurements in the case of the G/G_{max} values.

The obtained results have a lot of potential to predict dynamic properties, when laboratory measurements are not available and the number of tests of the laboratory schedule is not enough to obtain a complete geotechnical model of dynamic properties. Furthermore, the calibrated soil model can be used to perform seismic site-response analyses with the DEEPSOIL software and predict normalized shear modulus curves and material damping ratio curves of Bay of Campeche sand during initial stages of a project as well as urgent studies of regional seismic-site response that permit making technical decisions. Future actions may include dividing the Campeche Bay in geotechnical zones or consider reliability-based approach of the predictions. Naturally, the use of the calibrated model does not replace the best engineering practice of executing geotechnical exploration and dynamic testing.

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