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Eccentrically loaded circular surface foundation on loose and dense sand of limited thickness

Fondation de surface circulaire à charge excentrique sur sable lâche et dense d'épaisseur limitée

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ABSTRACT: Laboratory model test results for the ultimate capacity of a circular surface foundation of diameter B supported by a sand layer of limited thickness H underlain by a rigid rough base have been presented. Similar studies, although limited in number, for strip, rectangular and circular foundations with vertical centric loading have been published in the past. The present study is an extension of those in that it evaluates the effect of load eccentricity. The circular foundation was subjected to eccentric loading with load eccentricity = e. Tests were conducted on dense and loose sands at relative densities of compaction of 69% and 35%, respectively. The H/B and e/B ratios were respectively varied from 0.3 to 3 and 0 to 0.15. For a given e/B, the average ultimate load per unit area $[q_{u(H/B,e/B)}]$ decreases with the increase of H/B and becomes constant beyond a critical value of $H/B = (H/B)_{cr}$. Based on the model test results, empirical reduction factor relationships (R) have been developed for dense and loose sands to estimate the average ultimate load per unit area $[q_{u(H/B,e/B)}]$ from the case of the average ultimate load per unit area with eccentric loading and large thickness [$H/B = (H/B)_{cr}$] of sand layer $[q_{u(e/B)}]$. In other words, $q_{u(H/B,e/B)}/q_{u(e/B)} = f(H/B \text{ and } e/B) = R$.

RÉSUMÉ : Des résultats d'essais sur modèle de laboratoire pour la capacité portante ultime d'une fondation de surface circulaire de diamètre B soutenu par une couche de sable d'épaisseur limitée H reposant sur une base rigide et rugueuse ont été présentées. Des études similaires, bien qu'en nombre limité, pour les fondations en bandes, rectangulaires et circulaires à chargement vertical centré ont été publiées au passé. La présente étude est une extension de celles-ci qui évaluent l'effet de l'excentricité de la charge. La fondation du modèle circulaire a été soumise à une charge excentrique avec une excentricité = e. Des essais ont été menés sur des sables denses et meubles à des densités relatives de compactage de 69 % et 35 %, respectivement. Les rapports H/B et e/B variaient respectivement de 0,3 à 3 et de 0 à 0,15. Pour un e/B donné, la charge ultime moyenne par unité de surface $[q_{u(H/B,e/B)}]$ diminue avec l'augmentation de H/B et devient constante au-delà d'une valeur critique de $H/B = (H/B)_{cr}$. Sur la base des résultats des tests du modèle, des relations empiriques de facteurs de réduction (R) ont été développées pour les sables denses et meubles afin d'estimer la charge ultime moyenne par unité de surface $[q_{u(H/B,e/B)}]$ à partir du cas de la charge ultime moyenne par unité de surface avec une charge excentrique et une grande épaisseur $[H/B > (H/B)_{\mathrm{cr}}]$ de la couche de sable $[q_{u(e/B)}]$. En d'autres termes, $q_{u(H/B,e/B)}/q_{u(e/B)} = f(H/B \text{ et } e/B) = R$.

KEYWORDS: Critical state theory; direct simple shear; triaxial, discrete element method; granular material.

1 INTRODUCTION

Terzaghi (1943) developed the first comprehensive theory for the ultimate bearing capacity of shallow foundations. Following that, extensive studies have been conducted by several researchers on the topic for more than seven decades. In most of those studies, it was assumed that the soil supporting the foundation extends to a great depth. With that assumption, the failure surface of a surface strip foundation of width B supported by a sand layer will be as shown in Figure 1. In this case, the failure surface in the soil extends to a depth D measured from the bottom of the foundation. However, if the sand layer has a limited thickness H and is located or a rigid rough base, the nature of the failure surface in soil at ultimate load will be as shown in Figure 2. Mandel and Salencon (1972) have studied this problem analytically. For a given sand friction angle ϕ', the ultimate bearing capacity increases with the decrease in H/B. Figures 1 and 2 are for the condition where the foundation is subjected to centric load.

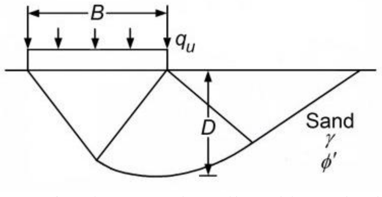

Figure 1. Failure surface in a granular soil at ultimate load supporting a surface strip foundation.

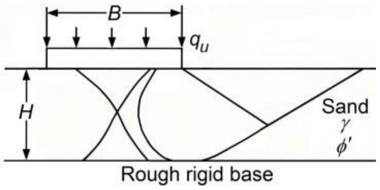

Figure 2. Failure surface in a granular soil of limited thickness supporting a surface strip foundation.

The present study is a related problem and provides the results of a laboratory model test program to determine the ultimate bearing capacity of a surface circular foundation resting on a sand layer of limited thickness with a rigid rough base located below it. The circular foundation is subjected to eccentric loading.

2 THEORETICAL DEVELOPMENT

The ultimate bearing capacity of a circular foundation subjected to a centric loading and located on the surface of a sand layer of limited thickness underlain by a rigid rough base can be expressed as (Mandel and Salencon, 1972; Meyerhof, 1974)

$$q_{u(H/B,e/B=0)} = \frac{1}{2}\gamma B N_{\gamma}^{*} F_{\gamma s}^{*} \tag{1}$$

where $q_{u(H/B,e/B=0)}$= ultimate bearing capacity when the thickness of the sand bed is equal to H; B = diameter of the foundation; e = load eccentricity; γ = unit weight of sand; N_{γ}^{*} = bearing capacity factor; $F_{\gamma s}^{*}$= shape factor.

The theoretical variation of N_{γ}^{*} as a function of H/B and soil friction angle ϕ', as provided by Mandel and Salencon (1972) is shown in Figure 3. For a given soil friction angle, ϕ', the value of N_{γ}^{*} decreases with the increase in H/B and reaches a minimum value $N_{\gamma}^{*} = N_{\gamma}$ at $H/B \geq D/B$. Meyerhof (1974) proposed the variation of $F_{\gamma s}^{*}$ as

$$F_{\gamma s}^{*} = 1 - m\left(\frac{B}{L}\right) \tag{2}$$

where L = length of the foundation and $m = f$ ($\phi'$and H/B) (Figure 4). Thus, for a circular foundation with $B = L$,

$$F_{\gamma s}^{*} = 1 - m \tag{3}$$

It is important to note that, for a circular foundation with $H/B \geq D/B$, $F_{\gamma s}^{*}$= 0.6.

The present paper is an extension of the results of the limited studies presently available on this subject (Milovic and Turnier, 1971; Meyerhof, 1974; Pfeifle and Das, 1979; Cerato and Lutenegger, 2006) in that the effect of load eccentricity on the ultimate bearing capacity has been evaluated by laboratory model tests.

3 LABORATORY MODEL TESTS

A poorly graded sand was used for the present tests. The general physical properties of the sand are as follows:

- Effective size, D_{10} = 0.33 m
- Uniformity coefficient, C_u = 1.42
- Coefficient of gradation, C_c = 1.14

Two series of tests were conducted with the sand in *dense* and *loose* states of compaction. For the sands in compacted state, the properties are:

- Series I – Dense sand:
  Relative density, D_r = 69%; friction angle, ϕ' = 40.9° (direct shear test); average unit weight = 14.36 kN/m^2
- Series II – Loose sand:
  Relative density, D_r = 35%; friction angle, ϕ' = 34° (direct shear test); average unit weight = 13.34 kN/m^2

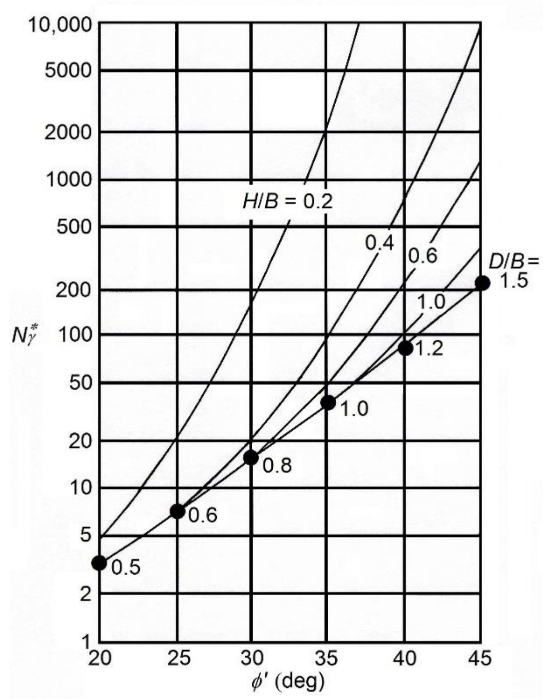

Figure 3. Mandel and Salencon theory—Variation of N_{γ}^{*} and ϕ' and H/B.

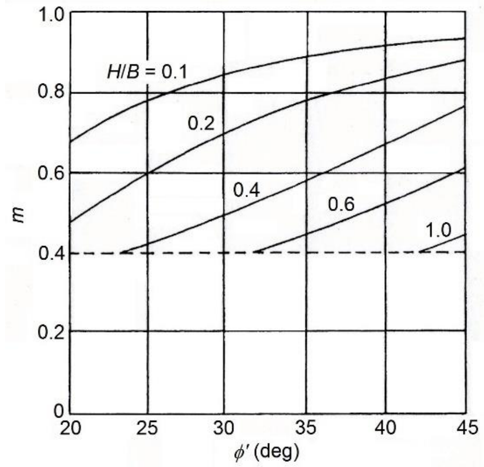

Figure 4. Variation of m with ϕ' and H/B [based on Meyerhof's theory (1974)].

Model tests were conducted in a square box with inside dimensions of 0.8 m × 0.8 m and height of 0.65 m. All four sides of the box were made from mild steel to avoid bulging during testing. The rigid rough base was prepared by placing a 0.8 m × 0.8 m mild steel plate at the bottom of the box. The top surface was made rough by applying a glue and sand mixture. Sand was poured into the test tank in layers of 25 mm from a fixed height by a raining technique to achieve the desired unit weight of compaction. The height of fall was fixed by making several trials in the test box to achieve the desired density. The model foundation was placed on the top of the sand layer. The diameter, B, of the model foundation was 100 mm. The bottom surface of the foundation was made rough by applying a glue-sand mixture. Load to the foundation was applied by a specially designed loading unit. The load could be applied to the model foundation in the range of 0 to 100 kN with an accuracy of 1 N. The settle-ment along the center line was measured by dial gauges placed on two sides of the model foundation. Tests were conducted with H/B = 0.3, 0.5, 1, 2, 3, and 5.5 with eccentricity ratio e/B = 0, 0.05, 0.10, and 0.15. *It is important to point out that H/B = 5.5 was considered to be the case with $H/B \geq D/B$ (great depth).*

For each test, the *average load per unit area* of the foundation was obtained as

$$q = \frac{\text{Total load Q}}{\text{Area of the model foundation}} \tag{4}$$

Plots were made for the average load per unit area versus average settlement, s, along the center line of the foundation. The ultimate average loads per unit area [i.e., $q_{u(H/B,e/B)}$ and $q_{u(e/B)}$] were determined from these plots. Note that the notations for the average ultimate load per unit area used in this paper are

- $q_{u(H/B,e/B)}$ for H/B = 0.3, 0.5, 1, 2, 3

and

- $q_{u(e/B)}$ for H/B = 5.5 (i.e., > D/B)

4 MODEL TEST RESULTS

The laboratory model test results for Series I and Series II are given in Tables 1 and 2 (Columns 1, 2, 3 and 4), respectively.

Table 1. Laboratory test result – series I (dense sand)

H/B (1)	e/B (2)	$q_{u(H/B,e/B)}$ (kN/m^2) (3)	$q_{u(e/B)}$ (kN/m^2) H/B = 5.5 (4)	R Experiment (5)	R Predicted (6)
0.3	0	880	116	7.59	7.68
0.3	0.05	810	104	7.79	7.65
0.3	0.1	690	88	7.84	7.62
0.3	0.15	565	77	7.34	7.60
0.5	0	425	116	3.66	3.65
0.5	0.05	390	104	3.75	3.62
0.5	0.1	330	88	3.75	3.59
0.5	0.15	270	77	3.51	3.56
1	0	194	116	1.67	1.73
1	0.05	170	104	1.63	1.70
1	0.1	144	88	1.64	1.67
1	0.15	110	77	1.43	1.65
2	0	128	116	1.10	1.18
2	0.05	120	104	1.15	1.15
2	0.1	104	88	1.18	1.12
2	0.15	92	77	1.19	1.09
3	0	119	116	1.03	1.06
3	0.05	111	104	1.07	1.03
3	0.1	94	88	1.07	1.00
3	0.15	83	77	1.08	0.98

Table 2. Laboratory test result – series I (loose sand)

H/B (1)	e/B (2)	$q_{u(H/B,e/B)}$ (kN/m^2) (3)	$q_{u(e/B)}$ (kN/m^2) H/B = 5.5 (4)	R Experiment (5)	R Predicted (6)
0.3	0	350	37	9.46	8.71
0.3	0.05	285	33	8.64	8.53
0.3	0.1	240	30	8.00	8.35
0.3	0.15	195	25	7.80	8.17
0.5	0	170	37	4.59	4.47
0.5	0.05	144	33	4.36	4.29
0.5	0.1	118	30	3.93	4.11
0.5	0.15	94	25	3.76	3.93
1	0	77	37	2.08	2.19
1	0.05	69	33	2.09	2.01
1	0.1	59	30	1.97	1.83
1	0.15	50	25	2.00	1.65
2	0	38	37	1.03	1.42
2	0.05	34	33	1.03	1.24
2	0.1	30	30	1.00	1.06
2	0.15	25.2	25	1.01	0.88
3	0	38	37	1.03	1.24
3	0.05	34	33	1.03	1.06
3	0.1	30	30	1.00	0.87
3	0.15	25.2	25	1.01	0.69

5 ANAYLSIS OF THE TEST RESULTS

Figures 5 and 6 show the variations of $q_{u(H/B,e/B)}$ and $q_{u(e/B)}$ with H/B (for e/B = 0 and 0.15) for dense and loose sand, respectively. From the plots, the following observations can be made:

- The experimental values of $q_{u(H/B,e/B)}$ for e/B = 0 (i.e. centric loading) deviated substantially from the theory via Eq. (1) and Figures 3 and 4.

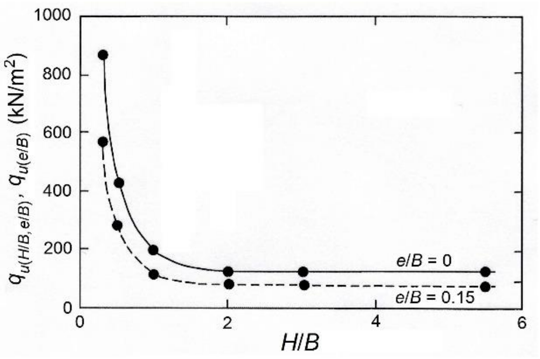

Figure 5. Variation of $q_{u(H/B,e/B)}$ and $q_{u(e/B)}$ with H/B (for e/B = 0 and 0.15)—Series I (dense sand)

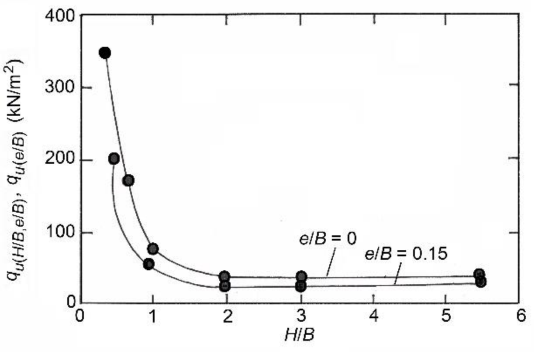

Figure 6. Variation of $q_{u(H/B,e/B)}$ and $q_{u(e/B)}$ with H/B (for e/B = 0 and 0.15)—Series II (loose sand)

- The experimental $(H/B)_{cr} = D/B$ values for eccentric loading are about 3 for dense sand and about 2.5 to 3 for loose sand. The theoretical D/B values for a strip foundation subjected to centric loading in dense (ϕ' = 40.9°) and loose (ϕ' = 34°) sands are about 1.2 and 1, respectively. Cerato and Lutenegger (2006), based on their model tests, suggested that $(H/B)_{cr}$ for circular foundations with centric load application can be approximated as 3.

The above-stated discrepancies between theory and model tests are primarily due to the interference effect of the rigid rough base as it related to the development of the failure surface in sand which cannot be resolved at this time. However, a practical solution may be to develop a reduction factor, R, which can be expressed as

$$R = f\,(H/B \text{ and } e/B) = q_{u(H/B,e/B)}/q_{u(e/B)} \tag{5}$$

Column 5 of Tables 1 and 2 show the experimental values of R as defined by Eq. (5).

The nondimensional reduction factor can be of the form

$$R = \left[a_1\left(\frac{H}{B}\right)^{a_2} + a_3\left(\frac{H}{B}\right) + a_4\right] \tag{6}$$

With the experimental values of R (Column 5, Tables 1 and 2), nonlinear regression analyses (NLREG) were performed to obtain the magnitudes of a_1, a_2, a_3 and a_4. NLREG performs statistical regression analyses to estimate the values of parameters for linear, multivariate, polynomial, logistic, exponential, and general nonlinear functions. The regression analysis determines the values of the coefficient that cause the function to best fit the observed data that are being provided, The values thus obtained from the present test results are:

- Dense sand: $a_1 = 0.78$, $a_2 = -1.79$, $a_3 = -0.56$ and $a_4 = 0.96$
- Loose sand: $a_1 = 1.16$, $a_2 = -1.57$, $a_3 = -3.62$ and $a_4 = 1.03$

Thus, substituting the values of a_1, a_2, a_3 and a_4 into Eq. (6),

Dense sand:

$$R = \left[0.78\left(\frac{H}{B}\right)^{-1.79} - 0.56\left(\frac{e}{B}\right) + 0.96\right] \quad (\text{for } H/B \le 3) \tag{7}$$

Loose sand:

$$R = \left[1.16\left(\frac{H}{B}\right)^{-1.57} - 3.62\left(\frac{e}{B}\right) + 1.03\right] \quad (\text{for } H/B \le 3) \tag{8}$$

For comparison purposes, the predicted values of R obtained using Eqs. (7) and (8) are shown in Column 6 of Tables 1 and 2. There is good agreement between the experimental (Column 5) and predicted (Column 6) values of R.

In a practical application, when required to estimate the *ultimate load* of a circular foundation, $Q_{u(H/B,e/B)}$, supported by a sand layer of limited thickness and subjected to eccentric loading, one can use the relationship

$$Q_{u(H/B,e/B)} = RQ_{u(e/B)} \tag{9}$$

where $Q_{u(e/B)}$ is the *ultimate load* on the foundation with eccentric load application with $H/B \ge D/B$. This can be estimated with the procedure available in the existing literature.

6 CONCLUSIONS

Laboratory model test results for the ultimate load carrying capacity of an eccentrically loaded circular surface foundation resting on dense and loose sand layers of limited thickness under-lain by a rigid rough base have been presented. Based on the test results, the following conclusions can be drawn.

- The average ultimate load per unit area $[q_{u(H/B,e/B)}]$ decreases with the increase in H/B and reaches a minimum value at $H/B \approx 3$.
- Based on the experimental results, a nondimensional reduction factor R has been derived which is a function of H/B ($H/B \le 3$).
- The reduction factor can be used to estimate $q_{u(H/B,e/B)}$ from $q_{u(e/B)}$.

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