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Experimental and numerical studies on pile groups under cyclic lateral loading

Études expérimentales et numériques sur des groupes de pieux sous charge latérale cyclique

Oliver Reul
Department of Geotechnical Engineering, University of Kassel, Germany, o.reul@uni-kassel.de

Christoph Niemann
Goldbeck Nord GmbH, Germany

Conleth O'Loughlin
Centre for Offshore Foundation Systems, University of Western Australia, Australia

Yinghui Tian & Mark Cassidy
Melbourne School of Engineering, University of Melbourne, Australia

ABSTRACT: The behaviour of soils, both fine- and coarse grained, under cyclic loading is complex. Cyclic lateral loads on piles in sand for example lead to either a densification or a loosening of the surrounding soil. However, the design of pile foundations under cyclic loading is still usually based on modifications to the more simplistic monotonic loading conditions. As part of a research project to develop a framework for quantifying cyclic effects on pile groups, centrifuge model tests on single piles and pile groups in dry silica sand have been carried out at the Centre for Offshore Foundation Systems (COFS) in Perth, Australia. In the scope of this paper, the results of the centrifuge tests are briefly discussed and compared to numerical simulations with the finite element method, which have been carried out to back-analyze selected tests. In the numerical analysis, a hypoplastic soil model with the intergranular strain extension has been applied to capture the behaviour of sand under cyclic loading.

RÉSUMÉ : Le comportement des sols, à grains fins et grossiers, sous charge cyclique est complexe. Les charges latérales cycliques sur les pieux dans le sable, par exemple, entraînent soit une densification, soit un ameublissement du sol environnant. Cependant, la conception des fondations de pieux sous charge cyclique est encore généralement basée sur des modifications des conditions de charge monotones plus simplistes. Dans le cadre d'un projet de recherche visant à développer un cadre pour quantifier les effets cycliques sur les groupes de pieux, des essais de modèles de centrifugation sur des pieux individuels et des groupes de pieux dans du sable de silice sec ont été réalisés au Centre for Offshore Foundation Systems (COFS) à Perth, en Australie. Dans le cadre de cet article, les résultats des essais de centrifugation sont brièvement discutés et comparés à des simulations numériques avec la méthode des éléments finis, qui ont été réalisées pour contre-analyser certains essais. Dans l'analyse numérique, un modèle de sol hypoplastique avec l'extension de la contrainte intergranulaire a été appliqué pour saisir le comportement du sable sous charge cyclique.

KEYWORDS: Physical modelling, numerical simulations, pile groups, cyclic loading, hypoplasticity

1 INTRODUCTION.

Cyclic loading of pile foundations is of importance in various fields of civil engineering such as offshore and onshore wind turbines (BSH 7005:2007-06) or structures for transport infrastructure (Berger et al. 2003). Cyclic loading may be caused by wind and waves (offshore wind turbines) or temperature induced constraints (integral bridges without joints and bearings). The design of pile foundations under cyclic loading is usually still based on modifications to the more simplistic monotonic loading conditions. A better understanding of pile-soil interaction during lateral cyclic loading is therefore required to establish more efficient design strategies for these types of foundations.

As part of a research project to develop a framework for quantifying cyclic effects on piles, centrifuge model tests on single piles and freestanding pile groups in dry silica sand have been carried out at the Centre for Offshore Foundation Systems (COFS) in Perth, Australia. Selected tests have been numerically back-analyzed using 3D finite element calculations applying a hypoplastic soil model to capture the behaviour of sand under cyclic loading.

2 CENTRIFUGE TESTS

The centrifuge tests were carried out at an acceleration of 100g using the 40g-tonne, 1.8 m radius beam centrifuge located at The University of Western Australia (Randolph et al. 1991). The test program comprised monotonic and cyclic lateral loading on single piles and cyclic lateral loading on pile groups.

The model pile group consisted of five piles ($n_p = 5$); two in the leading row, two in the trailing row and one center pile (Figure 1). Three piles were instrumented with six pairs of strain gauges at the depths shown in Figure 1. The centrifuge tests used circular model piles that were fabricated from aluminum to have a diameter, $D = 10$ mm, a length, $L = 130$ mm and a wall thickness, $t = 1$ mm. A cap at the base of the piles prevented soil entering the interior of the piles during installation. The surfaces of the piles were sand-blasted, such that the roughness of the pile-soil interface was comparable to bored piles in-situ. The embedment length of the piles was $L_p = 115$ mm ($L_p/D = 11.5$).

The pile cap was a rigid cruciform plate, fabricated from stainless steel to have an edge length of 125 mm and a thickness of 10 mm (Figure 1). Each pile was connected to the pile cap via a threaded connection, such that the connection between each pile and the pile cap could be considered as rigid. The pile cap had provisions for a number of pile connection locations, such that the same pile cap could be used in all tests with s/D (pile spacing ratio, see Figure 1) varying between 3 and 5.

The centrifuge tests were conducted in a fine to medium sub-angular silica sand with properties as summarized in e.g. Bienen et al. (2012). The air pluviation technique was used for sample preparation to achieve repeatable medium dense soil samples. The sample density was derived from measurements of the sample mass and volume. These measurements resulted in a relative density I_D = 45%.

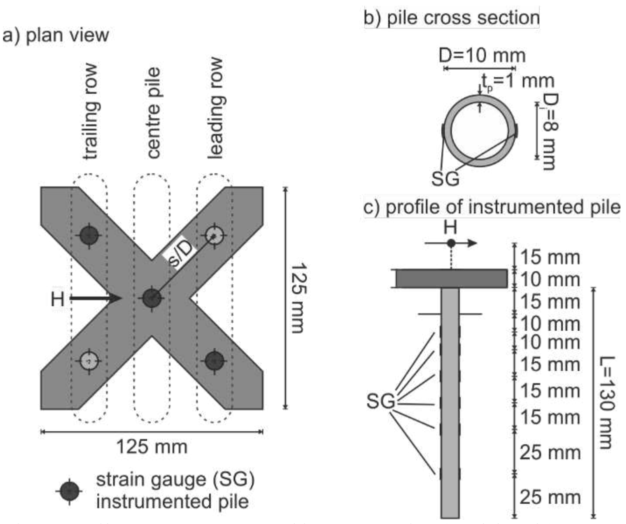

Figure 1. Pile group geometry and instrumentation (model scale)

Before any testing was undertaken, each centrifuge sample was subjected to a number of three spin up/down cycles (to 100g) to 'shakedown' the sample to a steady relative density. Two cone penetration tests (CPTs) were conducted in the first sample in advance of the cyclic tests using a 7 mm diameter model cone penetrometer. A detailed description of the experimental setup is provided in Niemann et al. (2017; 2018; 2019).

The test program is summarized in Table 1. Monotonic load tests were conducted under displacement control to determine the ultimate capacity of a single pile prior to the cyclic tests, from which the load amplitude in the cyclic tests was scaled. The cyclic tests were carried out under load control.

A total of six cyclic pile group tests were conducted in the centrifuge, with varying amplitudes for 1-way cyclic loading. The piles were jacked into the soil at 1g rather than in flight, as the in-flight installation resistance would have exceeded the 7 kN vertical capacity of the actuator. According to Li et al. (2010) the behaviour of 'pre-jacked' model piles is comparable to that of field scale bored piles. After pile installation, the centrifuge was accelerated to 100g and the pile was loaded. A loading frequency of 0.1 Hz was selected and the data were acquired at a sampling frequency of 10 Hz.

Table 1. Test program for cyclic loading of pile groups

		PG_02_3	PG_02_4	PG_02_5	PG_04_3	PG_04_4	PG_04_5
No. of cycles	N	500	500	500	500	500	500
Pile spacing ratio	s/D	3	4	5	2	4	5
Cyclic load magnitude	ζ_b*	0.2	0.2	0.2	0.4	0.4	0.4
Cyclic load symmetry	ζ_c*	0.0	0.0	0.0	0.0	0.0	0.0

*definition after LeBlanc et al. (2010)

3 FINITE ELEMENT ANALYSIS

3.1 *Finite Element Model*

The model consisted of hexahedral, eight-noded constant-strain elements. The number of elements (NE) and nodes (NN) varied depending on the investigated geometry, with a maximum of NE = 22500 and NN = 25400. In order to achieve comparable results from the test and the simulations, the dimensions of the finite element model were set the same as the dimensions of the centrifuge model (in prototype scale). Circular hollow piles (with closed ends) with a diameter of D = 1 m in prototype scale were used in the model tests. In the numerical simulation, the piles were modelled as solid profiles using continuum elements with linear-elastic material behaviour (Table 2).

In order to ensure the same flexural rigidity of the piles in the simulation as in the experiments, an equivalent Young's modulus, E, was calculated and used in the numerical simulations. The same applies for the Young's modulus of the pile cap, since the cruciform shape of the plate was not modelled in the simulation. In the centrifuge tests, the vertical load on the pile group was maintained at zero. The dead weight of the pile cap and the piles was ignored in the numerical modelling, tolerating a slight reduction of the stresses around the piles.

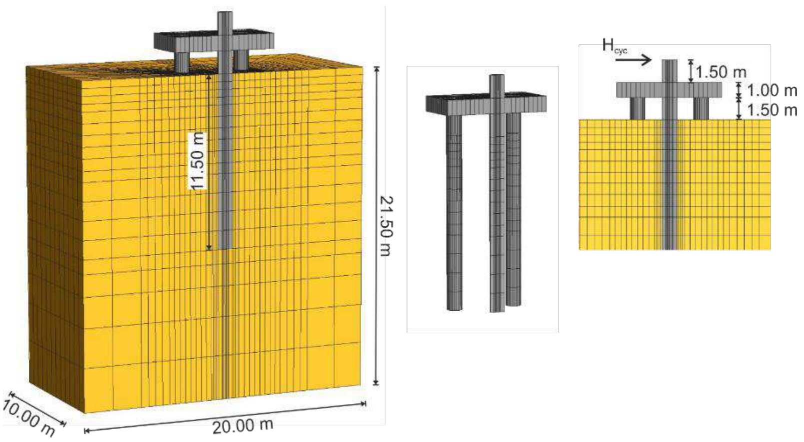

Figure 2 Numerical model for simulation of the centrifuge tests (s/D = 3)

Table 2 Material parameters piles and the pile cap

Structural elements			
Pile (aluminium)			
Unit weight	γ	0	kN/m³
Young's modulus	E	41300*	MN/m²
Poisson's ratio	ν	0.35	-
Pile cap (stainless steel)			
Unit weight	γ	-	kN/m³
Young's modulus	E	150000*	MN/m²
Poisson's ratio	ν	0.28	-

*reduced value

Interface elements were applied between the pile and soil in order to achieve a realistic pile-soil interaction. The shear strength of the interface has been modelled with the Coulomb friction model with the shear strength reduced compared to the shear strength of the surrounding soil using a factor R_{inter} = 0.5. The calculation steps are:

1. Initial stress state
2. Pile installation ('wished-in-place')
3. Installation of interface elements

4. Installation of pile cap
5. Start of cyclic loading

In Figure 2, the finite element mesh for the pile group with a pile spacing of $s/D = 3$ is shown. The numerical calculations have been carried out with the Finite Element program ‘Tochnog Professional’ (Tochnog Professional Company 2019b). For pre- and postprocessing the program ‘GiD 11.0.5’ (CIMNE 2013) was used.

3.2 *Hypoplastic soil model*

Since the combination of a hypoplastic soil model after Wolffersdorff (1996) and the intergranular strain extension (Niemunis & Herle 1997) is capable of modelling a stress-dependent soil stiffness and a varying stiffness for initial, un- and reloading, this constitutive model has been chosen to simulate the tests. Subsequently, the constitutive model is briefly discussed. For a comprehensive overview, the reader is referred to e.g. Wolffersdorff (1996) and Niemunis & Herle (1997).

The hypoplastic material law proposed by Wolffersdorff (1996) describes the change of stresses in the grain structure by means of a tensorial equation:

$$\dot{\boldsymbol{T}} = \boldsymbol{h}(\boldsymbol{T}, \boldsymbol{D}, e) = \boldsymbol{L}(\boldsymbol{T}, e) : \boldsymbol{D} + \boldsymbol{N}(\boldsymbol{T}, e)\|\boldsymbol{D}\| \tag{1}$$

where the Cauchy stress rate, $\dot{\boldsymbol{T}}$, is a function of the Cauchy stress tensor, $\boldsymbol{T}$, the stretching rate according to Euler, $\boldsymbol{D}$, and the void ratio, e, (Herle 1997). The tensors $\boldsymbol{L}$ and $\boldsymbol{N}$ consider the linear and non-linear stiffness. The main equations are summarized in Niemunis & Herle (1997).

In cases of cyclic loading at low strain levels (e.g. un- and reloading in oedometer tests), the stiffness increases and the stress dependency decreases (Wegener & Herle 2014). For these cases, Niemunis & Herle (1997) extends the basic formulation of hypoplasticity by the intergranular strains, δ (where the basic formulation underestimates the soil stiffness). The effect of intergranular strains can be understood as the deformation that occurs in the interface between soil particles within the grain skeleton.

The determination of the hypoplastic parameters was based on a literature survey as summarised in Table 3. The simulation tool ‘Incremental Driver’ (Tochnog Professional Company 2019a) has been used to run the simulation of the element tests.

Pucker et al. (2013) provide a comparison of measured and simulated results of oedometer tests for loose ($e_0 = 0.70$) and dense ($e_0 = 0.56$) samples (Figure 3). Furthermore, in Figure 3 the results of simulations carried out within the scope of this work applying the parameters provided by Bienen et al. (2015) and Labenski et al. (2016) are included. The results indicate a sufficient agreement in the results from laboratory tests and from the simulations. However, the soil stiffness after unloading of the sample seems to be significantly higher in the oedometer tests than in the simulations when the parameters provided in the literature (summarized in Table 3) are used.

In order to achieve a better agreement for the soil behaviour during unloading and reloading, which is important for cyclic loading, a further calibration of the parameters has been undertaken (Niemann 2020). The best fit with the measured results reported by Pucker et al. (2013) is shown in Figure 4.

For the loose sample (Figure 4a), the agreement between the measured and simulated results is reasonable. In the case of the dense sample (Figure 4b), although the initial compaction of the soil sample is significantly higher in the simulation than in the oedometer test, the soil stiffness after unloading is in reasonable agreement. In addition, the calibrated parameters have been chosen to describe the load-displacement behaviour of the pile group in the centrifuge tests sufficiently. A higher soil stiffness would result in lower lateral displacements of the pile group and thus a poorer agreement between measurements and simulation (Niemann 2020).

Table 3 Hypoplastic and intergranular parameters for UWA silica sand from literature and calibrated values

	Parameter	Pucker et al. (2013)	Bienen et al. (2015)	Labenski et al. (2016)	Calibration
Hypoplasticity after Wolffersdorff (1996)	φ_c (°)	30	30	30	30
	h_s (MPa)	1354	5370	1354	5370
	n	0.34	0.26	0.34	0.26
	e_{d0}	0.49	0.49	0.49	0.49
	e_{c0}	0.76	0.76	0.79	0.76
	e_{i0}	0.86	0.87	0.86	0.87
	α	0.18	0.30	0.18	0.15
	β	1.27	0.50	1.27	0.25
Intergranular strains after Niemunis & Herle (1997)	R	1E-4	1E-4	1E-4	1E-4
	m_R	5.16	5.00	2.2	2.2
	m_T	3.07	2.00	1.1	1.1
	β_R	0.58	0.50	0.936	0.03
	χ	5.74	6.00	1.485	5.74

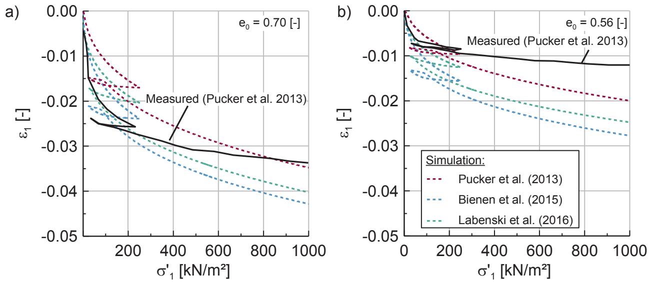

Figure 3 Comparison of measured and simulated oedometer test results using hypoplastic parameters reported in the literature

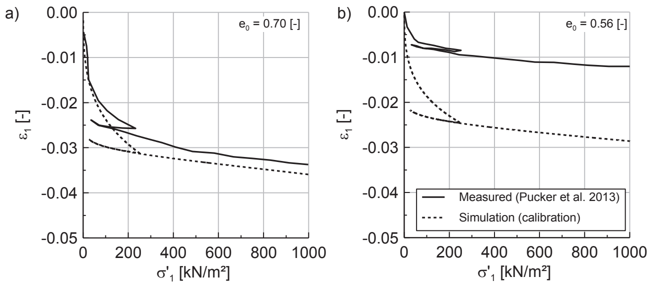

Figure 4 Comparison of measured and simulated oedometer test results using calibrated hypoplastic parameters

4 RESULTS

Figure 5 compares the load-displacement behaviour for all six centrifuge tests on pile groups (N = 500) with results from the numerical modelling ($N \leq 110$). The numerical m odelling agrees reasonably well with the centrifuge tests fo r the first monotonic loading and for a small number of c ycles. A rapid accumulation of the discrepancy can be see n with increasing cycle number, which is a common challe nge for current numerical modelling using the hypoplastic constitutive model (Le 2015).

In general, there is reasonable agreement in the load-displacement response between the model tests and the simulations. For most tests, the initial stiffness of the pile group in the simulations proved to be too high, leading to smaller displacements in the simulations compared to the model tests. Exceptions are tests PG_04_3 and PG_04_4, where the initial stiffness in the model tests and the simulations are similar. In addition, the simulated hysteretic loops indicate a higher stiffness of the system during unloading and reloading compared to the measurements.

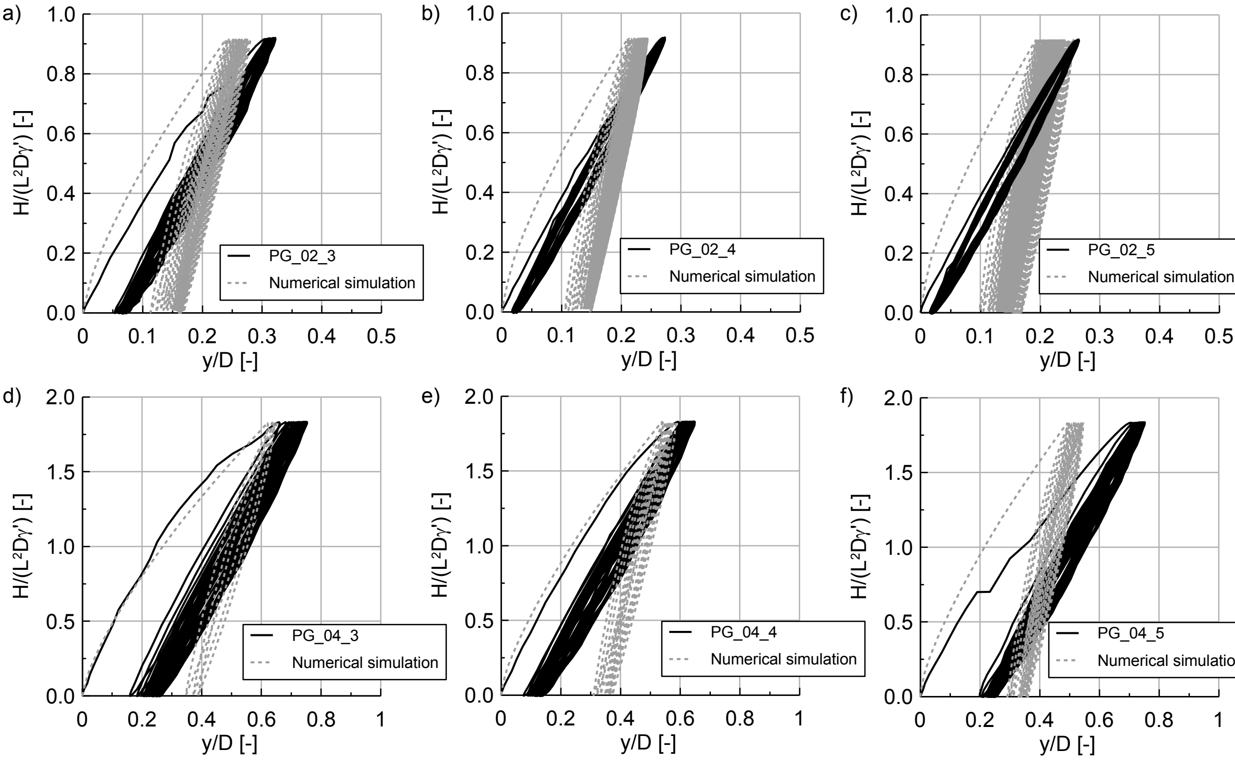

Figure 5 Measured and calculated load displacement response during cyclic loading

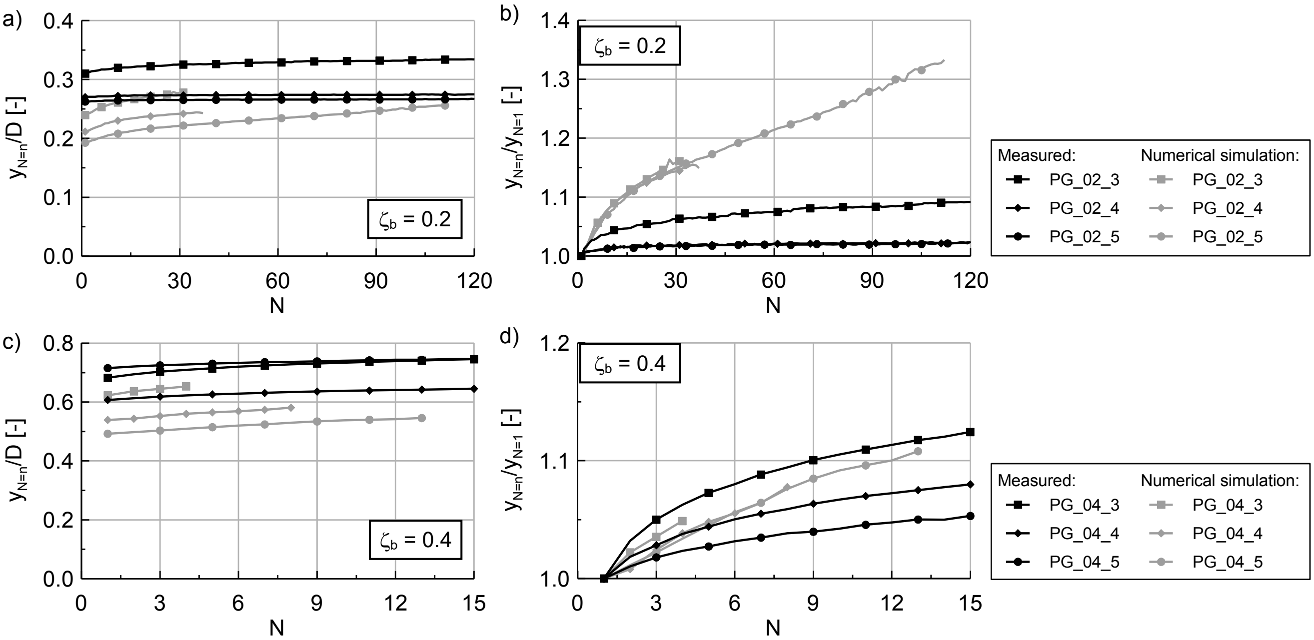

Figure 6 Measured and simulated evolution of lateral displacements during cyclic loading

A comparison of measured and simulated evolution of lateral displacements, y/D, and the accumulation-ratio, $y_{N=n}/y_{N=1}$, with the number of cycles, N, is provided in Figure 6 for cyclic magnitudes of $\zeta_b = 0.2$ and $\zeta_b = 0.4$. For all configurations, the simulations underestimate the lateral displacements of the pile groups (Figure 6a and c), due to the high stiffness adopted in the simulations. Although for $\zeta_b = 0.2$ the lateral displacements generally increase with decreasing pile spacing in both the centrifuge tests and the simulations, this tendency is not evident for the centrifuge tests at $\zeta_b = 0.4$.

At a cyclic magnitude of $\zeta_b = 0.2$, the accumulation-ratio derived from the numerical simulations significantly overpredicts the measured results (Figure 6b), whereas at the higher load magnitude (Figure 6d) the simulations get the rate of accumulation about right. Nevertheless, in contrast to the measurements, the numerical simulations seem to be unable to reproduce the influence of the pile spacing on the accumulation-ratio.

Investigations by Schmidt (1984, 1986) and Klüber (1988) have shown that the contribution of the individual piles of a laterally loaded pile group with a rigid cap (i.e. equal lateral pile head displacement) to the load transfer varies significantly. Based on these investigations, the approach to estimate the load distribution factor $\alpha_{Load,i}$ (Eq. 2) was developed (DGGT 2012).

In the tests carried out, the lateral load acting on each pile head, H_i, can be determined by differentiating the measured bending moment distribution. Depending on the pile position, the normalised load distribution factor, α_{Load}, is used:

$$\alpha_{Load,i} = \frac{H_i}{\sum_{j=1}^{n_p} H_j} \tag{2}$$

where H_i is the lateral load on one pile of the group and n_p is the total number of piles in the group.

The tests are analyzed considering mean values of α_{Load} from the two investigated cyclic load amplitudes (ζ_b = 0.2; ζ_b = 0.4). Evidently α_{Load} (and hence the lateral force at the pile head) reduces with the number of cycles in the leading row, but increases for the center pile and for piles in the trailing row. This indicates that a shift in the load distribution within the pile group takes place during cyclic loading, which can be ascribed to densification and loosening effects in the soil around the piles, a phenomenon that has already been observed by evaluating the cyclic changes of the maximum bending moments (Niemann et al. 2019). The load distribution factor α_{Load} due to cyclic loading can then be described as a logarithmic function:

$$\alpha_{Load}(N) = t_\alpha \cdot ln(N) + \alpha_{Load,1} \tag{3}$$

where $\alpha_{Load}(N)$ is the cyclic load distribution factor at cycle number N, t_α is the logarithmic fitting constant and $\alpha_{Load,1}$ is the load distribution during the first cycle.

The measured data on Figure 7 and Figure 8 are fitted by Eq. 3 using t_α = -0.003 to t_α = -0.006 for the leading row, $t_\alpha \approx$ 0.000 to t_α = 0.004 for the centre pile and t_α = 0.001 to t_α = 0.005 for the trailing row.

It is noteworthy that the low values of t_α lead to only minor changes in the head load distribution with cycle number, which is consistent with the findings of Klüber (1988) and Rakotonindriana (2009).

The cyclic changes of the load distribution, α_{Load}, from the simulations are plotted in Figure 7 (ζ_b = 0.2) and in Figure 8 (ζ_b = 0.4), respectively. In general, the leading row piles attract the highest amount of the load in the centrifuge tests as well as in the numerical simulations. While in the tests, the trailing piles take more load than the center pile, the simulations show the opposite distribution. For larger pile spacings the differences between the pile positions become smaller, i.e. the load is more evenly distributed within the group. In contrast to the tests, a clear trend in the development of the load distribution with the number of cycles cannot be identified for the simulation due to the limited number of cycles that could be calculated.

5 CONCLUSION

In general, the numerical model was capable of reproducing results from the centrifuge tests. However, in all tests the calculated results underestimated the lateral displacements derived from the centrifuge tests. Furthermore, the cyclic accumulation of the lateral displacements could be reproduced, especially for higher load magnitudes. A load dependency of the accumulation-ratio, as seen in the measurements, could not be reproduced properly in the simulation, leading to a too high accumulation-ratio at a small cyclic magnitude. To some extent, satisfactory correlations were found for the load distribution, especially at lower load magnitudes and within the first ten cycles.

Numerical simulations could help to provide a broader bandwidth of data for a general design tool. Consequently, it is necessary to enhance numerical models to achieve faster and more reliable calculations. Wichtmann (2016) developed a numerical calculation model, which could help to overcome the current limitations. The explicit nature of the so-called 'High Cycle Accumulation (HCA) Model' allows the consideration of high amounts of load cycles with varying magnitudes. Based on extensive parametric studies on calibrated reference models, a more general design approach could then be developed.

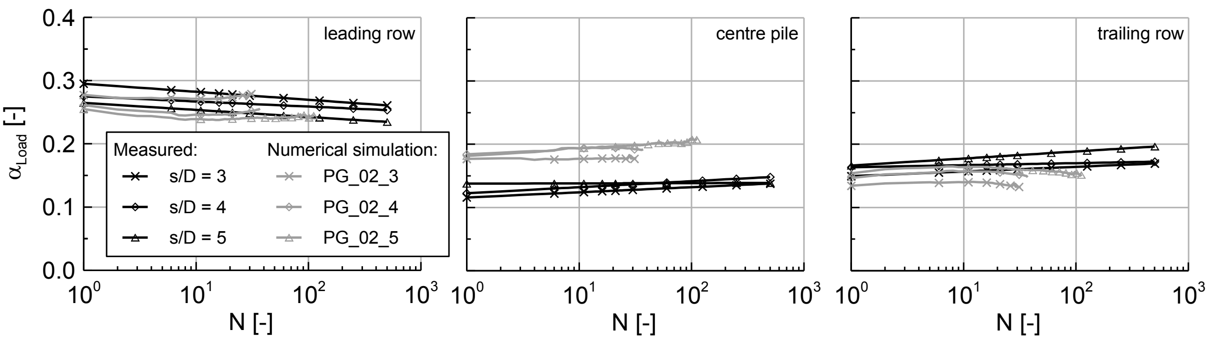

Figure 7 Changes in load distribution α_{Load} for ζ_b = 0.2 during cyclic loading

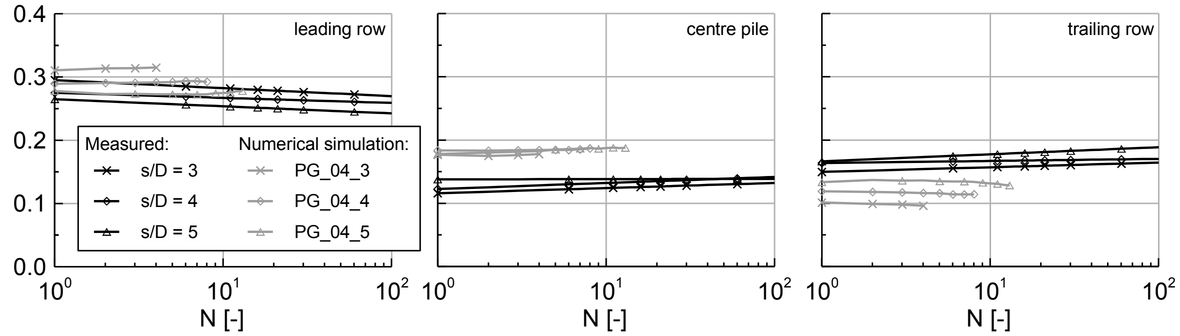

Figure 8 Changes in load distribution α_{Load} for ζ_b = 0.4 during cyclic loading

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