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The concept and design methodologies of resilient geotechnical and underground engineering

Concept et méthode de conception de RéTravaux géotechniques et souterrains

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ABSTRACT: Geotechnical and underground structures are usually designed based on elements or components rather than a system, and thus they are always lack of integrity, robustness and sufficient redundancy. Therefore, frequent geotechnical engineering collapses occur all over the world, and cause serious social impact and huge economic loss. In addition, the engineering function loss after collapse is difficult to recover. However, under the existing design theory frame, the system performance, the initial damage location and damage range caused by the local failure of geotechnical engineering are all lack of effective evaluation and control methods. In engineering field, three purposes of resilient design are high reliability, low disaster consequence and quick recovery, which indicate that resilient performance is a comprehensive safety evaluation index for an engineering project. However, in geotechnical engineering, the application and study of the resilience are very limited. Therefore, establishing a theory and design methodology to improve the resilient performance of geotechnical structures is of great significance for the infrastructure construction safety and resilient city development. In this study, several typical progressive collapse problems in geotechnical and underground engineering caused by low resilient performance were investigated, such as progressive collapses in deep excavation engineering, shield tunnel engineering and embankment engineering, etc.; the connotation of the resilient design concept to improve the resistance, adapting and recovery abilities of geotechnical and underground engineering under the condition of accidents and disasters was discussed; and the research progress on the resilience evaluation and resilient design in geotechnical and underground engineering was introduced.

RÉSUMÉ :les structures géotechniques et souterraines sont généralement conçues sur la base d'éléments ou de composants plutôt que d'un système, et sont donc toujours dépourvues d'intégrité, de robustesse et d'une redondance suffisante.Par conséquent, de fréquents effondrements de l'ingénierie géotechnique se produisent partout dans le monde, et causent un impact social grave et d'énormes pertes économiques.En outre, la perte de fonction technique après l'effondrement est difficile à récupérer.Cependant, dans le cadre de la théorie de la conception existante, les performances du système, la localisation initiale des dommages et la portée des dommages causés par la défaillance locale de l'ingénierie géotechnique sont autant de facteurs qui font défaut en ce qui concerne l'efficacité des méthodes d'évaluation et de contrôle.Dans le domaine de l'ingénierie, trois objectifs de la conception résiliente sont une haute fiabilité, une faible conséquence de catastrophe et une récupération rapide, ce qui indique que la performance résiliente est un indice global d'évaluation de la sécurité pour un projet d'ingénierie.Cependant, en ingénierie géotechnique, l'application et l'étude de la résilience sont très limitées.Par conséquent, l'établissement d'une théorie et d'une méthodologie de conception pour améliorer la performance résiliente des structures géotechniques est d'une grande importance pour la sécurité de la construction des infrastructures et le développement Urbain résilient.Dans cette étude, plusieurs problèmes typiques d'effondrement progressif en géotechnique et en génie souterrain causés par de faibles performances de résilience ont été étudiés, tels que les effondrements progressifs dans l'ingénierie des excavations profondes, l'ingénierie des boucliers et l'ingénierie des remblais, etc.;Les implications du concept de conception résiliente pour améliorer la résistance, l'adaptation et les capacités de récupération de l'ingénierie géotechnique et souterraine en cas d'accident et de catastrophe ont été discutées;Et les progrès de la recherche sur l'évaluation de la résilience et la conception résiliente en ingénierie géotechnique et souterraine ont été introduits.

KEYWORDS: Progressive collapse; Geotechnical Engineering; Underground Engineering; Resilience; Robustness

1 INTRODUCTION

Due to the design of geotechnical and underground structures generally proceeds element-by-element Zheng et al. (2011), they are always lack of integrity, robustness and sufficient redundancy. Therefore, engineering accidents, such as collapses of deep excavations, shield tunnels and embankments, etc., occurred frequently. However, few studies have been conducted on the evolution mechanism, simulation method and control theory of those progressive collapse problems, and this limits the development of Geotechnical and Underground Engineering.

In engineering field, three purposes of resilient design are high reliability, low disaster consequence and quick recovery, which indicate that resilient performance is a comprehensive safety evaluation index for an engineering project. However, in geotechnical engineering, the application and study of the resilience are very limited. Therefore, establishing a theory and design methodology to improve the resilient performance of geotechnical structures is of great significance for the infrastructure construction safety and resilient city development.

The resilient performance design aims to ensure geotechnical systems have enough robustness and redundancy to prevent from collapses or failures induced by the local damage or weakness. The research of resilient performance design of geotechnical systems includes: (1) identify the hazards and key elements in geotechnical system, (2) demonstrate the hazards whole evolution process of the progressive collapse triggered by the local damage, (3) investigate the propagation and termination mechanisms of the failure, and (4) propose control methodology to optimize the design and improve the resilient performance of geotechnical structures.

To prevent progressive collapse, three criterions can be considered to achieve the balance between construction cost and collapse loss of the geotechnical structure: (1) 1st level is to prevent the local failure; (2) 2nd level is to prevent propagation of failure triggered by local failure; and (3) 3rd level is to control the range of the progressive failure. A 30m deep excavation near the Nicoll Highway MRT station under construction along the Circle Line and near the Merdeka Bridge collapsed in Singapore in 2004. The failure started at the 9th level strutting connection. Yielding of the connection allowed the diaphragm walls to deform, overloading the struts in levels above, causing them to buckle. This triggered 130 m long progressive collapse of the diaphragm walls. The other examples include the excavation collapse at the Xianghu subway station on Hangzhou metro line 1 in 2008 in Hangzhou, China and flood walls and levees failure during New Orleans flooding in 2005 due to Hurricane Katrina. If these examples were designed using the proposed three level of criterions and considering the balance between construction cost and failure loss, the collapse range could be reduced.

The resilient performance of a geotechnical structure also aims to quickly restore its basic function in a short time following disaster. One example to further illustration this point is the deep-sea Su-ai tunnel located in intense seismic area of China. The shape memory alloys are used to recover the dislocation and reduce the opening of the tunnel joints in a short time after earthquakes to prevent from the inflow of sand and water.

In this study, several typical progressive collapse problems in geotechnical and underground engineering caused by low resilient performance were investigated, such as progressive collapses in deep excavation engineering, shield tunnel engineering and embankment engineering, etc. The connotation of the resilient design concept to improve the resistance, adapting and recovery abilities of geotechnical and underground engineering under the condition of accidents and disasters was discussed.

2 PROGRESSIVE FAILURE OF DEEP EXCAVATION

2.1 *The progressive collapse mechanism and control method of excavation retained by cantilever piles*

2.1.1 *Progressive collapse mechanism*

To investigate the mechanism of partial collapse (sudden failures of certain retaining piles) and progressive collapse of cantilever retaining piles, several model tests and numerical simulations were conducted.

As shown in the Fig. 1, the model test results show that partial collapse can cause a sudden increase in the bending moments of adjacent piles via an arching effect. The load-transfer coefficients are defined to be equal to the peak increase ratios of the maximum bending moments in adjacent piles (peak moments caused by collapse over the values before the collapse). Within a certain range of the extent of partial collapse, a larger partial collapse extent will produce larger load transfer coefficients and have a greater influence over a larger area. When the maximum load-transfer coefficient T_{max} is larger than the bearing capacity safety factor of the piles, the partial failure will lead to progressive collapse, as shown in Fig. 2. The influential factors of the progressive collapse mechanism, such as the partial collapse extent, excavation depth, and capping beam, were also investigated.

As shown in Fig. 2, a continuous capping beam can reduce T_{max}. For the same retaining structure, a higher soil strength will result in larger load transfer coefficients. Moreover, when the lateral stiffness of the retaining piles is lower, T_{max} will be smaller and the range of influence will be larger. In an excavation with a corner effect, when the number of failed piles is above a certain threshold, T_{max} decreases with an increasing number of failed piles. The corner effect can be recognized as a cause of the natural termination of progressive collapse.

During the progressive collapse process, a previously failed pile could cause new stress arching acting on the next adjacent pile, which is an evolution mechanism for progressive collapse. Simultaneously, the soil behind certain nearest intact piles could be loosened. When this unloading effect becomes sufficiently large, the soil arching effect cannot cause the failure of additional piles. Hence, the progressive collapse ceases, as shown in Fig. 3.

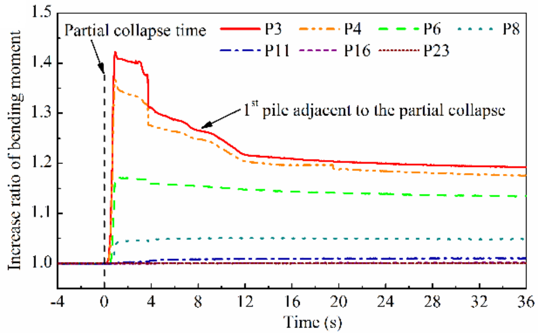

Figure 1. Load-transfer coefficients caused by partial collapse

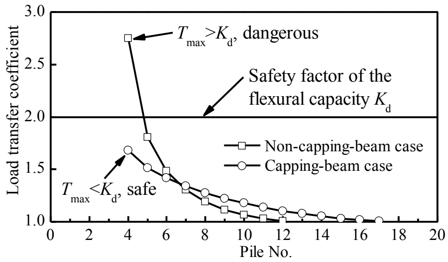

Figure 2. Comparison of the load transfer coefficients between the capping-beam case and the non-capping-beam case after the failure of 3 piles.

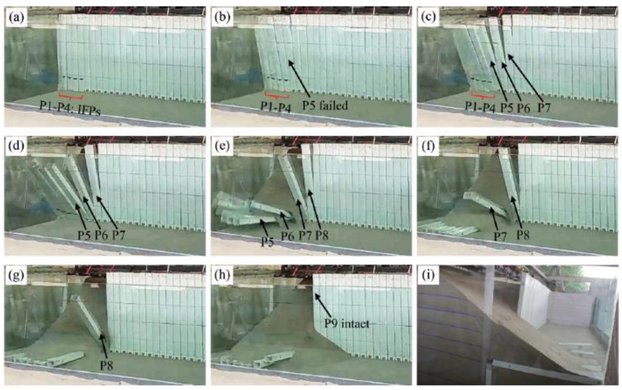

Figure 3. Progressive collapse process of cantilever piles.

2.1.2 *Control theory and design method*

Based on the progressive collapse mechanism mentioned before, the method of alternately setting progressive failure blocking units (abbreviated as blocking units method) to control the development of progressive failure is proposed, as shown in Fig. 4. Combining with the load transfer coefficient curve, the design principles of blocking units under different reinforced pile stiffness can be derived. As shown in the Fig. 5, the reasonable design of blocking units can effectively control the progressive failure caused by the partial failure within the setting range, such as when there is three reinforced piles in the blocking unit.

However, when the number of reinforced piles in the blocking unit was insufficient, progressive failure had crossover effect, which means the progressive failure continued to develop across the blocking unit and in turn led to the failure of the blocking unit, as shown in the Fig. 6. In practice, the escape and safety passages in excavations can also be designed in conjunction with the progressive failure blocking unit.

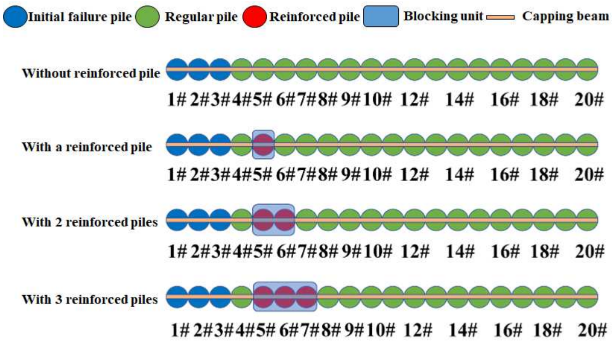

Figure 4. Calculation cases for single blocking unit condition

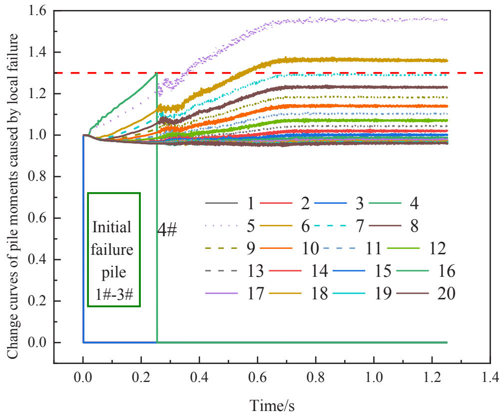

Figure 5. Change curves of pile moments for 3 reinforced piles

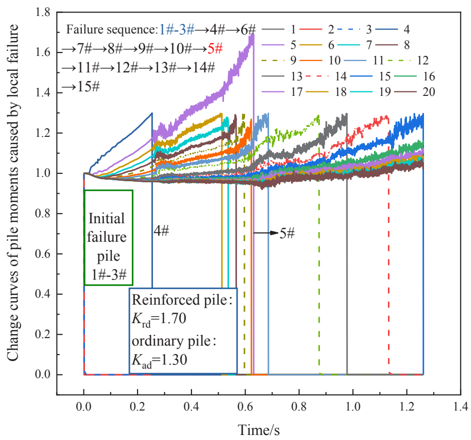

Figure 6. Change curves of pile moments for 1 reinforced piles

2.2 *A Control Measure of the Progressive Collapse of Excavations Retained by a Multilayer Strutting System*

Xianghu Station was the origin station of Hangzhou Metro Line 1#. The excavation depth and width of this station are approximately 16m and 20.5m, respectively, supported by 800mm thick diaphragm wall and 4 levels of pipe struts (diameter is 609mm and thickness is 16mm). At about 3:20pm on 15th November 2008, the western diaphragm wall of the excavation collapsed with the subsidence area of approximately 100m long, 40m wide and 7m deep.

There are several technical and administrative factors that contributed to this collapse Zhang and Li (2010). A most important and direct cause is over-excavation. In the collapse area, the excavation proceeded to the bottom when most struts at the 4th level had not been installed. Furthermore, the western diaphragm wall of the excavation also suffered the excessive vehicle load of the Fengqing Avenue. Consequently, the western diaphragm wall was broken at the point below the 3rd level struts. Then the collapse occurred, as shown in Fig. 7, but if the supporting system had higher redundancy, the collapse would be relatively slower and local.

Figure 7. The messy struts after the collapse

The connections of struts and diaphragm could significantly influence the integrity and robustness of the system. Fig. 7 shows that the bracket was two short steel angles welded on the embedded steel plate of the diaphragm. The pipe struts were just put on the brackets without welding or riveting to the diaphragm.

Numerical simulation of this collapse with different types of diaphragm-strut connection using PFC 2D was performed. When the western diaphragm is broken by shear force, if the struts have not been fixed to the diaphragm tightly and the connection can't bear any shear force or tension, the simulated collapse scenario is shown in Fig. 8 (a). It can be seen that the struts fell down and a slip surface was developed subsequently in soil and passed the point where diaphragm wall was broken, which is very similar to the actual situation. For comparison, a model in which the connections of struts and diaphragms are strong enough has also been calculated. Fig. 8 (b) shows the scenario when the calculation reaches a relatively equilibrium state, where the struts remain fixed with diaphragm wall at the both ends after the diaphragm wall was broken. Since the struts remained fixed to diaphragm wall at both ends, the horizontal struts would not fall and thus make it possible for the workers on the construction site at the bottom of formation to have more opportunities to evacuate. Compared Fig. 8 (a) with 8 (b), the connection between struts and wale is quite essential to prevent an overall failure of supporting system.

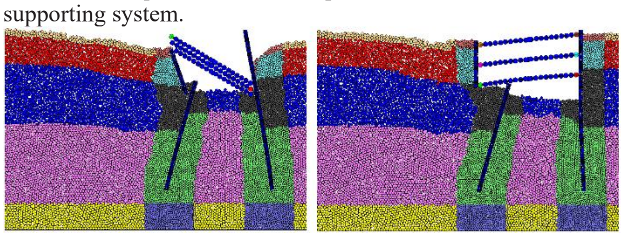

Figure 8. Collapse scenario when the diaphragm-strut connections are weak or strong: (a) weak connections and (b) strong connections

3 PROGRESSIVE FAILURE OF SHIELD TUNNEL

3.1 *Trigger of the progressive failure*

In this section, model tests were conducted to study the mechanisms behind the progressive failure of tunnel being triggered by local failure of tunnel.

3.1.1 *Trigger mechanism*

The model tests were conducted at Tianjin University, and the detailed information of the tests were presented in Zheng et al (2016). In the model tests, the local failure segment would be removed manually to simulate the local failure of the tunnels. The internal forces of the segments and the earth pressures were all monitored after the local failure of the tunnel, and the position of the load cells and strain gauges were shown in Fig. 9.

Fig. 10 showed the result of the model test when the cover depth of tunnel was 4 times of the tunnel diameter. Fig. 10 (a) collected the increasing ratio of the bending moment, λ_m, of the segment, which was calculated by:

$$\lambda_m = {M_a}/{M_b} \quad (1)$$

where M_a and M_b were the bending moments of the segments after and before the local failure of the tunnel. Fig. 10 (b) showed the increasing ratio of the earth pressure, λ_e, of the segment, which was calculated by

$$\lambda_e = {P_a}/{P_b} \quad (2)$$

where P_a and P_b were the earth pressures acting on segments after and before the local failure of the tunnel. It was found that the increasing ratio of the bending moment of the segment that was closer to the local failure segment was larger, and the increasing ratio of the earth pressure acting on the segment that was closer to the local failure segment was smaller.

As shown in Fig. 10 (b), after the local failure of the tunnel, the soil pressure acting on the segment near the failed segment decreased, although the earth pressure acting on the waist of ring 10# increased after the local failure due to the transient arching effect of the soil which disappeared in about 0.1s after the local failure. The pressures acting on the rings near the failure ring were different with that under the stated working conditions. The circular rings with weak joint between segments had no advantages under the changed working conditions, and the moments of the segments of adjacent rings increased which may excess the capacities of the segments. Consequently, the progressive failure of the adjacent segments may be induced.

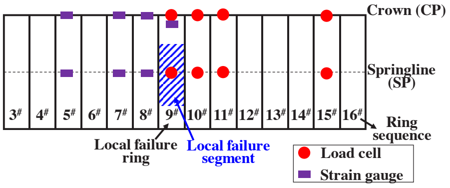

Figure 9. Sketch of model tunnel

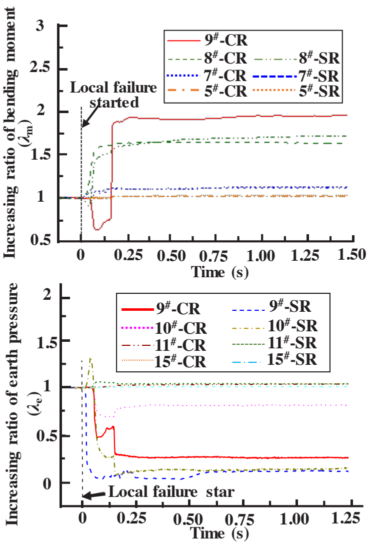

Figure 10 Result of the model test: (a) Increase ratio of bending moment after local failure and (b) Increase ratio of earth pressure after local failure

3.1.2 *Measures against the escalation of local failure*

In preventing the local failure of the tunnel from developing to large-scale progressive failure, it was important to increase the capacities of the adjacent rings or to reduce the impact of the local failure to the adjacent rings. In this section, two types countermeasures were proposed to protect the tunnel from some inevitable local failures.

(1) Increasing the strength of adjacent rings

Local failure would change the pattern of the pressure acting on the adjacent rings, and subsequently, the bending moment in adjacent rings would increase. Increasing the reinforcement ratio of the adjacent rings could increase the capacities of the segments, and of course, increase the performance of the adjacent rings under local failure of the tunnel.

However, it was hard to foresee the location that would suffer local failure. In practical engineering, temporary steel retaining structure could be adopted at locations where the risk of local failure of the tunnel was high, like the rings being built connecting passages.

(2) Soil improvement

To reduce the impact of the local failure to the adjacent rings, replacing the covering soil above the tunnel with light materials was an effective measure to reduce the earth pressure acting on the rings, like EPS material used in part of the tunnel of Shanghai metro ling 1. Grouting treatment could be used on the soil at the side of the tunnel to reduce the flowability of the soil. These two measures were not conflicting with each other, and could be adopted in same engineering to protect the structures.

3.2 *Propagation of the progressive failure*

In practical engineering, it was very hard to predict the locations of local failures. Once the effective measures against the local failures were not applied in time, the development of the local failure of the tunnel would threaten the safety of the whole tunnel. Therefore, it was necessary to clarify the mechanism behind the development of the progressive failure of tunnel.

3.2.1 *Propagation mechanism*

The progressive failure of the tunnel, a muti-body structure, involves large deformation of the soil and structures, and soil-structure interaction; therefore, the mechanism can be more complex. In this section, a large deformation modelling technique, i.e., the coupled Eulerian-Lagrangian (CEL), was introduced to simulate the progressive failure behavior of the tunnel.

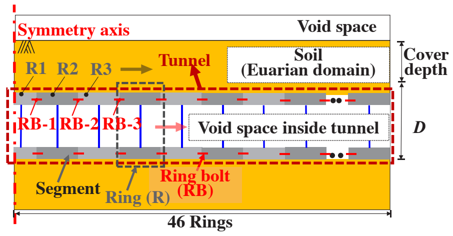

Figure 11. CEL model

In the CEL model, as shown in Fig. 11, by enforcing the failure of a few rings, the progressive failure in the tunnel was successfully reproduced. It was found that local failure in the tunnel caused a redistribution of the earth pressure that acted on it; the rings close to the location of the failure received higher loads and therefore may fail first. The ring bolts played negative roles in the development of the progressive failure. They provided the paths for load transfer between the failed rings and the working rings, which aggravated the propagation of the failure.

3.2.2 *Countermeasures against the propagation of the progressive failure*

Based on the analysis in Section 3.1, it was known that the strength of the rings and the bolts between rings were important roles in the development of the progressive failure of the tunnel. Therefore, the study of the countermeasures should focus on the performances of them.

For ease of clarification, we defined the damage potential, F_p, to evaluate the scope of the failure Zheng et al. (2020a):

$$F_p = {n_f}/{n_0} \quad (3)$$

where n_0 was the scope of local failure of the tunnel; n_f was the total number of failed rings of the tunnel at end of the progressive failure. Larger F_p implied that more rings were brought to failure in the progressive failure of the tunnel.

(1) Weakening the ring bolts

During the development of the progressive failure of the tunnel, the load of the earth pressures and the failed rings was transmitted to the working rings by ring bolts. Therefore, weakening the ring bolt was helpful in reducing the loss of the progressive failure of the tunnel, as shown in Fig. 12.

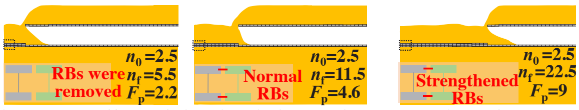

Figure 12. Effect of the ring bolts on the propagation of progressive failure of tunnel: (a) No ring bolts, (b) Normal ring bolts and (c) Strengthened ring bolts

However, weakening the ring bolts may cripple the integrality of the tunnel and then bring other problems, e.g., the uneven settlement and the leakage of water from the weak spots. Therefore, comprehensive judgements should be made on the safety of the tunnel if this measure was decided to be adopted.

(2) Increasing the strength of the rings

Segments was the basic components that bearing the load of the soils. Increasing the strength of the segments was not only helpful in protecting the tunnel from the local failure, but also an effective measure to reduce the scale of the progressive failure of the tunnel (as shown in Fig. 13). However, considering the high expense to increase the strength of all the segments of the tunnel, such countermeasures may be suitable for implementation only at locations where the tunnel is likely damaged or at higher risks.

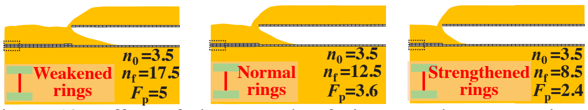

Figure 13. Effect of the strength of rings on the propagation of progressive failure of tunnel: (a) Weakened rings, (b) Normal rings and (c) Strengthened rings

(3) Enhancing the ductility of the rings

Enhancing the ductility of the rings could increase the deformability of the rings. Once the local failure occurred, due to the high deformability of the rings, the distribution of the changed earth pressure could be optimized, because more rings would participate in bearing the abnormal loads and reach their full potential strengths in resisting the propagation of the failure, as shown in Fig. 14. In other words, the synergy among the rings was enhanced and the robustness of the tunnel was improved.

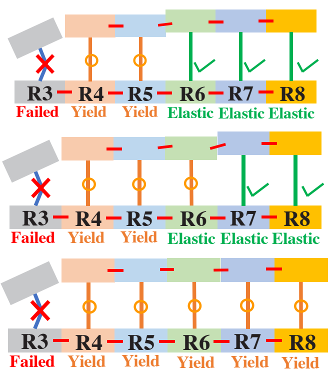

Figure 14. Working state of the rings: (a) Working state of weakened-ductility rings , (b) Working state of normal-ductility rings and (c) Working state of enahnced-ductility rings

In practical engineering, enhancing the ductility of the rings can be achieved by either employing construction materials with good plastic deformability Zheng et al. (2020a) or by pre-setting energy-dissipating units (e.g., plastic hinge). Such treatments improve the robustness of the tunnel while not significantly increasing the engineering cost; therefore, they may be suitable for widespread use.

4 PROGRESSIVE FAILURE OF EMBANKMENT

4.1 *Research status of composite foundation stability*

The composite foundation is prone to instability under an embankment load, as shown in Fig. 15. According to different types of composite foundations under flexible loads, different calculation theories are required to avoid instability. A correct understanding of failure modes is the base of the establishment of these calculation theories.


Figure 15. Failures case histories of embankments

4.2 *Failure mechanism under embankment load*

Incompressible dipping substrata are commonly encountered in engineering practice. Compared to horizontal underlying strata, the inclined underlying stratum increase the risk of collapse of embankments reinforced with columns because it weakens the restraint of the column base. The instability failure of rigid piles-supported embankments can be divided into internal failure and external failure. Zheng et al. (2017) proposed the internal failure mode and the secondary bending failure theory of column-supported embankments. Bending failure first occurs in the piles beneath the embankment toe near the interface between the clay and the sand layer. It causes the rapid decline of the tensile stress and bending moment at the failure position. At the same time, the stress redistribution of piles and soils leads to the increase of tensile stress and bending moment of adjacent piles, further resulting a progressive failure of adjacent piles. With an increase in the surcharge, a continuous failure occurs in the upper part of piles due to a greater release of energy, leading to the propagation of a slip surface and the global instability of the embankment, as shown in Fig. 16.

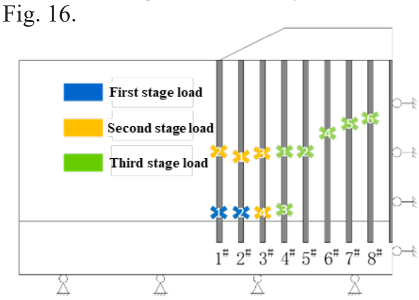

Figure 16. Progressive failure of rigid columns

When the embedment of pile is weak and the strength of soil around the pile is low, the pile is easy to overturn and cause tilting failure Zheng et al. (2020b). For column-supported geosynthetic-reinforced embankment, Zheng et al. (2020c) investigated the effectiveness of geosynthetics on improving the embankment stability when the underlying stratum is inclined. The influence of geosynthetic tensile stiffness on the ultimate surcharge and failure mechanism is studied. A deep-seated failure with column tilting occurs when the geosynthetic tensile stiffness is low, whereas a lateral sliding occurs when the geosynthetic tensile stiffness is high. Geosynthetics effectively remedy an insufficient embedment induced by an inclined underlying stratum and ensure the embankment stability.

4.3 *Resilient performance-based design*

Zheng et al. (2020d) considered the difference of the contribution of rigid piles at different positions under the embankment to the stability of the embankment. The concept of the key pile and the non-consistent design method are put forward. By improving the characteristics of a few key piles, the overall stability of embankment can be improved effectively and economically.

Based on the bending failure of rigid pile, Zheng et al. (2020e) proposed that the ductility of plain concrete piles could be enhanced by reinforcement. After changing the behavior of the pile, the tension strength of the pile is improved and the stability of the composite foundation is enhanced. The failure mechanism of piles at different position under embankment is different. Thus, a method of subdivision non-equi-strength design is proposed to reinforce the piles in the flexural shear zone and the flexural zone. This method basically achieves the stability of embankment supported by reinforcing all piles, as shown in Fig. 17.

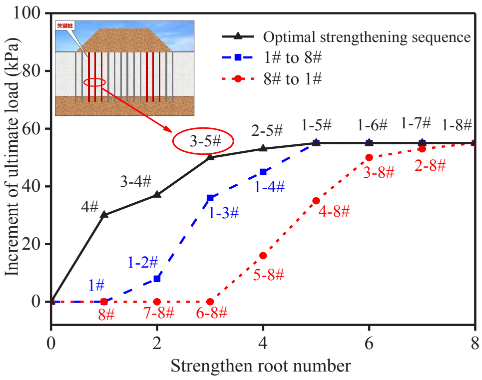

Figure 17. Non-uniform strength design method

Zhou et al. (2019) proposed that the titling failure usually happened when the underlying layer is inclined. With the increase of the embedded depth of rigid pile, the failure mode of pile changes from titling failure to bending failure. The contribution of pile varies with different locations. The stability of embankment can be improved economically and effectively by making full use of the characteristics of piles at different positions only by increasing the embedded depth of some piles at key positions, as shown in Fig. 18.

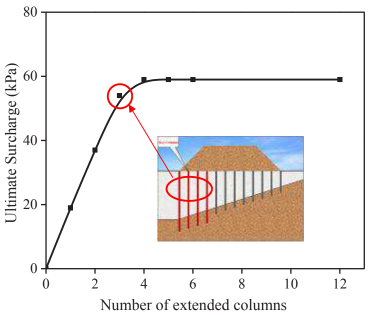

Figure 18. Non-uniform length design method

5 CONCLUSION

In this study, several typical progressive collapse problems in geotechnical and underground engineering caused by low resilient performance were investigated. The connotation and some research of the resilient design concept to improve the resistance and adapting abilities of geotechnical and underground engineering under the condition of accidents were discussed. Geotechnical structures always lack integrity and sufficient redundancy. The occurrence frequency and consequences of progressive collapses in Geotechnical Engineering could be even higher than those in Structural Engineering. Therefore, establishing a theory and design methodology to improve the resilient performance of geotechnical structures is of great significance for the infrastructure construction safety and resilient city development.

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