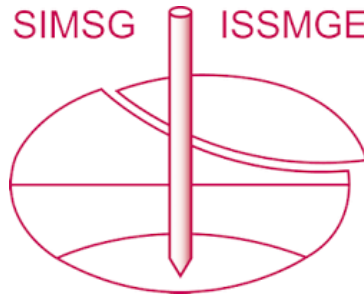


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The paper was published in the proceedings of the 20th International Conference on Soil Mechanics and Geotechnical Engineering and was edited by Mizanur Rahman and Mark Jaksa. The conference was held from May 1st to May 5th 2022 in Sydney, Australia.

Shear strength estimation of very coarse mine waste using Barton's empirical criterion and a Bayesian analysis

Estimation de la résistance au cisaillement des déchets miniers très grossiers à l'aide du critère empirique de Barton et une analyse bayésienne

Sandra Linero-Molina & Luis-Fernando Contreras
SRK Consulting Australasia, Australia

Justin Dixon
Fortescue Metals Group, Australia

ABSTRACT: Evaluating the shear strength of mine waste rock containing particles of metre scale is a challenging task because commercial laboratory testing devices can only accommodate samples composed of particles a few centimetres in size. The shear strength empirical model of Barton & Kjærnsli (1981) is therefore frequently used to assess the nonlinear shear strength of very coarse granular material for stability assessment of mine waste dumps. According to the model, the stress-dependent structural component of the shear strength is dictated by the uniaxial compression strength of the parental rock, the degree of roundness of the constitutive particles, D_{50} particle size and the porosity of the material. This structural component is added to the basic friction angle determined on sawn surfaces of the rock to determine the material shear strength. In this article, we discuss practical methods for collecting the information required as input to the model. We also show the implementation of a Bayesian approach for the inference of the model parameters using the results of large-scale direct shear testing and the improvement of the estimation by considering additional information on the material. The approach allows for the customisation of the model parameters to consider a material characterised by high particle strength and a pronounced particle size-shape correlation, due to the sedimentary origin of the parental rock.

RÉSUMÉ : L'évaluation de la résistance au cisaillement des stériles miniers contenant des particules à l'échelle métrique est une tâche difficile, car les appareils d'essai de laboratoire commerciaux ne peuvent accueillir que des échantillons composés de particules de quelques centimètres. Le modèle empirique de résistance au cisaillement de Barton et Kjærnsli (1981) est donc fréquemment utilisé pour évaluer la résistance au cisaillement non linéaire d'un matériau granulaire très grossier pour la conception de la stabilité des décharges de déchets miniers. Selon le modèle, la composante structurelle dépendante de la contrainte de la résistance au cisaillement est dictée par la résistance à la compression uniaxiale de la roche parentale, le degré de rondeur des particules constitutives et la taille des particules D_{50} et la porosité du matériau. Ce composant structurel est ajouté à l'angle de frottement de base déterminé sur les surfaces sciées de la roche pour déterminer la résistance au cisaillement du matériau. Dans cet article, nous discutons des méthodes pratiques pour collecter les informations nécessaires pour alimenter le modèle. Nous montrons également la mise en œuvre d'une approche bayésienne pour l'inférence des paramètres du modèle en utilisant les résultats d'essais de cisaillement direct à grande échelle et l'amélioration de l'estimation en considérant des informations supplémentaires sur le matériau. L'approche a permis de personnaliser les paramètres du modèle afin de considérer un matériau caractérisé par une résistance élevée des particules et une corrélation granulométrique-forme prononcée, en raison de l'origine sédimentaire de la roche parentale.

KEYWORDS: Non-linear shear strength, very coarse granular material, Bayesian inference.

1 INTRODUCTION

Mine waste rock in open-pit mining are materials with non-significant economic value that needs to be removed from the pit during mining production to gain access to the ore and that is transported to waste dumps for storage (Hartman & Mutmansky 2002, Hawley 2017). The particle size distribution (PSD) and the maximum particle size (D_{max}) of a waste rock material will be naturally variable due to spatial variation in the material properties and drill and blast practices. In large-scale mining, where miners increasingly opt for large capacity trucks, wastes with blocks up to 2 m in diameter are not unusual. Waste dumps may reach hundreds of metres in height (Linero et al. 2007, Palma et al. 2009). They are traditionally built in lifts by end-dumping material over the dump face. Due to large dump deformations, the primary geotechnical parameter required for the stability design of waste dumps is the steady-state shear strength.

Evaluating the steady-state shear strength of mine waste rock containing particles of metre scale is challenging because commercial laboratory testing devices can only accommodate

samples composed of particles a few centimetres in size. To overcome testing limitations, the shear strength is frequently estimated using the shear strength empirical model of Barton & Kjærnsli (1981) (B-K criterion), which considers the nonlinearity of the shear strength envelope, characterising the behaviour of very coarse granular materials submitted to very high loads (Leps, 1970).

In the B-K criterion, a stress-dependent structural component of the shear strength is parametrised with the equivalent roughness R and equivalent strength S . The structural component is added to the basic friction angle (ϕ_b) of the parental rock to determine the shear strength of the waste rock material. Barton modelled this criterion using R and S as equivalent to his joint roughness coefficient JRC and joint wall compression strength JCS in his shear strength criterion for rock joints (Barton 1973).

The research presented in this paper explores the customisation of the parameters R and S of the B-K criterion, taking advantage of the availability of large scale testing results. A mine waste rock material composed of particles with a maximum diameter of 120 mm was investigated and high-pressure large-scale direct shear testing (DST) was conducted.

Bayesian statistics was then implemented to explore the best R and S values to represent the behaviour of the material. The analysis was then enhanced to illustrate the potential of the method to consider other sources of material information.

The Bayesian approach allowed, not only customisation of the parameters R and S for the specific material investigated, but also further insight into the performance of the model.

2 MATERIAL CHARACTERISTICS

The material evaluated, referred to as overburden, consists of Neogene alluvial and colluvial sediments that locally overlie channel iron deposits at the Solomon Iron Ore Mine in the central Hamersley Ranges within the Pilbara region of Western Australia. The sediments originated from weathering, erosion, and transportation of Precambrian banded iron formation rocks (Morris 1993). The sediments are mainly composed of gravel and sand-sized fragments, with cobbles of up to 120 mm in diameter, and less than 2% fines. The D_{50} particle size range from 2 mm to 24 mm according to gradation information obtained through operational verification data. The overburden particles are sound and strong with losses in the Los Angeles test of 24.6%, and water absorption 1.9%. The specific gravity of the material is variable and was also found to be size-correlated. It is 3.82 for the fraction below 4.75 mm and 3.20 for the fraction between 4.74 mm and 75 mm. The plastic limit for the fines is around 5% and the liquid limit varies between 19% and 27%. The as-sampled natural moisture content was around 2%. The overburden particles are sub-angular to sub-rounded with shapes ranging from slabs to sub-equant blocks, with the peculiarity to be size-shape correlated. Hence, larger particles are flatter and platier than smaller particles, which tend to be more equant to rod-shaped in comparison. The results of the particle shape investigation are described by Linero et al. (2017).

The shear strength of the material was investigated using a large-scale direct shear apparatus with a 720 mm square shear box. The characteristics of the equipment and sample construction are described by Linero et al. (2020).

Samples with the prototype PSD and other four variants called M1, M2, M3, and MB were tested. The samples were designed to examine the effect of gradation modifications on material behaviour as part of an investigation that will be described in a different paper. Particle size distributions of the samples tested are illustrated in Figure 1. Testing conditions and friction angles at large displacement, for the different materials and normal stresses implemented, are summarised in Table 1.

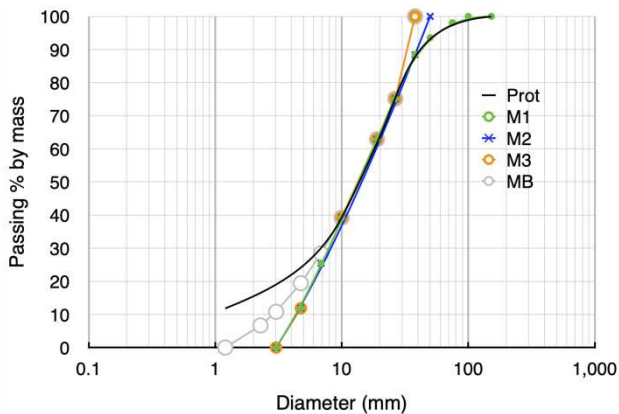


Figure 1. Particle size distribution of the prototype and the variant samples tested in the large-scale direct shear apparatus.

3 BARTON-KJÆRNSLI STRENGTH MODEL

The non-linear B-K criterion is represented by Eq. 1, where φ is the effective friction angle, φ_b is the basic friction angle of the rock, and R is the equivalent roughness and S is the size-dependent equivalent strength of the waste particles.

$$\varphi = \varphi_b + R \log_{10} \left(\frac{S}{\sigma_n} \right) \quad (1)$$

Table 1. Summary of test conditions and results.

PSD ID	σ_n MPa	D_{max} mm	G_s	γ_{max} kN/m ³	γ_{min} kN/m ³	n %	φ_{DST} (°)
Prot	0.5	100	3.32	24.6	20.1	38	38.7
Prot	1.0	100	3.32	24.6	20.1	36	40.1
Prot	2.0	100	3.32	24.6	20.1	38	37.5
Prot	3.0	100	3.32	24.6	20.1	37	35.5
M1	0.5	100	3.26	22.6	18.1	41	38.3
M1	1.0	100	3.26	22.6	18.1	41	37.5
M1	2.0	100	3.26	22.6	18.1	42	36.5
M1	3.0	100	3.26	22.6	18.1	42	33.2
M2	0.5	50	3.26	22.7	17.7	43	38.5
M2	1.0	50	3.26	22.7	17.7	42	37.5
M2	2.0	50	3.26	22.7	17.7	40	37.5
M2	3.0	50	3.26	22.7	17.7	42	34.2
M3	0.5	38	3.26	22.8	17.7	40	38.5
M3	1.0	38	3.26	22.8	17.7	39	38.7
M3	2.0	38	3.26	22.8	17.7	42	36.8
M3	3.0	38	3.26	22.8	17.7	44	36.4
MB	0.5	38	3.30	23.3	18.4	40	38.3
MB	1.0	38	3.30	23.3	18.4	40	37.0
MB	2.0	38	3.30	23.3	18.4	41	35.4
MB	3.0	38	3.30	23.3	18.4	44	34.8

σ_n = Normal stress γ_{max} = Maximum density
 n = Initial porosity γ_{min} = Minimum density
 D_{max} = Maximum particle diameter G_s = Specific gravity
 φ_{DST} = Direct shear friction angle at large displacement

3.1 Basic friction angle

The φ_b can be estimated in tilting tests using dry, sawn surfaces of the parental rock (Barton & Kjærnsli 1981). In the mining context, it is more often estimated from direct shear tests conducted on saw-cut samples selected from drill cores. This information is usually readily available in the geotechnical drillhole database, as it is also used for rock mass characterisation for pit design purposes. Because of the relative small size of the particles comprising the overburden, core samples for saw-cut testing were not available, so φ_b could not be explicitly investigated. It was known, nevertheless, that it probably varied between 20° and 40° as this was the range containing the values observed for all rock units investigated in the area from which the overburden developed.

3.2 Stress dependent structural component of the shear strength

The factor R can be determined from the empirical chart proposed by Barton & Kjærnsli (1981) as a function of the degree of particle roundness and the porosity (n) of the dumped waste rock material. It ranges between 0 and 15 for loose arrangements of rounded, very smooth particles to dense arrangements of very angular and rough particles, respectively. The parameter R captures the fact that the more angular, rough and well packable a material is, the higher its shear strength. In the current version of the model, R does not allow for differentiation based on the characteristic aspect ratio of the rock fragments, which is now known to play a significant role in material strength, due to its direct impact on particle rolling resistance/ability during shearing (Azéma et al. 2016, 2017, Linero et al. 2019). For the overburden material R can be defined by Eq. 2 as a function of n .

The equation was obtained as the best fitting curve for this material on the B-K chart:

$$R = 20.2e^{-4.1n} \quad (2)$$

The porosity of dumped waste material can be estimated by reconciliation of the haul-weight and the volume of large-scale dumped piles, or actual waste dumps. In the latter case, LiDAR information from pre-mining and at-the-time-of-the evaluation topography are compared. Large-scale determination of porosity was not conducted in this study. Nevertheless, determinations of maximum and minimum density conducted in the laboratory, for the samples tested in the direct shear apparatus, suggested that porosity may range between 0.26 and 0.46 for the overburden.

The factor S can be determined from the empirical chart proposed by Barton & Kjaernsli (1981) as a function of the unconfined compressive strength (UCS) of the parental rock and the characteristic particle size of the material, usually arbitrarily adopted as D_{50} . The parameter S captures the fact that smaller particles tend to be more sound and crushing resistant than larger fragments (Silvani 2007, Zhang et al. 2015). Therefore, the curvature of the shear strength envelope is less pronounced in the case of finer, sand-size material, than in the case of materials containing coarser particles. In the current version of the method, the evaluation of S differentiates between triaxial and plain shearing because it has been observed that crushing of particles is more pronounced when the material is submitted to triaxial loading. Nevertheless, in practice, the estimation for the triaxial condition is usually adopted as a conservative precaution, as the loading conditions inside a dumped slope are certainly complex and variable along the potential failure surface. For the triaxial loading case, S is defined by Eq. 3, obtained as the best fitting curve of the B-K chart.

$$S = 0.70 UCS(D_{50})^{-0.24} \quad (3)$$

The UCS data are usually available in the geotechnical drillhole database of the mine. Additional data can be sourced by testing selected core samples representing the rock analysed, or by correlation with point load tests (PLTs) or Schmidt hammer test results. In this investigation, because of the small size of the particles, drill core samples for testing UCS were not available. The strength of the particles was investigated by conducting PLT and UCS tests on 25 mm high core samples (mini cylinders) drilled from exceptionally large fragments. A correction (Turk & Dearman 1986) was used to standardise the results for a length-to-diameter ratio of 2 and 50 mm core diameter. The information from the mini-cylinders allowed the calibration for converting PLT values to UCS values. The information collected is presented in the boxplot in Figure 2.

In a boxplot, the lower and upper box boundary represents the 25th and 75th percentile of the data respectively. The line inside the box represents the median. With this, the four quartiles Q1 to Q4 can be identified. The box illustrates the interquartile range $IQR = Q3 - Q2$ where 50% of the data are located. The 'minimum' of the distribution is defined as $Q1 - 1.5 * IQR$, and the 'maximum' is defined as $Q3 + 1.5 * IQR$. Values higher or lower are considered outliers.

A representative particle size distribution needs to include fragments of the maximum particle size of the material evaluated. Consequently, the sampling procedure for material gradation determination needs to be defined according to the maximum particle size of the material. For coarse wastes with maximum particle sizes in the range of metres for example, samples of about 20 tonnes of material are required that are largely processed on-site with only the material below 75 mm being quartered and transported to the laboratory for assessment of the

gradation of the finer fraction. In this case, the weight of the fragments larger than 0.30 m is determined from the measurements of the fragment's dimensions. Another less precise but practical methodology is using an image processing algorithm. A calibration to access the potential discrepancies is recommended in this case as the method tends to overlook the finer material.

The challenge at a feasibility level study stage is that there is typically no blasted waste rock available and hence PSD testing on actual waste rock is not possible. In these scenarios, blasting fragmentation empirical assessments and site benchmarking are required to be adopted with an appreciation of the associated limitations. During data collection, a representative number of samples are required to be collected and assessed to account for material variability. The reliability of the data source should be reflected in the values. Several data sources are typically considered to assess a representative D_{50} value; the adopted order of priority for the data source is: large scale bulk sampling and laboratory PSD (i.e. sample size > 1 tonne); Split-Net analysis of waste rock dump, fill embankment and existing stockpile material if available; small-scale laboratory sampling with an estimate of oversize during sampling; blasting fragmentation empirical assessment based on known site-specific material properties and a typical blast design pattern; benchmark against a similar site (Dwumfour et al. 2020).

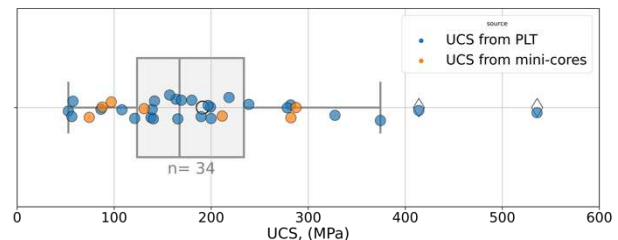


Figure 2. Boxplot of the UCS data collected for the overburden. Filled circles represent the data points. Outliers are identified with diamonds. The average value is represented by the empty circle. The jitter along the categorical axes is included to prevent the overlap of data points.

Several methodologies have been implemented for the determination of the gradation of the overburden across the years, some linked with the utilisation of the material for the construction of engineered structures. They refer to Split-Net determinations and laboratory determinations on medium to large-scale samples. To determine the gradation of the prototype material tested in the direct shear test, for example, the material was machine-excavated from the deposit and direct packed into heavy-duty one-cubic-metre bags for transport to the laboratory, limiting re-handling and segregation. The sample in the laboratory was processed by hand following the philosophy described in standards. On another occasion, a rotary divider was also used in the laboratory for reducing the sample in a heavy-duty bag into subsamples of about 20 kg each for manual processing. Both methodologies provided very similar results. The available D_{50} information collected across the years is summarised in the boxplot in Figure 3.

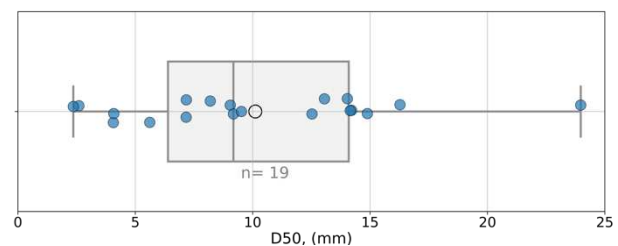


Figure 3. Boxplot of the D_{50} data collected for the overburden. Filled circles represent the data points. The average value is represented by the

empty circle. The jitter along the categorical axes is included to prevent the overlap of data points.

4 INFERENCE OF THE B-K PARAMETERS

4.1 Bayesian approach using direct shear test results

The advantages of the Bayesian approach for inference of parameters are described in detail by Contreras et al. (2018) in the context of intact rock strength. The Bayesian analysis enables the estimation of the more likely parameter values given three elements: (1) a model, (2) the data input to the model and (3) any prior information on the parameter values. The model corresponds to a mathematical function that represents the performance of a system of interest. The data correspond to measurements of the actual performance of the system that are compared with the model predictions to define errors. The prior information typically consists of valid ranges of credible values of the uncertain parameters to be inferred. These three elements (model, data and prior information) are combined in a probabilistic function that contains the set of parameters for inference. This function corresponds to a posterior probability distribution within the Bayesian concept and gives probability values for particular sets of parameters. The objective of the analysis is to define the sets of parameters that produce the highest probability values (minimum errors). Generally, the parameters are defined by evaluating the posterior function with a technique known as Markov chain Monte Carlo (MCMC) analysis (Kruschke 2015).

A simplified representation of the Bayesian model for the strength characterisation of the overburden is shown in Figure 4.

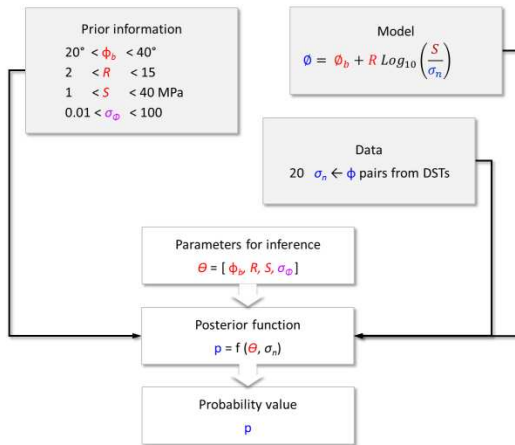


Figure 4. Conceptual Bayesian model for inference of the friction angle of overburden using only large-scale DST results.

The model corresponds to the B-K strength model represented by Eq. 1. The data consist of the large-scale direct shear test results that include measurements of ϕ for particular values of σ_n , and the prior information consists of the known ranges of credible values for the uncertain parameters in the B-K equation.

The construction of the posterior function uses the Bayes' rule concept and involves the evaluation of the likelihood of the dataset, which is combined with the prior component to define the so called posterior function. This function includes the parameters for inference grouped in the vector θ in the diagram of Figure 4. The evaluation of the likelihood of the data entails the evaluation of errors defined as the differences between model predictions and data measurements. Figure 5 illustrates how errors are measured in the Bayesian analysis.

A normal distribution is centred in the model prediction and is used to quantify the probability of each data point. In this way,

small errors would have higher probability values and vice versa. The product of the probabilities of the data points gives the probability of the dataset for a particular selection of parameters. The objective of the method is to find the set of parameters (ϕ_b , R , S) that result in higher probability values of the dataset, i.e. smaller errors. For this search, the method should try normal distributions with different widths and for this reason, the standard deviation of the normal distribution used to quantify the errors of ϕ (σ_ϕ) is an additional parameter that needs to be investigated.

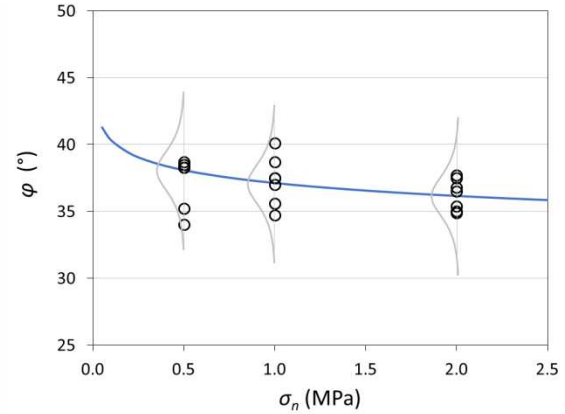


Figure 5. Measurement of errors with a normal distribution centred in the model prediction.

The specification of the prior distributions is based on the characteristics of the datasets and consists of setting up values sufficiently vague to avoid constraining the result. The vague priors were defined with uniform distributions with ranges from 20° to 40° for ϕ_b , 2 to 15 for R , 1 MPa to 40 MPa for S and 0.01 to 100 for σ_ϕ . The posterior probability distribution is evaluated by sampling the parameters with an MCMC algorithm. The methodology outlined in Figure 4 was implemented in the Python programming language (PSF, 2001). The analysis is carried out with the affine-invariant ensemble sampler algorithm implemented in the emcee Python package developed by Foreman-Mackey et al. (2013).

The results of the analysis are summarised in the corner plot of Figure 6 and the envelope graph of Figure 7.

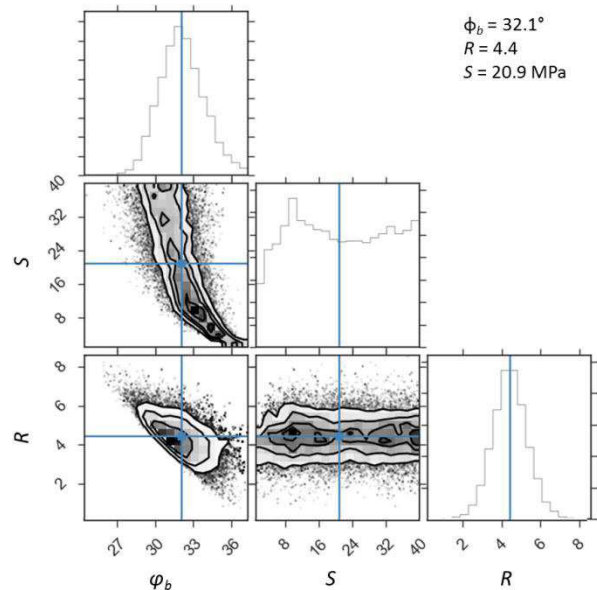


Figure 6. Corner plot of inferred parameters with the Bayesian analysis of DST results.

The scatter plot shows the correlation between the inferred parameters, and the histograms define the ranges of likely values. These results suggest a strong negative correlation between ϕ_b and R and a moderate negative correlation between ϕ_b and S . The scatter plots also indicate that S appears to be unbounded as the prior used for this parameter is transferred to the result without a preferential value. This result suggests the need to support the estimation of S with data from UCS and D_{50} determinations to avoid the bias caused by the prior in the current analysis.

The sampled values of the parameters represented in the scatter plots produce a spread of the B-K envelopes around the mean fit as indicated in the graph of ϕ versus σ_n shown in Figure 7. The graph includes the data points and the band of envelopes corresponding to the 95% highest density interval (HDI) of the parameters, which reflects their uncertainty.

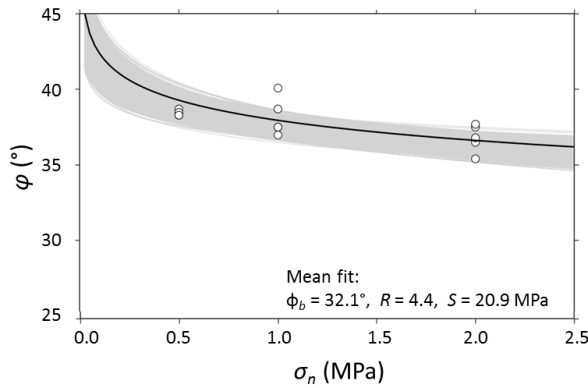


Figure 7. Uncertainty of the B-K envelopes represented by the 95% HDI of parameter values in Figure. 7.

4.2 Improved strength estimation with additional data

To define the parameter S aiming for greater definition on the shear strength results the Bayesian model was extended to include Equation 3 to calculate S from UCS and D_{50} data to prevent the bias caused by the assumed prior in the analysis. The additional data included the 34 UCS values and the 19 D_{50} determinations presented in Figures 2 and 3. The conceptual model of the Bayesian analysis with the additional data inputs is shown in Figure 8.

In this case, the parameters for inference have been expanded to include mean values of UCS and D_{50} and the standard deviations σ_{UCS} and σ_{D50} to quantify the errors when the inferred parameters are compared with their respective values in the datasets. The prior information has also been updated based on the datasets by defining ranges sufficiently wide to avoid constraining the results.

The results of the inference analysis considering the additional data are summarised in the simplified corner plot of Figure 9. This graph excludes the standard deviations of the errors, which are normally called the nuisance parameters, and replaces UCS and D_{50} with the respective calculated S values. The results show a sharp definition of the inferred parameters and also confirm the strong negative correlation between ϕ_b and R identified with the previous analysis. The negative correlation is purely a statistical condition imposed by the data points to ensure a good fit with the model. The value of S is now well defined above the range assumed in the previous analysis based on judgement. The new results also indicate a minor reduction of ϕ_b and confirm the value of R obtained initially. The variability of the inferred parameters is reflected in the uncertainty of the envelopes of ϕ as shown in Figure 10 with the band of envelopes for the 95% HDI of parameters.

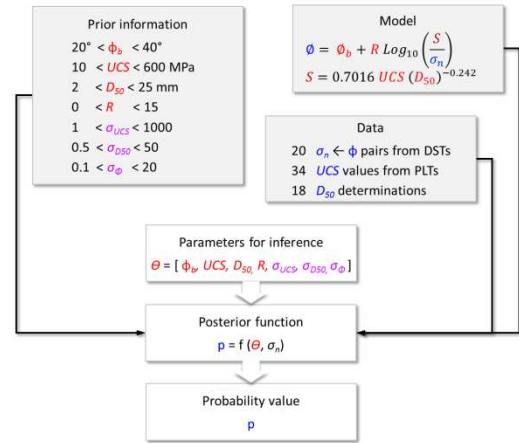


Figure 8. Conceptual Bayesian model for inference of the friction angle of overburden using large-scale DST results plus additional UCS and D_{50} data.

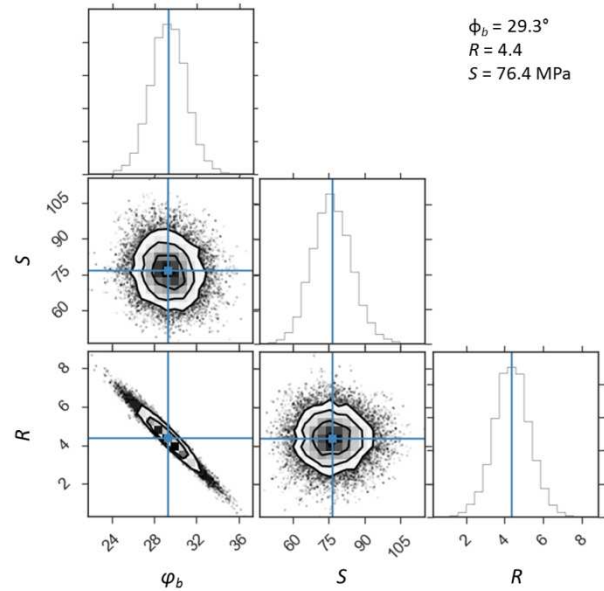


Figure 9. Corner plot of inferred parameters with the Bayesian analysis of DST, UCS and D_{50} results.

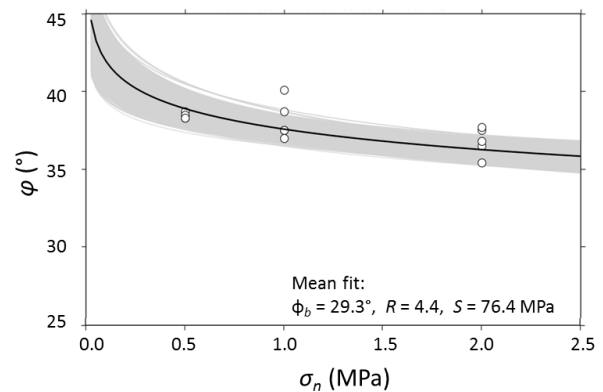


Figure 10. Uncertainty of the B-K envelopes represented by the 95% HDI of parameter values in Figure. 9.

5 CONCLUSIONS

Due to the variable nature inherent to mine waste rock materials, large-scale strength determinations in the laboratory can only represent the prototype sample tested. The implementation of a Bayesian approach allows the use of all the available information to characterise the material strength at project scale beside the direct strength determinations.

The Barton-Kjaernsli (B-K) empirical criterion was used to represent the shear strength of the overburden material. The Bayesian approach of statistical analysis was used to estimate the more likely B-K parameter values, ϕ_b , R and S to represent the material, honouring all the information available.

To show the benefits of including different sources of information in the analysis, a first evaluation was conducted using only the large-scale direct shear test results available. The evaluation was improved with a second analysis incorporating other sources of information, in this case, *UCS* and *D₅₀* determinations. The main observations from these analyses are:

- The Bayesian approach allowed the assessment of the quality of the information used for the evaluation. The first analysis suggested that the information used was insufficient to define adequately all the B-K parameters, and that additional data was necessary for a better definition of the parameter S . The second analysis showed a good definition of the model parameters and provided strength results with higher confidence.
- The Bayesian analysis provided valuable information in terms of the variability and correlation of the B-K model parameters ϕ_b , R and S . A negative correlation between ϕ_b and R was observed for the overburden material, which is purely a statistical condition imposed by the data points to ensure a good fit with the model.
- The analysis facilitates the identification of the more relevant factors requiring more investigation to improve the characterisation of the material strength. Once the additional information required is available the parameters can be updated and the reduction of their uncertainty can be verified. The results of the evaluation indicated that the shear strength in the B-K model is less sensitive to the parameter S suggesting that greater attention needs to be given to the evaluation of ϕ_b and R .
- The analysis allowed the inference of the best set of parameters of the B-K criterion, to represent a material composed of size-shape correlated particles, for which the applicability of the standard B-K charts for estimation of parameters was unknown.
- The Bayesian analysis indicated that the B-K parameters that best describe the overburden are $\phi_b = 29.3^\circ$, $R = 4.4$ and $S = 76.4$ MPa. The obtained value of R reflects adequately the ability of the material to pack well minimising void space, which is dictated by its particle shape characteristics, whatever these characteristics are.

6 ACKNOWLEDGEMENTS

This research was developed from the first author's PhD research at the University of Newcastle and the second author's PhD research at the University of Queensland. Special thanks to Dr Nick Barton for his corrections and valuable comments during the preparation of this paper.

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