Proceedings of the 20th International Conference on Soil Mechanics and Geotechnical Engineering– Rahman and Jaksa (Eds)

An investigation of probabilistic analysis in relation to the design of pile groups

Une étude de l'analyse probabiliste en relation avec la conception des groupes de pieux

Mark Randolph
University of Western Australia, Perth, Australia, mark@randolphonking.com

Stephen Buttling
National Geotechnical Consultants, Brisbane, Australia

ABSTRACT: There are two main ways in which pile groups are analysed; one in which piles are analysed as discrete elements, and another in which the soil is modelled as a continuum, such that pile-soil-pile behaviour is intrinsically taken into account. PIGLET is an example of the latter type and has recently been updated to account for the effects of non-linear axial and lateral response of individual piles. At the same time most geotechnical engineering design involves the use of "design values" of essential parameters and much effort has been expended on deciding how representative values should be determined. In reality most practising geotechnical engineers ignore the application of statistics, and treat the design values as unique numbers, rather than representations of random distributions of the variables. The principles have been well described, and procedures such as Monte Carlo simulation, or the First Order Reliability Method (FORM), can be used to consider the reliability of a design. A front end has been written for PIGLET to allow such probabilistic analyses for pile groups, and this is described and illustrated with worked examples.

RÉSUMÉ : Les groupes de pieux sont analysés de deux manières principales; un dans lequel les pieux sont analysés comme des éléments discrets, et un autre dans lequel le sol est modélisé comme un continuum, de sorte que le comportement pieu-sol-pieu est intrinsèquement pris en compte. PIGLET est un exemple de ce dernier type, et a récemment été mis à jour pour tenir compte des effets de la réponse axiale et latérale non linéaire des pieux individuels. Dans le même temps, la plupart des conceptions techniques géotechniques impliquent l'utilisation de «valeurs de conception» de paramètres essentiels et beaucoup d'efforts ont été consacrés à la détermination de la manière dont les valeurs représentatives devraient être déterminées. En réalité, la plupart des ingénieurs géotechniciens ignorent l'application des statistiques et traitent les valeurs de conception comme des nombres uniques, plutôt que comme des représentations de distributions aléatoires des variables. Les principes ont été bien décrits et des procédures telles que la simulation de Monte Carlo ou la méthode de fiabilité du premier ordre (FORM) peuvent être utilisées pour évaluer la fiabilité d'une conception. Un frontal a été écrit pour PIGLET pour permettre de telles analyses probabilistes pour les groupes de pieux, et cela est décrit et illustré avec des exemples travaillés.

1 INTRODUCTION

On a day-to-day basis, engineering design of geostructures is carried out using parameters defined by unique numerical values that are deemed to be representative of the body of soil affected by the design. This is despite the fact that it is reasonably well known that the soil parameters are not single values, but are random variables, with significant spatial variability, even within a single defined geological unit. Based on conversations with many practising geotechnical engineers, there is a reluctance to embrace the principles of statistical analysis that are best used due to limitations in the education of engineers, even in current times, when the subject is taught by Mathematics Departments using stock examples, frequently based in medicine or similar disciplines, and engineering students fail to detect their relevance to their chosen subject. Another comment frequently heard is that there is not enough data to make use of statistics.

Duncan (2000) gave detailed reasoning as to why an understanding of the reliability of a design through a probabilistic analysis was a useful addition to a deterministic analysis, and also explained how concepts like 6-sigma could be used to produce robust and repeatable estimates of parameter distributions when no data exists. It should also be remembered that one of the prime aims of design standards is to produce consistent levels of reliability in which clients and the public can have confidence. The very heavy reliance of deterministic methods on characteristic values that are left largely to engineering judgement has been demonstrated to fall far short of producing consistent levels of reliability (Bond and Harris 2008). Much effort has been directed towards restricting the range of characteristic values that might be selected, such as Schneider (1997) and Orr (2017).

A 'front end' has therefore been written for a popular geotechnical analysis program, in this case PIGLET, to allow a probabilistic analysis to be carried out quickly and conveniently, such that the reliability of the design can be evaluated. This will be described, with a worked example, to illustrate the advantages of the technique.

2 BACKGROUND.

When designing foundations for infrastructure or buildings, it is common practice for geotechnical engineers to use a program such as PIGLET (Randolph, 2020) to evaluate the 3D response of a pile group to the imposed 3D loading. The piles are represented by cylindrical hollow columns, which may have zero internal diameter, with potentially different axial and lateral modulus values both above and below the ground surface. The soil is modelled as a linear elastic continuum with a shear modulus that varies linearly with depth. The recent revision to the software allows the simulation of non-linear axial and lateral response of individual piles, as well as permitting different pile bending stiffness in the x:z and y:z planes, and a continuous normalized degree of rotational fixity between pile and pile cap, rather than choosing between the extreme values of 'pinned' or 'fixed' conditions as in the past.

In a typical design situation, a set of 3D forces will be applied to the centre of the foundation, representing the load combinations required by an applicable standard. A 3D geometrical model of the pile group will be created, specifying pile internal and external diameters, modulus values, pile centres and rakes. The program will then analyse the group rapidly, reporting the deflections and forces in each direction under each load case, as well as stiffness values for unit loads and deflections in the three orthogonal directions. Typical outputs include deflected shapes, bending moments and shear forces in the *x:z* and *y:z* planes, allowing other software such as structural programs, which typically use linear elastic springs on a rigid base to represent soil behaviour, to be adjusted to produce a more realistic output.

3 PROBABILISTIC ANALYSIS

As mentioned in the Introduction, in everyday work situations geotechnical engineers seek a unique characteristic value for the magnitude of any parameter, which is then considered to be representative of that parameter throughout the geological unit to which it refers. This is despite the fact that it is widely known and accepted that most parameter values vary considerably in space, and also in time. Two problems arise from this.

The first is in the selection of the characteristic value. The Eurocodes suggest "*a cautious estimate of the value affecting the limit state*", while AS 2159:2009 states that "*in general the value of a geotechnical parameter should be a conservatively assessed value of that parameter*". Neither of these give much guidance on the selection of the characteristic value, and it has been shown by Bond & Harris (2008) from research carried out with about 100 engineers, that the subjective estimates that result can cover a range of up 300 to 500% of the minimum chosen. When the characteristic value is multiplied by a strength or resistance reduction factor to obtain a design value, and that reduction factor is a number less than unity and defined to the second decimal place, this process can never produce consistent results.

The second is that the process does not require any consideration of the uncertainty inherent in any parameter value. If shear strength of a soil is considered, then there are many different ways in which this can be determined, including high quality laboratory tests, correlations with quality in-situ tests such as pressuremeter and DMT, correlations with CPT, correlations with SPT, or simply looking up a value in a published table. Each will have a different level of uncertainty, which will also be related to the quantity of data available. Then there are also the uncertainties associated with spatial variability. As shown by Christian et al (1994), it is quite possible, when comparing two geotechnical designs, for the one with the higher factor of safety to also have a higher probability of failure, if the data uncertainty is greater.

In order to overcome this deficiency, it is helpful to carry out a probabilistic analysis, and to investigate the uncertainty in the result based on the assessed uncertainties in the selected parameters. This can be done using the First Order Reliability Method (FORM) which has been conveniently programmed in EXCEL as shown by Low and Tang (1997) and many subsequent publications. Alternatively, a Monte Carlo simulation technique can be used, in which a distribution is selected to represent the data, and these distributions are randomly sampled for values to use in an analysis. The process is repeated many times, such as 10,000, such that the appropriate distribution is built up for each parameter, and the resulting distribution of output can be determined. For geotechnical engineering applications, distributions such as normal or log-normal are common, but any justifiable distribution can be used.

PIGLET has therefore been modified by the introduction of a 'front end' in PYTHON script, which allows the entry of mean values for each selected random variable, together with a standard deviation to represent the spread. The procedure is as follows:

1. Assign number of MC analyses (up to 100,000 in the case study) and also target ULS (or SLS) criterion.
2. Assign deterministic values (taken from PIGLET spreadsheet) of all key input data to be varied:
    1. Axial G_0 (shear modulus at surface)
    2. Axial G_{ref} (shear modulus at about 2/3 embedment of piles)
    3. Axial capacity (identical for all piles)
    4. Lateral G_0
    5. Lateral G_{ref} (shear modulus at about mid-critical depth)
    6. Lateral non-linear $(u/D)_{50}$ – normalised lateral pile head displacement at which secant stiffness is 50% of initial tangent value
    7. All 38 load cases (see explanation below)
3. Establish normalised means (unity), coefficients of variation and 6 x 6 correlation matrix (leading diagonal unity) for the soil input data
4. Use Python functions to evaluate 6 x 6 covariance matrix from Step 3
5. Use Python function to achieve, say, 100,000 sets of random multi-variate data
6. [Check statistics of sampled data]
7. For each of 10,000 analyses, evaluate actual input data for PIGLET, including limits on each; for example:
    1. non-negative modulus intercepts at ground level
    2. non-negative modulus gradients
    3. reference modulus values between 50 and 150% of deterministic values
    4. axial capacity between 50 and 200% of deterministic value
    5. lateral $(u/D)_{50}$ between 40 and 250% of deterministic value
    6. Conduct PIGLET analysis for each case in turn
    7. Store actual input data for each set and also resulting lateral displacement; count number of times criterion not achieved and evaluate Reliability Index, β.

4 CASE STUDY

In order to test the method, it has been applied to the analysis of a rail bridge structure for a waterway crossing. This is illustrated in Figure 1. The significant features of this case study include:

1. Very high imposed (live) load to permanent (dead) load ratio
2. Significant flood loading from water flow and matted floating debris, under 1:100 and 1:2000 AEP floods

Figure 1. View of completed bridge from case study.

3. Significant scour under 1:100 (1.5 m) and 1:2000 (1.7 m) AEP flood flows
4. Rail alignment not perpendicular to flow

5. Three load cases to consider, including serviceability limit state, and ultimate limit states for scour from a 1:100 AEP flood but with trains still running, and for scour from a 1:2000 AEP flood with trains assumed to be stopped.

This resulted in 14 SLS load cases, 30 ULS with 1:100 AEP flood load cases, and 18 ULS with 1:2000 AEP flood load cases. The design included 10 driven precast concrete piles in a 3:4:3 arrangement under a rectangular pile cap (see Figure 2). The 38 1:100 AEP flood load cases were selected for this case study, since they included a ULS limit on lateral movement of the pier of 50 mm. Deterministic and probabilistic analyses were carried out using the non-linear option in PIGLET with results discussed below.

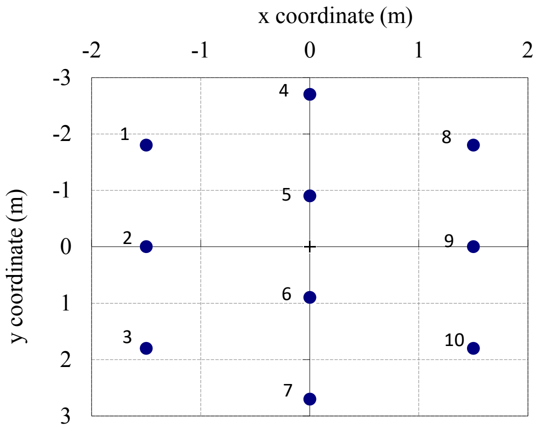

Figure 2. Layout of pile group.

4.1 *Load and soil parameter variations*

A simplified approach was to take the permanent (dead) load as the average V for all 38 load cases, which was about 5100 kN. That was then varied according to a Coefficient of Variation (CoV) of 0.1. All other loads (including variations in V above or below the dead load) were then varied, adopting a mean value of 50% of the specified (i.e. factored deterministic) load, but with a CoV of 0.4 (i.e. 20% of the specified deterministic load).

A typical load case P is then a vertical load of 5100 kN times a 'random normal' factor with mean and CoV of (1, 0.1) plus imposed (live) loads (including variations of vertical load from the mean of 5100 kN) multiplied by a 'random normal' factor with mean and CoV of (0.5,0.2).

For the soil parameters, the Poisson's ratio of the soil (0.5), the free-standing and embedded piles lengths (3 m and 20 m respectively), and the pile modulus values (28.4 GPa) were all input as deterministic values. This left the data listed in Table 1 as random variables, characterized by their means and the tabulated values of Coefficient of Variation. The cross-correlation values for the six variables are given in Table 2.

Since normal distributions cover the full range from $-\infty$ to $+\infty$, limits were placed on the applicable values of the variables as follows.

1. Both G(0) values and the axial pile capacity were restricted to be between 0.5 and 2 times the mean value.
2. The axial G(13m) and lateral G(3m) values were restricted to be between 0.5 and 1.5 times the mean value.
3. The $(u/D)_{50}$ was restricted to be between 0.04 and 2.5 times the mean value, noting that a smaller numerical value implies a greater non-linear response and hence a larger displacement.

With the above limits and probabilistic distributions, a typical Monte Carlo simulation gave results for the key input soil parameters as summarised in Table 3.

Table 1. Random variables and their CoV values.

Variable	Description	CoV
Axial G(0)	Surface intercept	0.25
Axial G(13m)	Value at around 2/3 embedment depth	0.25
Axial_V_{ult}	Axial pile capacity	0.15
Lateral G(0)	Surface intercept	0.5
Lateral G(3m)	Value at about mid-depth of critical pile length	0.25
$(u/D)_{50}$	Lateral non-linearity control parameter	0.2

Table 2. Cross-correlation matrix.

1	0.5	0.5	0.3	0.3	0
0.5	1	0.5	0.3	0.3	0
0.5	0.5	1	0.3	0.3	0
0.3	0.3	0.3	1	0.5	0
0.3	0.3	0.3	0.5	1	0
0	0	0	0	0	1

Table 3. Results of Monte Carlo simulation.

Variable	Max	Min	Avg	StDev	Units
Axial G(0)	14000	700	7015	1750	MPa
Axial gradient dG/dz	2935	985	1905	263	MPa/m
Axial V_{ult}	3072	900	1800	272	kN
Lateral G(0)	7000	0	3500	1700	MPa
Lateral gradient dG/dz	2505	0	958	482	MPa/m
$(u/D)_{50}$	0.0356	0.008	0.02	0.004	-
Computed max lateral disp.	96.2	18.6	28.2	5.6	mm

4.2 *Outcomes*

Deterministic analysis gave maximum axial load and lateral displacement of an individual pile in Load Case 7, with maximum axial load of 1374 kN in pile #4 and maximum horizontal deflection of 28.1 mm for pile #2. Bending moments and shear forces were all reasonable for the precast pile selected.

Running the Monte Carlo simulation 100,000 times for the 38 load cases took around 150 minutes on a standard notebook computer. The results are discussed below.

Initially, the probabilistic analysis was undertaken without varying the deterministic design loads. The ULS design limit for the lateral displacement limit of 50 mm was found to be exceeded by 0.33% of the simulations, giving a 'Probability of Failure' of 0.0033; this is equivalent to the Reliability Index proposed by Hasover & Lind (1974) of 2.72. This compares with the value of 3.1 recommended in ISO 2394:2015 for Class 2 structures with a high cost of safety measures. Typical bridges are considered as Class 3 structures and require a Reliability Index of at least 3.3.

This would suggest that further work would be required to reduce the uncertainty in the parameters, for example by improving site investigation, perhaps using a pressuremeter or dilatometer to measure appropriate stiffness values and ranges. To explore the effect of reducing the uncertainty, a further analysis was run, reducing the coefficient of variation for the lateral G(0) value from 0.5 down to 0.25. That gave $\beta = 3.19$.

The effect of replacing the deterministic loads by probabilistic load estimations was significant. Since the 'live' loads were modelled with a mean of 50% of the deterministic loads, and CoV of 40% (i.e. 20% of original deterministic loads), the PDF shows maximum frequency of calculated maximum lateral displacement shifted to around 0.017 m, compared with 0.026 to 0.027 m if the loads are not varied (note the original fully deterministic outcome was about 0.028 m). The Reliability Index increased to 3.60 (for lateral G(0) CoV of 0.5), and to 4.11 for the reduced lateral G(0) CoV of 0.25.

These values are consistent with published information on CoV values for geotechnical parameters, which can be found, for example, in Bond & Harris (2008), Duncan (2000), and Nadim et al (2005).

A summary of the PDFs for all analyses is shown in Figure 3.

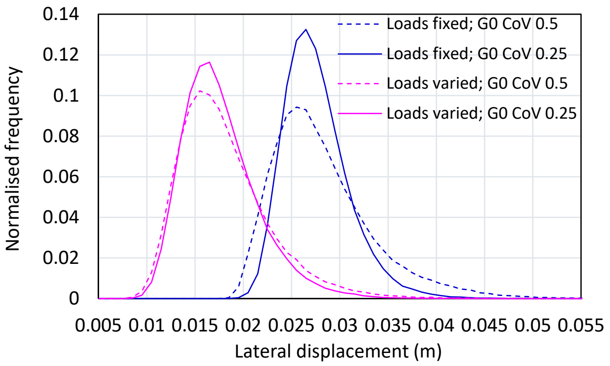

Figure 3. Probability density functions of lateral displacement from case study.

5 DISCUSSION

The 'front end' written using Python script (see Appendix) calling the PIGLET DLL to allow probabilistic analyses to be performed using Monte Carlo simulation has been programmed successfully. Tests have been run with up to 100,000 trials, which were completed within a reasonable time scale.

With the load variability ignored, so adopting deterministic design loads, the Reliability Index, β with relatively modest coefficients of variability on the key soil parameters was somewhat lower than recommended values for this type of structure. Reducing the CoV for one of the key input variables led to sufficient increase in the Reliability Index. However, it is evident that greater understanding and improved quantification is needed for the probability density functions for input data such as soil modulus, axial capacity and even the degree of non-linearity of the lateral pile response.

The test case used involved the substructure design of a rail over water bridge, as shown in Figure 1. The loaded freight trains are likely to create imposed loads that are an unusually high proportion of the permanent loads. In addition, since this was a dedicated railway line, not open to general traffic, it is likely that a realistic assessment of the imposed load variability would show that the traditional load factors on imposed load used to obtain deterministic loads are unduly conservative in this case. It is therefore considered that more realistic assessment would lead to lower values of CoV for the imposed loads, resulting in higher values of Reliability Index. If these values were to be in excess of the required value of, say, 3.3, then economies could be made in the bridge design. For a project such as this, with 23 similar bridges, there could be significant savings to the project.

The further step of using non-deterministic values of load, even with a relatively high CoV of 0.4 for the live component, gave significant increases in Reliability Index, which may be attributed to taking the mean live loads as 50% of the deterministic values, hence reducing the average value of maximum displacement, even if the spread was slightly larger.

Further work is needed to examine in detail the effect of reduced variability in the major imposed (live) load components where these are well known, while still accommodating variability in other, less well controlled, components such as traction or braking force.

It is hoped that geotechnical engineers will embrace the concepts contained in this study, applying them more generally. The use of Monte Carlo simulation through Python script has been shown to be carried out readily, and this could be applied to a number of other similar applications. Just to get away from the idea that geotechnical parameters are fixed numbers, and accept that they are random distributions, would be a major step forward, and could lead to rethinking of a number of standard geotechnical design situations, such as slope stability and retaining wall design.

6 CONCLUSIONS

The main conclusions from this study are:

1. It has been shown that PIGLET can be modified to carry out probabilistic analyses using Monte Carlo simulation.
2. The influence of variability of shear modulus of the soil, both axial and lateral, and the axial pile capacity, were investigated in terms of their effect on the Reliability Index.
3. It was shown that reasonable adjustments to the variability (CoV), had sufficient effect on the Reliability Index to allow optimisation of a design.
4. It was found that a probabilistic approach to specification of design loads resulted in a reduction of calculated average displacements and a significant increase in Reliability Index.

7 REFERENCES

AS 2159. 2009. *Piling-Design and installation.* Standards Australia, Sydney.

Bond A. and Harris A. 2008. *Decoding Eurocode 7.* Taylor & Francis, London.

Christian, J.T., Ladd, C.C and Baecher, G.B. 1994. Reliability and probability in stability analysis. *Journal of Geotechnical Engineering, ASCE,* 120, 1071-1111

Duncan J.M. 2000. Factors of safety and reliability in geotechnical engineering. *Journal of Geotechnical and Geoenvironmental engineering, ASCE.* 126 (4), 307-316.

Hasover, A.M. and Lind, N. 1974. An exact and invariant first-order reliability format. *Journal of Engineering Mechanics, ASCE.* 100 (1), 111-121

ISO 2394. 2015. *General principles on reliability for structures.* The International Organisation for Standardization, Geneva.

Low, B.K. and Tang, W.H. 1997. Efficient reliability evaluation using spreadsheet. *Journal of Engineering Mechanics, ASCE.* 123 (7), 749-752

Nadim, F., Einstein, H., and Roberds, W. *Probabilistic stability analysis for individual slopes in soil ad rock*. Landslide Risk Management (eds. Hungr, Fell, Coulture and Eberhardt) Taylor and Francis, London.

Orr, T.L.L. 2017 *Defining and selecting characteristic values of geotechnical parameters for designs to Eurocode 7.* Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards. 11, 103-115

Randolph, M.F. 2020. *PIGLET – Analysis and design of pile groups*. Software Manual.
Schneider, H.R. 1997. *Panel discussion: Definition and determination of characteristic soil properties*. 14th ICSMGE, Paris, 2271-2274

APPENDIX

Example Python Code

```
# Monte Carlo section
p_samples=100000                    # Number of MC simulations
p_MCvar=6               # Total number of MC soil input variables to be modified

# Set up output array for (a) Soil(3), Soil(4), lateral displacement; (b) Count
MC_sum=np.zeros((p_MCvar+1,p_samples),dtype=np.float)
MC_target=0.05                  # SLS maximum displacement

# Monte Carlo adjustment of axial shear modulus profile
N=Pigletdata['Number of piles']
ax_ref_depth=0.
for i in range(N):
    ax_ref_depth=ax_ref_depth+Pigletdata['Group   data']['Pile
        '+str(i+1)][4]             # Average embedment

ax_ref_depth=0.65*ax_ref_depth/N
G_ax_ref=Pigletdata['Soil data']['Axial: Surface shear modulus,
    G(0)']+ax_ref_depth*Pigletdata['Soil   data']['Axial:   Shear
    modulus gradient, dG/dz']

#print(G_ax_ref)
ax_cap=Pigletdata['Group data']['Pile 1'][9]
ax_cap=1800                          # Overriding spreadsheet value

# Monte Carlo adjustment of lateral shear modulus profile
lat_ref_depth=3              # Setting depth for reference shear modulus (approx. mid critical depth)
G_lat_ref=Pigletdata['Soil data']['Lateral: Surface shear modulus,
    G(0)']+lat_ref_depth*Pigletdata['Soil   data']['Lateral:   Shear
    modulus gradient, dG/dz']
#print(G_lat_ref)
u_D_50=Pigletdata['Soil non-linear data']['Lateral: Normalised
    disp. to halve stiffness']

# Input means (unity), standard devs (coeff. of var.) and correlation matrix between six soil variables
soil_mean=[1.,1.,1.,1.,1.,1.]
soil_std_dev=np.array([0.25,0.25,0.15,0.5,0.25,0.2])
soil_cor_mat=np.array([[1.,0.5,0.5,0.3,0.3,0.],
    [0.5,1.,0.5,0.3,0.3,0.],
    [0.5,0.5,1.,0.3,0.3,0.],
    [0.3,0.3,0.3,1.,0.5,0.],
    [0.3,0.3,0.3,0.5,1.,0.],
    [0.,0.,0.,0.,0.,1.]])
# compute covariance matrix
soil_cov_mat=np.outer(soil_std_dev,soil_std_dev)*soil_cor_m
    at

# Sample data (note "sampled_data" is a 6 x 10,000 array)
sampled_data=np.random.multivariate_normal(mean=soil_m
    ean, cov=soil_cov_mat, size=p_samples)

# Compute statistics of sampled data (as a check)
data_means=np.mean(sampled_data,axis=0)
data_std_dev=np.std(sampled_data,axis=0)
data_covariance=np.cov(np.transpose(sampled_data))
data_cor_mat=np.corrcoef(np.transpose(sampled_data))
#print(data_means,data_std_dev,data_covariance,data_cor_mat)

# Monte Carlo adjustment of loads for each loading case
Dead_load=np.zeros(6,dtype=np.float32)
Mean_load=sum(Loads[0,:])/Nloads
Dead_load[0]=Mean_load
Loads_tmp=np.zeros((7,Nloads),dtype=np.float32)
Loads_mean=np.zeros((6,Nloads),dtype=np.float32)          #
Essentially unfactored basic design loads
mu_1=1          # Mean factor for dead loads
std_1=0.1      # Standard deviation of dead loads
mu_2=0.5              # Mean factor for live loads
std_2=0.2            # Standard deviation of live loads
for jl in range(Nloads):
    Loads_mean[0:6,jl]=mu_1*Dead_load+mu_2*(Loads[0:6
        ,jl]-Dead_load)

# Compute adjusted soil data and loads for each sample

# Main Monte Carlo loop
MC_count=0
start_time=time.perf_counter()
for i in range(p_samples):
# Axial soil data
    Soil[0]=max(0,min(2,sampled_data[i,0]))*Pigletdata['Soil
        data']['Axial: Surface shear modulus, G(0)']
    G_tmp=max(Soil[0],0.5*G_ax_ref*(1+max(0,min(2,sam
        pled_data[i,1]))))
    Soil[1]=(G_tmp-Soil[0])/ax_ref_depth
#      print(Soil[0],Soil[1],G_tmp)
#      Soil[0]=6054.
#      Soil[1]=1917
    Soil[2]=Soil[0]+Soil[1]*(ax_ref_depth/0.65)

    for jp in range(Npiles):
        Group[9:9+Nloads,jp]=max(0.5,min(2,sampled_data
            [i,2]))*ax_cap
    # Lateral soil data
    Soil[3]=max(0,min(2,sampled_data[i,3]))*Pigletdata
        ['Soil data']['Lateral: Surface shear modulus, G(0)']
    G_tmp=max(Soil[3],0.5*G_lat_ref*(1+max(0,min(2,sam
        pled_data[i,4]))))
    Soil[4]=(G_tmp-Soil[3])/lat_ref_depth
#      print(Soil[3],Soil[4],G_tmp)
    Soilnl[2]=max(0.4,min(2.5,sampled_data[i,5]))*u_D_50

# Vary loads
    for jl in range(Nloads):
        fac_1=max(0.7,nr.normal(mu_1,std_1))
        fac_2=max(0.,nr.normal(mu_2,std_2))
        Loads_tmp[0:6,jl]=fac_1*Dead_load+fac_2*(Loads[
            0:6,jl]-Dead_load)
        Loads_tmp[6,jl]=Loads[6,jl]
# Call PIGLET DLL
    ap.PIGLET(Nscope,np.ctypeslib.as_ctypes(Soil),
        np.ctypeslib.as_ctypes(Soilnl),np.ctypeslib.as_ctype
        s(Pile),np.ctypeslib.as_ctypes(Group),np.ctypeslib.as
        _ctypes(Loads),np.ctypeslib.as_ctypes(Pigout),np.ct
        ypeslib.as_ctypes(Sumtable),
        Outpathbytes,Pathlength)

#    lat_disp=max(Pigout[11+Nloads:11+2*Nloads,1])
# x-direction
    lat_disp=np.sqrt(max(Pigout[11+Nloads:11+2*Nloads,1]
        **2+Pigout[11+Nloads:11+2*Nloads,3]**2))        #
        resultant
```

```
# Capture outcomes including soil input data
        if lat_disp > MC_target:
             MC_count=MC_count+1
        MC_sum[0,i]=Soil[0]
        MC_sum[1,i]=Soil[1]
        MC_sum[2,i]=Group[9,0]
        MC_sum[3,i]=Soil[3]
        MC_sum[4,i]=Soil[4]
        MC_sum[5,i]=Soilnl[2]
        MC_sum[6,i]=lat_disp
        MC_loads[6*i:6*i+6,range(Nloads)]=Loads_tmp[0:6,ran
               ge(Nloads)]
        MC_loads[6*p_samples:6*p_samples+6,range(Nloads)]=
               MC_loads[6*p_samples:6*p_samples+6,
               range(Nloads)]+Loads_tmp[0:6,range(Nloads)]
        MC_loads[6*p_samples+6:6*p_samples+12,range(Nload
               s)]=MC_loads[6*p_samples+6:6*p_samples+12,
               range(Nloads)]+Loads_tmp[0:6,range(Nloads)]**2

# Standard deviation of loads
MC_loads[6*p_samples+6:6*p_samples+12,
        range(Nloads)]=np.sqrt((MC_loads[6*p_samples+6:
        6*p_samples+12,range(Nloads)]-
        MC_loads[6*p_samples:6*p_samples+6,
        range(Nloads)]**2/p_samples)/p_samples)
# Mean of loads
MC_loads[6*p_samples:6*p_samples+6,range(Nloads)]=
        MC_loads[6*p_samples:6*p_samples+6,range(Nloads)]/p
        _samples

# Reliability index
beta=-stats.norm.ppf(MC_count/p_samples)
```