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Probabilistic back analysis of a Copenhagen metro station

Rétro-analyse probabiliste d'une station de métro de Copenhague

Ellinor Olbekk, Francesco Petrella & Antonio Cañavate-Grimal
Arup, Technical University of Denmark

Irene Rocchi
DTU

ABSTRACT: As part of the Cityringen metro line extension project in Copenhagen, Denmark, an extensive, high-quality structural and geotechnical monitoring program was implemented to measure the impact of deep excavations on the urban built environment. This paper presents the back analysis of one of the 17 stations using a probabilistic approach. The station construction required a 22m deep excavation and the use of a temporary secant piles-retaining structure. The ground conditions comprise glacial till, meltwater sand and Copenhagen Limestone. The retaining structure was built with a hybrid bottom-up construction sequence whereby the permanent roof slab was cast prior to the main excavation and temporary steel struts were used to support the wall at the lower levels. The novelty of this work is a Bayesian probabilistic approach combined with a polynomial chaos surface response method. The methodology not only provides a set of most probable parameters from the analysis of the inclinometer readings at low numerical cost, but also an assessment of their reliability, consistent with the model and observations errors. Such probabilistic approach may increase the opportunities to use the observational method that is regarded as one of the means to achieve more sustainable infrastructures. Furthermore, the analysis yields a sensitivity analysis identifying the governing parameters at each excavation stage. The adopted approach enables the discussion on the compatibility between the different observations and the suitability of the chosen design model.

RÉSUMÉ : Dans le cadre du projet d'extension du métro circulaire de Copenhague (Cityringen), au Danemark, un programme de monitorage structurel et géotechnique de haute qualité a été implémenté afin de mesurer l'impact des excavations profondes sur les bâtiments en milieu urbain. Ce rapport présente la rétro-analyse de l'une des 17 gares du Cityringen en utilisant une approche probabiliste. La construction de la gare en question impliquait la réalisation d'une tranchée de 22 m de profondeur et l'utilisation d'une structure de soutènement par pieux sécants. Les conditions du sol comprenaient du til glacial, du sable de fonte, et du calcaire de Copenhague. La structure de soutènement a été construite par une séquence de construction hybride bottom-up où la dalle de couverture a été coulée avant la fouille principale et des butons métalliques ont été utilisés pour soutenir le mur aux niveaux inférieurs. La nouveauté de cette construction se trouve dans l'approche probabiliste bayésienne combinée á la méthode de surface de résponse par chaos polynomial. Cette méthode offre la double possibilité d'obtenir les paramètres les plus courants venant de l'analyse des lectures de l'inclinometre á faible cout numérique et d'établir leur fiabilité, en cohérence avec le modèle et les erreurs d'observation. Cette approche probabiliste augmente les opportunités d'utiliser la méthode d'observation considérée comme l'un des moyens d'obtenir des infrastructures plus durables. De plus, elle produit une analyse de sensibilité qui identifie les paramètres directeurs à chaque étape de l'excavation. L'approche adoptée permet de discuter sur la compatibilité entre les différentes observations et la pertinence du modèle de conception choisi.

KEYWORDS: Copenhagen limestone, deep excavation, Bayesian back analysis, observational method, sustainable infrastructure.

1 INTRODUCTION.

Back analysis is used in geotechnical engineering to assess the design model parameters from the observed behaviour during construction. Back analysis can be used to validate design assumptions and make predictions of the following construction stages. Back analysis is routinely performed as a curve fitting exercise in which the parameters are changed manually until a good agreement by eye between the model and the observations is achieved. This practice can lead to highly subjective outcomes and is generally highly time consuming. Consequently, the implementation of scheduled back analysis during construction is not common practice and back analysis is relegated to a mere academic exercise. However, the use of probabilistic techniques such as the Bayesian inference (Tarantola 2005 and Gelman *et al.* 2013) can provide tools to expedite the analysis and make it less subjective. These techniques can be implemented into any of the observational method frameworks presented by Hardy et al. (2017) offering potential savings in construction programme and costs as well as a rigorous and clear allocation and treatment of construction risk.

This work presents the case study of a deep excavation, where the deflection of temporary walls measured during construction were used as an input for a back analysis using a Bayesian probabilistic approach combined with a polynomial chaos surface response method (Cañavate *et al*, 2015). The results obtained for the case study are compared with those used for design. A discussion follows on the key learnings obtained, which need to be incorporated in order to implement such probabilistic back analysis within a project and allow for employing the observational method.

2 CASE DESCRIPTION

The case study used for the back analysis was the metro station Aksel Møllers Have of the Cityringen metro line in Copenhagen. The construction of Cityringen metro line commenced in 2009, and opened to public in 2019. The metro line has 16.5 km twin-tube tunnels excavated by four Earth Pressure Balanced TBMs

(Eskesen et al., 2014) and added 17 new stations with five associated shafts to the existing metro system in Copenhagen.

The station Aksel Møllers Have is located in a park, see Figure 1, with a multi-story building in close proximity to the north-western longitudinal wall of the shaft. A small gazebo is located close to the opposite longitudinal wall.

2.1 *Structural details*

The metro station Aksel Møllers Have was constructed as a cut-and-cover hybrid bottom-up excavation, where the central part of the roof slab was cast prior to the main excavation works. A schematic illustration of the station is shown in Figure 2. This figure also shows the location of the longitudinal wall inclinometers which are referred to using the prefix IN.

The retaining walls consist of 27 m deep, 1 m diameter reinforced concrete secant piles with spacing of 0.785 m (longitudinal walls). At certain locations the secant pile wall was constructed with double piles (additional row of piles along the outer perimeter of the wall), primarily between IN23 and the northern head wall as well as in the vicinity of IN25. Three levels of temporary struts were used during excavation. The struts were 25 mm thick Ø812.5mm circular steel pipes installed with a 5.5 m spacing and prestressed with a 550 kN load.

Figure 1. Station location

2.2 *Ground conditions*

The typical geological profile in Copenhagen consists of heavily over-consolidated glacial deposits overlying the Copenhagen Limestone east of the Carlsberg fault and the Bryozoan Limestone west of the Carlsberg fault -refer to Frederiksen et al. (2003) for more details on the Copenhagen geology-. The glacial deposits encountered at Aksel Møllers Have (see Figure 2(b)) comprise the Upper Clay Till (ML1), Meltwater Sand (DS) and Lower Clay Till (ML2). The limestone is typically divided into different units, the highly permeable Upper Copenhagen Limestone (UCL HP), intact Upper Copenhagen Limestone (UCL) and Middle Copenhagen Limestone (MCL). The characteristic geotechnical parameters adopted as part of the geotechnical model were based on a site-specific ground investigation and are provided in Table 1.

2.3 *Ground water details*

The primary aquifer at Aksel Møllers Have comprises the Meltwater Sand and limestone units, which can be considered as hydraulically connected despite being locally divided by the Lower Clay Till (ML2). A secondary aquifer is found locally in the units above the Meltwater Sand. To model the excavation process, only the primary aquifer was considered. A ground water level of +1.9 m DVR90 assuming hydrostatic water pressure was applied for modelling purposes.

During construction, pumping wells within the excavation area ensured that the ground water level was kept at least 1 m below the excavation level. The groundwater was treated and recharged into a set of wells located around the station perimeter to avoid lowering the water table in the surroundings.

2.4 *Construction sequence*

The station box construction sequence is described below:

Stage 01 - Excavate to working level +10.75 m DVR90 and cast secant piles;
Stage 02 - Excavate to level +7.57 m DVR90;
Stage 03 - Cast roof slab in central part of excavation (top of slab at level +9.77 m DVR90) and install struts on both sides;
Stage 04 - Excavate to level +2.30 m DVR90;
Stage 05 - Install strut level one at +3.80 m DVR90;
Stage 06 - Excavate to level -2.70 m DVR90;
Stage 07 - Install strut level two at -1.20 m DVR90;
Stage 08 - Excavate to level -8.00 m DVR90;
Stage 09 - Install strut level three at -6.5 m DVR90;
Stage 10 - Excavate to formation level at -12.55 m DVR90.

The precise dates of the construction stages were not available to the authors.

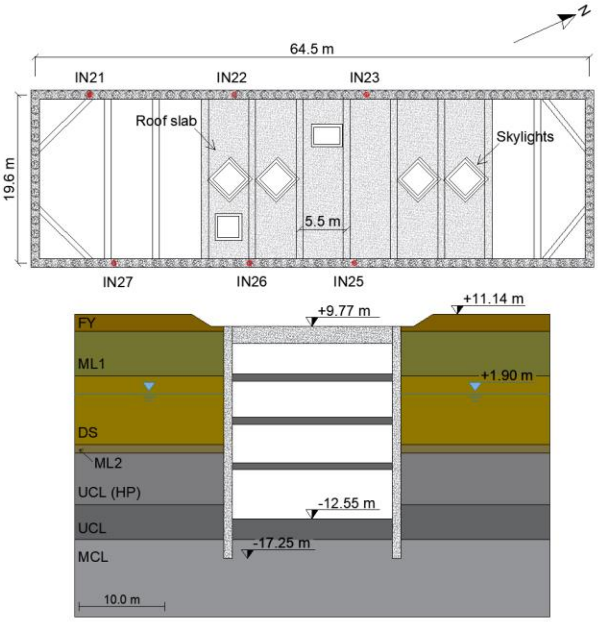

Figure 2. Temporary retaining system: (a, top) Plan drawing (b, bottom) cross section drawing (DVR90 datum)

Table 1. Soil unit weight, Mohr Coulomb and Young's modulus characteristic parameters at Aksel Møllers Have

Strata	γ [kN/m^3]	φ' [°]	c' [kPa]	E [MPa]
ML1	22	34	0	148
DS	21	38	0	175
ML2	22	34	0	403
UCL(HP)	21	40	50	800
UCL	21	45	100	1500
MCL	21	45	100	1500

3 MONITORING

An extensive monitoring program was adopted as a risk-mitigating tool as Cityringen runs through an urban environment. The monitoring system was described in detail by Falbe-Hansen et al. (2018). The back analysis presented in this paper was based on the horizontal displacements of the retaining walls as monitored by inclinometers.

3.1 *Inclinometers*

The Aksel Møllers Have station retaining walls were monitored by a total of eight inclinometers; three located on each longitudinal wall and one on each head wall (see Figure 2). The in-place inclinometers had sensing gauges at 3 m intervals, consisting of two MEMS accelerometers measuring the tilt in two perpendicular directions: towards the excavation and in the wall direction. The monitored behaviour was transmitted to the database with a minimum frequency of four hours. If the alert and alarm levels were to be approached, the frequency was set to increase. If the levels were exceeded, emergency and contingency actions would be initiated. According to the manufacturer of the inclinometers, the sensor accuracy is 0.05 mm/m for a ±10° range.

The back analysis presented relies on the inclinometer IN26 as this inclinometer was considered the most representative of free-field conditions – it was located on the wall opposite to the multistorey building, it was on a part of the wall without double piles and furthermore deemed to not be influenced by corner effects (see Figure 2).

3.2 *Observed behaviour*

The inclinometer readings started at stage 3 due to the construction of the capping beam and the initial wall movement as a cantilever was not measured.

The observed behaviour of the inclinometer IN26 over the time is shown on Figure 3. The displacements corresponding to each construction stage had to be interpreted based on the available time series of inclinometer readings, as exact dates of the various construction stages were not available to the authors of this paper (adding subjectivity to the back analysis). Figure 4 shows the interpreted displacements (perpendicular to the wall direction) at each construction stage from installation of the roof slab to the final excavation stage. The roof slab and strut levels are also indicated on the figure. Table 2 presents as a reference a comparison of the maximum horizontal displacements measured at the longitudinal walls.

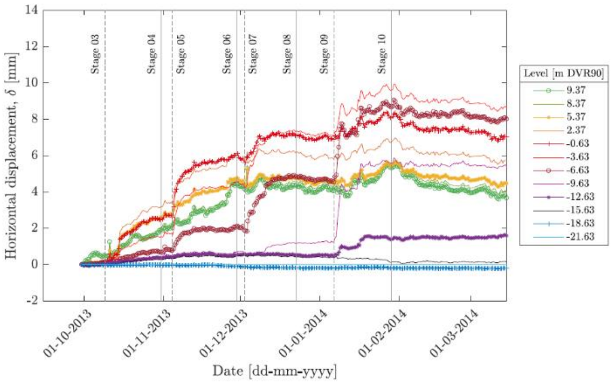

Figure 3. Observed behaviour timeseries of inclinometer IN26

4 PROBABILISTIC BACK ANALYSIS

The deterministic back analysis aims to evaluate the set of model parameters which best agree with the available observations. This approach assumes implicitly that the model can perfectly reproduce observed phenomenon and the observations are error free. The probabilistic back analysis considers the model parameters as random variables and the solution of the problem is given in form of probability distribution functions. The Bayesian probabilistic approach allows to consider the model and observation errors and, according to Tarantola (2015), it is the appropriate methodology to perform back analyses.

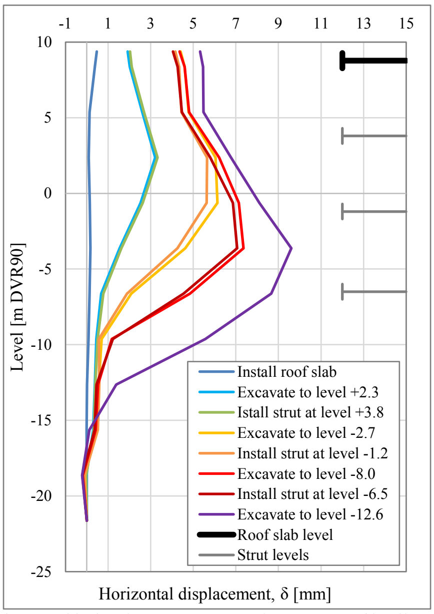

Figure 4. Observed behaviour at construction stages of inclinometer IN26

Table 2. Observed maximum horizontal displacements (long walls)

Inclinometer	δ_{max} [mm]	Level [m DVR90]
IN21	9.3	-3.63
IN22	7.0	-3.63
IN23	7.1	-0.63
IN25	5.1	-9.41
IN26	9.6	-3.63
IN27	6.4	-3.63

4.1 *Back analysis set up*

The back analysis presented in this work followed the methodology by Cañavate *et al.* (2015) in which the probabilistic calculations are run on a surrogated model (also known as response surface or metamodel) constructed on a polynomial chaos basis (refer to Ghanem & Spanos, 1991 and Le Maître & Knio, 2010) to alleviate the numerical burden. The use of a polynomial surrogate is deemed appropriate as deep urban excavations are designed to be distant from failure and the wall response is considered smooth (i.e. small parameter variations yield to small changes in the response). Mathematically, the smooth response can be regarded as a mildly non-linear problem and the polynomial approach suitable.

The use of polynomials surrogate models might be impacted by overfitting (using a polynomial degree so high that it fails to yield an adequate response to different sets of data) or underfitting (using a polynomial degree so low that some of the original model features are missed). To overcome this issue, Cañavate et al. (2015) take advantage of the mildly non-linearity of the problem and propose an iterative strategy using low degree polynomials to narrow down at each iteration the search to the areas in which the solution has higher probability of occurrence. The suitability of a low degree polynomial to mimic the original model response increases at each iteration.

The wall was modelled using the program FREW (Oasys, 2021) which is specifically intended for the analysis of flexible earth retaining structures. The ground layers are modelled as Mohr Coulomb materials and the structural elements are regarded elastic. The Mohr Coulomb failure criterion is deemed suitable for this analysis considering the observed small strain movement of the retaining wall. Furthermore, this criterion is often adopted for retaining wall problems in Copenhagen and was used for design of the Aksel Møllers Have station. The back analysis was performed using a FREW functionality currently under development.

The work done for this paper involved numerous simulations (not presented in this paper) investigating the model sensitivity to many different model parameters, including ground layering, ground stiffness, ground strength, structural parameters and water levels. Each simulation was run with a defined lower and upper boundary of each model variable. The chosen boundaries were based on the available geotechnical and structural information. The general observation comparing these different simulations was that the ground stiffness is governing the wall behaviour. For that reason, the Young's moduli of the six geological units were chosen as variables to back analyse in this paper. Table 3 presents the prior (or initial) Young's moduli ranges to be updated using the observed wall deflection. The Young moduli are identified using the prefix E.

Stage 08 in which the excavation reached level -8.0 m DVR90 was chosen to perform the back analysis. Figure 5 shows the observed deflections at Stage 08 and 10 -maximum excavation stage- together with the observation error 95% confidence interval based on the inclinometer manufacturer specification. The deepest sensing gauge was installed 4.6 m below the wall toe ensuring that the bottom of the inclinometer is fixed. Due to the wall stiffness, the initial cantilever movement before Stage 03 was assumed negligible and the observed movements are representative of the overall wall deflections.

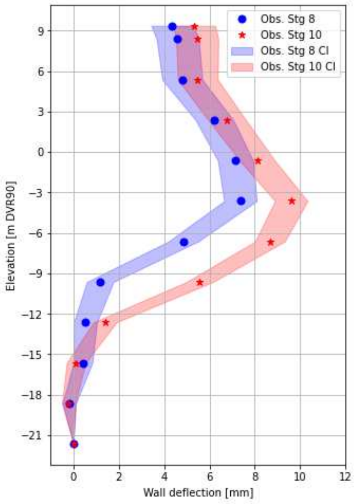

Figure 5. Inclinometer IN26 readings at stage 8 and 10 with 95% observation confidence interval.

4.2 *Results*

As the model parameters are considered random variables, the model response is also random. The contribution of each parameter to the response variance can be seen as an indication of the response sensitivity. Table 3 presents the influence of each model parameter to the total variance. For the back analysis in this paper, the stiffness of units Upper Clay Till (ML1), Meltwater Sand (DS) and Lower Clay Till (ML2) together with the intact Upper Copenhagen Limestone (UCL) explain 99% of the total variance. Table 3 presents the contribution of each parameter to the total variance. This result allows to identify the influential parameters by quantifying their relative importance to the model behaviour.

Figure 6 shows the results of the back analysis (or calibration) of the model at Stage 08 using Bayesian inference. It can be readily seen that the results are given in terms of 95% confidence intervals and mean response. The prior parameter estimation reduces to narrower ranges (posterior) as presented in Table 3. The posterior ranges yield to the confidence interval (CI) presented in Figure 6 (*Parameter CI* in Table 3). An intuitive explanation would be that the Bayesian inference tries to find the best overlap between response and the observation confidence intervals accounting for the initial ranges and the model constraints. The probabilistic approach can provide an estimation of the fitness of the model to represent the phenomenon. On Figure 6, the modelling error is also presented (*Model CI*). The modelling error gives an estimation of the discrepancy between the model and the observations. Figure 6 shows that the model can represent well the bottom of the wall however might be not as accurate for the central part. An additional model error is also considered, the surrogation error (Surrogation CI). The surrogation error is the approximation error made when using a metamodel instead of the original model. In this case, the surrogation error is negligible.

Table 3. Back analysis input and output

Parameter	Prior		Posterior		Mean estimate	MAP estimate	Influence [%]
	min	max	min	max			
E1, ML1	80	500	104	292	188	135	13.7
E2, DS	80	500	198	327	258	225	62.0
E3, ML2	80	500	194	470	332	491	12.7
E4, UCL(HP)	500	2000	500	2000	1190	1776	0.2
E5, UCL	500	6000	1212	4533	2638	1286	10.6
E6, MCL	500	6000	1223	5975	3675	5975	0.8

The maximum a posteriori or MAP is an estimation of the parameters which makes minimum the discrepancy between the observations and the model response. Implicitly, the MAP estimation assumes that the model is perfect and the observations are free of error. In this case, the MAP is a least square estimation. On the other hand, the mean estimate is the mean value of the range of parameters which maximises the overlapping between the observation and the response confidence intervals. Note that the model response computed using the MAP might fit better to the observations than the response from the mean estimate. That might be due to the MAP being a combination of extreme parameters and hence unlikely to happen. The mean estimate is consistent with the initial parameter range estimation. For that reason, the parameter range selection should be assessed from best and worst credible ground values based on factual data. The MAP estimate should be read in conjunction with the parameter bounds.

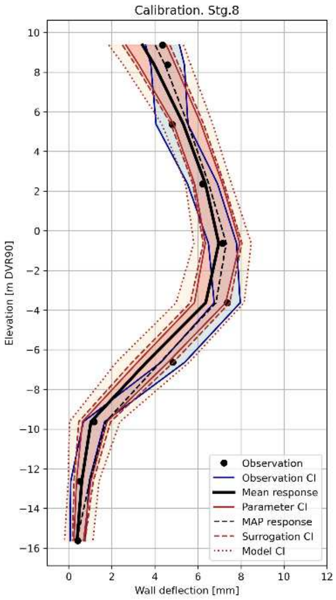

Figure 6. Stage 08 back analysed deflection curve.

5 DISCUSSION

One of the back analysis goals is to provide an early design model parameter estimation from the initial construction stages monitoring data to assess the future performance. Figure 7 presents a comparison between the computed model deflections -using the design characteristic parameters (see Table 1) and the back analysis results (mean estimate from Table 3)- and the observed deflections. It can be readily seen that the characteristic parameters (*FREW ck*) yield a softer response than the back analysed parameters (*FREW BA*). The inclinometer data suggests that the bottom of the wall is well fixed on the ground. The model using the back analysed parameters can mimic well the bottom fixity and provides a reasonable estimation of the upper wall movements. However, the estimation of the central wall movements on Stage 10 is poor. The model using the design characteristic model yields a better estimation of deflections for Stage 10 than Stage 08.

This simple comparison exercise makes clear that back analysis parameters should not be taken at face value and a lot of engineering judgement should be put into the model interpretation. For example, IN26 observations showed larger relative displacements between stages 08 and 10 than observations from the other inclinometers at Aksel Møllers Have. The authors had not access to a detailed construction log and could not incorporate it into their model. Furthermore, the verticality observed in the topmost three observations (see Figure 4) could not be properly replicated in the model.

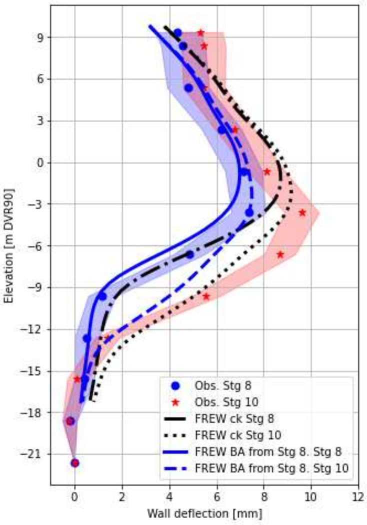

Figure 7. Comparison of model deflections at Stages 08 and 10 (using the design characteristic and the back analysed parameters from Stage 08) with observed deflection.

6 CONCLUSIONS

This paper presents a probabilistic back analysis based on inclinometer data from a Copenhagen metro station retaining wall. The described process includes considerations made by the authors on the back analysis process as well as the back analysis results.

The probabilistic back analysis gave reasonable results in terms of the best estimate soil stiffness. Much of the strength in the adopted process is however considered to lie in the probabilistic assessment of the variable influence. This knowledge would alleviate the typically time consuming back analysis process by providing a quantified relative importance of the model parameters. The associated posterior variable distributions provide useful insight which might increase confidence when defining future design parameters.

Engineering judgement can however not be excluded from the back analysis process. Results will only be as reliable as the provided data allows, and careful review of monitoring (input) data is necessary to ensure validity of the back analysis results (output). A detailed insight in the construction process is deemed of high importance to provide confidence in the validity of monitoring data. However, where such confidence is achieved, the Bayesian probabilistic approach to back analysis will equip the user with a faster and more informative result which might facilitate wider use of the observational method in construction projects.

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8 REFERENCES

Cañavate-Grimal, A., Falcó, A., Calderón, P., Payá-Zaforteza, I. 2015. On the use of stochastic spectral methods in deep excavation inverse problems. Computers and Structures 159 (2015) 41–60.

Eskesen, S. D., Paulatto, E., Stubbs, J. 2014. Construction challenges for urban tunnelling - The Copenhagen Metro Circle Line. Geomechanik und Tunnelbau 7.5 223–229.

Falbe-Hansen, K., Paulatto, E. and Juliao, I.A. 2018. Monitoring Cityringen Metro project in Copenhagen, Denmark. *Proceedings of the Institution of Civil Engineers-Civil Engineering* 171.5, 39-47.

Frederiksen, J.K., Brendstrup, J., Eriksen, F.S., Gordon, M.A., Knudsen, C., Jørgensen, M.E., and Møller, H.M. 2003. Engineering Geology of Copenhagen. *Bulletin of Engineering Geology and the Environment.* 62(3), 189-206.

Gelman, A., Carlin, J.B., Stern, H.S., Dunson, D.B., Vehtari, A., Rubin, DB. 2013. Bayesian data analysis. CRC press

Ghanem RG, Spanos PD. 1991 Stochastic finite elements: a spectral approach. New York: Springer.

Hardy, S., Nicholson, D., Ingram, P., Gaba, A., Chen, Y., Biscontin, G. 2017. New observational method framework and application. ICSMGE 2017 - 19th International Conference on Soil Mechanics and Geotechnical Engineering, 2017-September 1995-1998. https://doi.org/10.17863/CAM.26630

Le Maître OP, Knio OM. 2010. Spectral methods for uncertainty quantification with applications to computational fluid dynamics. New York: Springer; 2010.

Oasys. 2021. FREW 19.4 Help guide. London: Oasys Ltd.

Tarantola, A. 2005. A. Inverse problem theory and methods for model parameter. Philadelphia: SIAM.