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Probability-based geotechnical site characterization scheme for deep foundations in onshore Niger Delta Areas of Nigeria

Schéma de caractérisation de site géotechnique basé sur la probabilité pour les fondations profondes dans les zones onshore du delta du Niger au Nigeria

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ABSTRACT : Geotechnical site characterization requires the determination of site-specific field parameters which are known to exhibit pronounced variability. In this paper, in-situ and laboratory test data of a project site located in the mangrove swamp of the Niger Delta region of Nigeria are used to characterize the site based on probability concepts. The characterization involves the application of random field theory in estimating the spatial variability parameters, using CPT data from different test locations. Of particular interest are the two key deep foundation design quantities – cone tip resistance, q_c, and undrained shear strength, s_u. The modeling of spatial variability entails the statistical description of the soil properties in terms of the statistical quantities: mean μ, variance $\sigma 2$, and correlation structure θ, in the vertical and horizontal directions. In this study however, only the vertical spatial variation of the design quantities is modelled, with the correlation structure estimated by fitting the theoretical correlation to the experimental correlation function. The cosine exponential correlation model is found to yield good level of accuracy in estimating the correlation length or scale of fluctuations. The generated soil profile portrays a sequence of organic sensitive silty Clay underlain by alternating layers of inorganic Clay and gravely Sand. The estimated scale of fluctuations for the upper organic sensitive Clay is small for both design quantities, which is indicative of rough field up to a depth of about 5m. Beyond this depth, the correlation length increased with depth implying strong spatial correlation in the design quantities within soil layers at great depths. The site-specific estimates of the vertical scale of fluctuation of both the unit cone resistance and the undrained shear strength have been presented and compares closely to literature globally and hence scheme recommended for deep foundation development in the Nigerian Niger Delta Areas.

RÉSUMÉ: La caractérisation géotechnique du site nécessite la détermination de paramètres de terrain spécifiques au site qui sont connus pour présenter une variabilité prononcée. Dans cet article, les données d'essais in situ et en laboratoire d'un site de projet situé dans la mangrove de la région du delta du Niger au Nigeria sont utilisées pour caractériser le site sur la base de concepts de probabilité. La caractérisation implique l'application de la théorie des champs aléatoires dans l'estimation des paramètres de variabilité spatiale, en utilisant des données CPT provenant de différents emplacements de test. Les deux grandeurs de conception des fondations profondes sont particulièrement intéressantes : la résistance à la pointe du cône, q_c,, et la résistance au cisaillement non drainé, s_u. La modélisation de la variabilité spatiale implique la description statistique des propriétés du sol en termes de grandeurs statistiques : moyenne μ, variance $\sigma 2$ et structure de corrélation θ, dans les directions verticale et horizontale. Cependant, dans cette étude, seule la variation spatiale verticale des grandeurs de conception est modélisée, la structure de corrélation étant estimée en ajustant la corrélation théorique à la fonction de corrélation expérimentale. Le modèle de corrélation exponentielle en cosinus donne un bon niveau de précision dans l'estimation de la longueur de corrélation ou de l'échelle des fluctuations. Le profil de sol généré dépeint une séquence d'argile limoneuse sensible aux matières organiques reposant sur des couches alternées d'argile inorganique et de sable graveleux. L'échelle estimée des fluctuations pour l'argile sensible organique supérieure est petite pour les deux quantités de conception, ce qui indique un champ rugueux jusqu'à une profondeur d'environ 5m. Au-delà de cette profondeur, la longueur de corrélation augmente avec la profondeur, ce qui implique une forte corrélation spatiale dans les grandeurs de conception dans les couches de sol à de grandes profondeurs. Les estimations spécifiques au site de l'échelle verticale de fluctuation de la résistance du cône unitaire et de la résistance au cisaillement non drainé ont été présentées et se comparent étroitement à la littérature mondiale et donc au schéma recommandé pour le développement de fondations profondes dans les zones du delta du Niger au Nigéria.

KEYWORDS: site characterization, random field theory, spatial variability, correlation model, isotropic field.

1 INTRODUCTION

Soils are natural materials formed by a weathering process with physical properties varying from one place to another as a result of the physical and chemical changes they undergo during formation (Mitchel and Soga, 2005). The conventional tool used in accounting for spatial variability in geotechnical engineering practice is reliance on factors of safety and local experience. However, the hypothesis of randomness is now commonly adopted as a more rational method of resolving issues of variability. Consequently, efforts at characterizing spatial variability of soil properties have relied more on statistical and probabilistic methods (Onyejekwe, 2012).

Application of statistical analysis in the evaluation of soil behavior in Nigeria is rare but there is no doubt that using such approach offers a wide range of opportunities for advancement of geotechnical engineering practice in the country. Research in this area is scanty and only few works exist on the application of probabilistic methods in analyzing geotechnical engineering behavior and validating analytical models of Nigerian soils, (e.g., Ejezie and Harrop-Williams, 1984). Most of these studies applied concepts formulated at the early stages of development of the probabilistic and reliability methods and therefore, did not adequately evaluate the effect of spatial variability. This study seeks to broaden the application of statistical analysis by developing models of the spatial variability of the soil properties of a project site located in a mangrove swamp within the Niger Delta region of Nigeria using the random field theory.

2 RANDOM FIELD THEORY

2.1 *Spatial Variability*

The random field model of spatial variability assumes spatial dependence such that the soil properties $X(x_1)$ and $X(x_2)$ are expected to exhibit some sort of interdependence that decreases with their separation distance. The interdependence of the properties at the different points in the field is depicted using joint bivariate distribution $f_{x_1,x_2}(x_1,x_2)$. However, for three or more points, the complete probabilistic description of the random process becomes complex and difficult to use in practice. The characterization problem is, nevertheless, simplified by assuming a Gaussian process and stationarity of data which allows the complete joint distribution to be quantified by the mean vector and covariance matrix, and makes the distribution independent on stationary position but dependent only on relative positions of points (Fenton and Griffiths, 2008). Assumption of stationarity implies that the statistical properties of the random field remain the same when the spatial origin changes position.

To facilitate the application of random field theory, a transformation of the variables by decomposition is often performed to convert the non-stationary field to a stationary or nearly stationary field. The decomposition transformation technique idealizes the soil property as comprising of a deterministic trend component and a fluctuating or variable component expressed in form of an additive equation (Baecher & Christian 2003; Uzielli 2006):

$$\psi(z) = t(z) + \xi(z) \tag{1}$$

The objective in the decomposition process is primarily to obtain an estimate and remove the deterministic component, $t(z)$, while ensuring that the residual random component, $\xi(z)$, remains stationary. Analysis of inherent variability involves modeling the residual component of the soil property by statistical means. The residual component is assumed to have a spatial structure defined by the scale of fluctuation, θ, and the autocovariance function, $C(\tau)$, where τ is the distance between observation points (Oguz and Huvaj, 2019)

The modelling of the soil parameters relies much on two essential statistical properties of the random field namely: autocovariance, c_k, and autocorrelation coefficient, ρ_k, at lag, k,. In practice, c_k and ρ_k are estimated from the samples obtained from a population. The sample autocovariance c_k^* and the sample autocorrelation coefficient, at $lag\ k, r_k$ are defined as follows (Jaksa 2006):

$$c_k^* = \frac{1}{n}\sum_{i=1}^{n-k}(X_i - \bar{X})(X_{i+k} - \bar{X}) \tag{2}$$

and

$$r_k = \frac{c_k^*}{c_0} = \frac{\sum_{i=1}^{n-k}(X_i-\bar{X})(X_{i+k}-\bar{X})}{\sum_{i=1}^{n}(X_i-\bar{X})^2} \tag{3}$$

$\bar{X}$ = average of the observations $X_1, X_2, \ldots, X_n,\ and\ 0 \le k \le n$

The plot or graph of c_k^* for lags $k = 0,1,2,\ldots$ represents the sample autocovariance function (ACVF), or auto-variogram while the plot of r_k for lags $k = 0,1,2,\ldots K$ represents the sample autocorrelation (ACF), where K is the maximum number of lags for $r_k$calculations (e.g., $K = n/4$)

The modelling of the spatial variability of geotechnical material requires a minimum of three parameters: the mean, μ; a measure of variance, σ^2 (standard deviation,σ, or coefficient of variation); and the scale of fluctuation, θ, that associates the correlation of properties with distance (Vanmarcke, 1983). Large values of θ, for a particular property, signifies that the property slowly fluctuates with distance about the mean, suggesting a more continuous deposit, while a small θ is an indication of the property fluctuating rapidly about the mean, suggesting a more randomly varying material (Jaksa, 2006).

2.2 *Scale of Fluctuation*

The scale of fluctuation, θ, is a measure of the distance within which points in a domain are significantly correlated and it conveniently describes the spatial variability of a soil property (Vanmarcke, 1983). The correlation between two points depends on how the separation distance compares with θ. Given the importance of θ in the spatial variability description of soil property in a random field, extensive research work aimed at developing more rational approaches in determining accurate estimates of the scale of fluctuation have been carried out (Fenton and Griffiths, 2005; Griffiths et al. 2009; Hicks and Spencer, 2010; Cassidy et al., 2013). Small values of θ obtained from any of the models indicate that the correlation function decays rapidly to zero with increasing τ (meaning that the correlation between the two points under consideration are rapidly smaller) resulting in a rougher random field. As $\theta \to 0$, all points within the domain become uncorrelated and the field becomes extremely rough. Conversely, increasing values of θ is an indication that the property field is smoother meaning that the field is showing less variability converging to a uniform field when $\theta \to \infty$ Lloret-Cabot et al. (2013). In practice, estimation of θ is done by fitting the theoretical correlation to the experimental correlation function (Uzielli et al., 2006; Zhu and Zhang, 2013; Oguz and Huvaj, 2019).

Table 1: Some common correlation models

Correlation Model	Expression	Scale of Fluctuation $\boldsymbol{\theta}$
Simple exponential	$\rho(\tau) = exp[-\lvert\tau\rvert/b]$	$2b$
Gaussian exponential	$\rho(\tau) = exp\{-\pi[\lvert\tau\rvert/c]^2\}$	$\sqrt{\pi}c$
Second-order autoregressive process	$\rho(\tau) = exp^{-\lvert\tau\rvert/d}(1 + \lvert\tau\rvert/d)$	$4d$
Cosine exponential	$\rho(\tau) = exp^{-\lvert\tau\rvert/\alpha}cos(\tau/\alpha)$	α

Source: Lloret-Cabot et al. (2013)

3 RESEARCH METHODOLOGY

3.1 *Study Area and Data Acquisition*

The area of study falls within the Tertiary Niger Delta which occurs at the southern end of Nigeria bordering the Atlantic Ocean and extends from about longitudes 3^0-9^0E and latitude 4^0 30'-5^0 20'N.

Figure 1. Location of study area

The study used data from a geotechnical site investigation report of a refinery project with a total of 96 data sets available, out of which 20 (16 CPT and 4 borehole data), were carefully selected for the study to present an equally spaced CPT grid, suited for random field theory application. A summary of the data set is presented in Table 2

Table 2. Study data set

Data	Refinery			
CPT	CP 10	CP 14	CP 19	CP 24
	CP11	CP16	CP20	CP26
	CP12	CP17	CP22	CP27
	CP13	CP18	CP23	CP28
Borehole	BH8	BH9	BH10	BH11

3.2 *Method of Data Analysis*

The soil profile generated from the borehole log and CPT data identified five distinct soil layers and analyses were performed for each of the soil layers. To ensure statistical homogeneity or stationarity within the domain for a seamless application of the random field theory, the entire soil profile within the zone of influence was divided into number of statistically homogeneous or stationary sections, and the data within each layer subjected separately to statistical analysis (Phoon et al. 2003).

Data from each CPT test hole was evaluated to determine the value of geotechnical parameter at the different strata of the soil profile. The examination of data of each CPT to determine the value of the realization at any strata was carried out using the following steps (Jaska, 2006):

- Examine the data of the parameter across the depth and transform the non-stationary data into stationary data. Where the data exhibited a trend, decomposition was required otherwise linear transformation into a weak stationary field was performed.
- Decomposition involved separating the trended data into a slowly changing trend component and a random or residual component. The ordinary least square (OLS) method was used to estimate the trend.
- To ensure stationarity of the residuals, eyeballing and the Kendal's τ test was used to examine for stationarity. With stationarity confirmed, it was assumed that the residuals are normally distributed.
- Calculate the sample autocovariance and autocorrelation functions using Equations (2) and (3) respectively.
- Estimate the correlation length, θ, by fitting a theoretical model from Table 1 to the plot of sample ACF over lag distance
- Calculate the Bartletts distance (i.e., distance over which the samples are autocorrelated).

The vertical spatial variability was analyzed by estimating the correlation distance within each soil layer for the cone tip resistance and undrained shear strength.

4 RESULTS AND DISCUSSION

4.1 *Soil Profile Generation*

The CPT data were analyzed with the aid of NovoCPT® (2019) software, which generates soil profiles using the charts developed by Robertson (1986), Robertson (1990 SBT) and Jefferies & Been (2006). Comparative examination of the soil behavior pattern in Figure 2, initial soil profile in Figure 3 and CPT data shows that some of the soil layers are mixtures of "clays" and "sands". There was need therefore to classify the soil layers into major soil groups. The initial soil profile generated was adjusted through a three-step approach that allowed thin layers to be merged into neighboring layers (Salgado et al., 2015).

- SBT chart band approach – merging thin layers into adjacent layers by consideration of the secondary soil type(s) classification
- Soil group approach – merging thin layers into adjacent layers of the same soil group
- Average q_c approach – merging thin layers into adjacent layers with similar average q_c

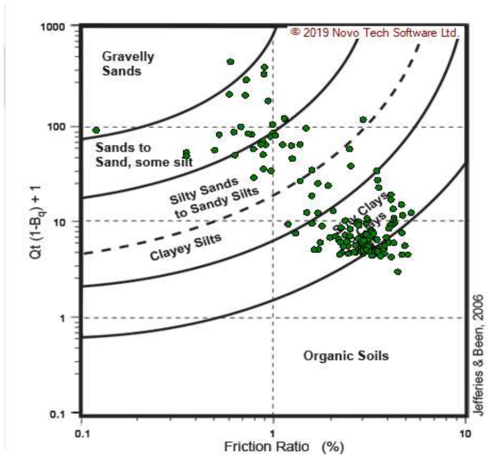

Figure 2: Jefferies & Been 2006 Soil type

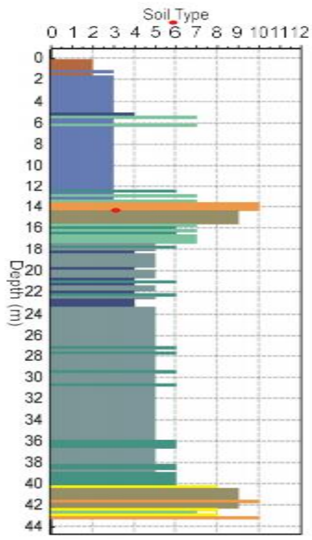

Figure 3: NovoCPT generated soil profiles

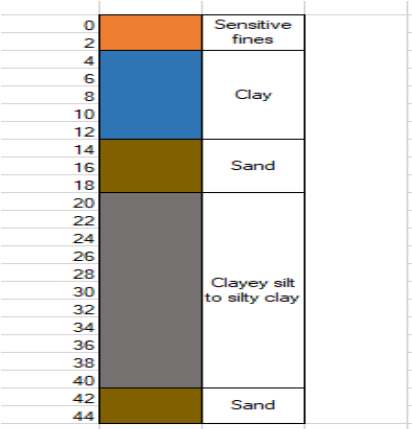

Figure 4: Adjusted Soil Profile

The final soil profile generated for the study, as shown in Figure 4, is composed of five main soil groups.

4.2 *Laboratory and in-situ test results*

The summary of in-situ and laboratory test results for undrained shear strength is presented in Table 3 which shows the mean of the undrained shear strength increasing with depth as the coefficient of variation (COV) decreased as depth increased.

Table 3: Statistics of undrained shear strength

Soil layer Description	No of data	Mean (kPa)	Std Dev	COV (%)
Sensitive organic clay	306	17.79	8.58	48.2
Clay	481	34.14	13.97	40.9
Clayey silt to silty clay	858	64.47	25.58	39.7

4.2 *Random Field Characterization*

4.2.1 *Vertical spatial variability of Cone Tip Resistance* q_c

The random field modeling for vertical variability of q_c is illustrated using data from the CP26 test hole. The plot of q_c against depth from the CPT interpretation software is shown in Figure 5. Each stratum was examined for spatial trend and where trends existed it was removed using Ordinary Least Square (OLS) method.

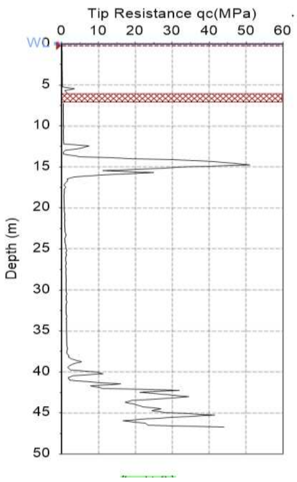

Figure 5: Cone tip resistance plot of CP26

The data exhibited both linear and quadratic trends. Figure 6 shows a trend model with residual over the depth band shown in Figures 7. The clay soil layers followed a linear trend while the sand and gravelly layers followed a quadratic trend. For the strata lying within 12.5-25m depth, the residual, r_{qc} is obtained by subtracting the trend value from the measured value as in equation (4).

$$r_{qc} = q_c - (9.3999 - 7.203x + 1.9175x^2 - 0.0951x^3) \quad (4)$$

Results of the Kendall's τ test had all the soil layers τ values within the range ± 1 and closer to 0 indicating stationarity of data. To estimate the correlation length, the autocorrelation functions (ACF) were computed for separation distances $\tau_k = k\Delta z$ for lags $k = 0,1,2, \ldots n/4$ where $\Delta z = 0.25m$ (i.e., sampling interval) and n=number of data points within layer. The plot of sample ACF over the separation distances with the cosine exponential theoretical model superimposed, for soil layer at 16.5-37.5m depth is shown in Figure 8.

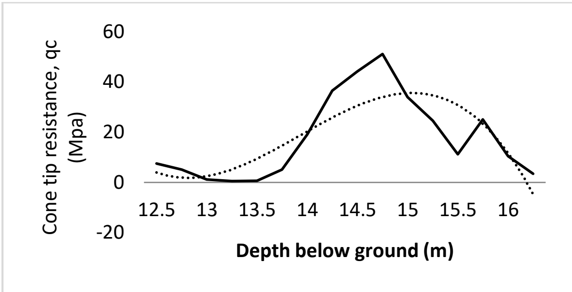

Figure 6: Measured cone tip resistance with quadratic trend for at 12.5-16.25m depth

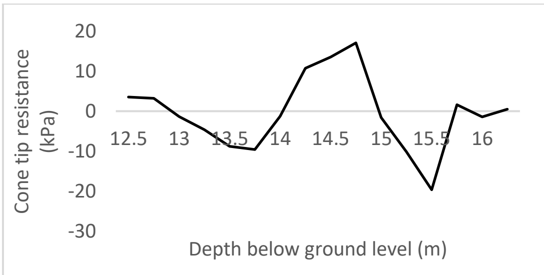

Figure 7: Residuals of qc, after trend removal at 12.5-25m depth

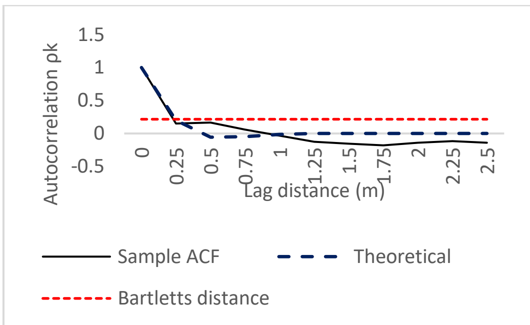

Figure 8: Sample and model ACF from residual of q_c, at 16.5-37 .5m depth

Summary of results of vertical variability analyses of CP26 test hole is presented in Table 4.

Similar analyses were repeated on the data from each of the other 15 CP test holes and the estimated vertical scale of fluctuations are presented in Table 5 for the clay soil units and Table 6 for the sand and gravelly soil units.

Table 4: Summary of results of vertical variability analyses on CP26 test hole data

Depth range of stratum (m)	Value of Parameter (m)	Bartlett's distance	Scale of Fluctuation, $\boldsymbol{\theta}$ (m)
0.25-5.25	α=0.478	±0.428	0.250
6.25-12.25	α=0.105	±0.392	0.135
12.5-16.25	α=0.345	±0.490	0.248
16.5-37.5	α=0.182	±0.216	0.176
37.75-46.5	α=0.261	±0.490	0.128

Table 5: Vertical Scale of Fluctuation, θ_v, of cone tip resistance, $q_{c,}$ in clay units

Sensitive fines	0.25 - 5.25m		
Clay		6.25 – 12.25m	16.5 – 37.5m
Variable	$\theta_V\ (m)$	$\theta_V\ (m)$	$\theta_V\ (m)$
CP10	0.246	0.162	0.186
CP11	0.195	0.173	0.212
CP12	0.184	0.168	0.179
CP13	0.224	0.161	0.210
CP14	0.246	0.165	0.202
CP16	0.214	0.142	0.180
CP17	0.238	0.158	0.204
CP18	0.147	0.154	0.199
CP19	0.210	0.145	0.201
CP20	0.167	0.172	0.212
CP22	0.247	0.174	0.216
CP23	0.195	0.165	0.183
CP24	0.246	0.175	0.188
CP26	0.250	0.135	0.176
CP27	0.135	0.188	0.216
CP28	0.165	0.167	0.181
Mean (m)	0.207	0.163	0.197
Std Dev (m)	0.038	0.014	0.014
COV (%)	18.6	8.4	7.4

Table 6: Vertical Scale of Fluctuation of cone tip resistance in sand and gravelly units

Sand	12.5 – 16.25m	
Gravely Sand		37.75 - 46.5m
Variable	$\theta_V\ (m)$	$\theta_V\ (m)$
CP10	0.265	0.182
CP11	0.237	0.198
CP12	0.321	0.180
CP13	0.237	0.210
CP14	0.246	0.200
CP16	0.231	0.242
CP17	0.322	0.183
CP18	0.280	0.203
CP19	0.280	0.225
CP20	0.288	0.220
CP22	0.313	0.250
CP23	0.284	0.240
CP24	0.249	0.254
CP26	0.248	0.128
CP27	0.135	0.125
CP28	0.268	0.220
Mean (m)	0.262	0.204
Std Dev (m)	0.045	0.038
COV (%)	17.2	18.8

Based on the results of the vertical variability analysis for the cone tip resistance, the following are deduced

a) The estimated vertical scale of fluctuation or correlation length for the clay soil layers are within the range 0.135-0.250m with coefficient of variation (COV) relatively high for the upper layers and less for the deeper layers indicating a more strongly spatial correlated cone tip resistance as depth increases. The sand and gravelly layers do not exhibit any notable trend as the correlation length seem not to be influenced by the depth of occurrence of the cohesionless layer.

b) The decrease with depth of the COV in the clay units Table 8), is a further indication that the soil parameters become more predictable with depth as soil deposits go into more stable state with increasing depth during the soil formation process.

4.2.2 *Vertical spatial variability of Undrained Shear Strength* s_u

The vertical variability of undrained shear strength is illustrated with the data for CPT test holes CP26 following similar steps as in the cone tip resistance. Data stationarity was confirmed, and the cosine exponential correlation model was used to fit the sample ACF as shown in Figure 9 with the summary of the analyses presented in Table 7.

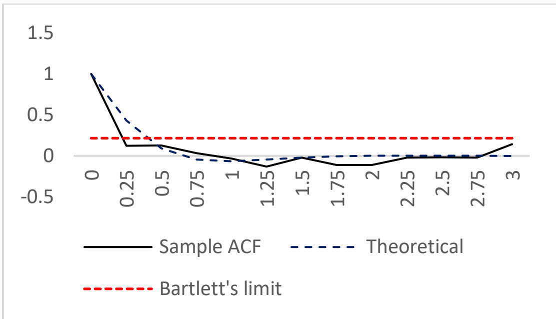

Figure 9: Sample and model ACF from residuals of s_u at 16.5-37 .5m depth

Table 7: Summary of vertical variability analyses of s_u with CP26 test hole data

Depth range of stratum (m)	Value of Parameter (m)	Bartlett's distance (m)	Scale of Fluctuation, $\boldsymbol{\theta}$ (m)
0.25-5.25	α=0.300	±0.428	0.191
6.25-12.25	α=0.400	±0.392	0.388
17.00-37.25	α=0.800	±0.216	0.875

Similar analyses were repeated on the data from each of the other 15 CP test holes and the estimated vertical scale of fluctuations are presented in Table 8.

The results of vertical spatial variability of the undrained shear strength also show that the COV decreases with depth, a further confirmation that the soil property is more spatially predictable as depth increases.

Table 9 presents the COV of the soil parameters for all the soil units investigated in the study showing decreasing COV as depth is increasing.

The estimated vertical correlation lengths for other soil layers of clay and sand are generally smaller than those reported in literature of other regions. The correlation lengths for the mangrove swamp compared with those in literature is presented in Table 10

Table 8: Vertical scale of fluctuation, θ_v of undrained shear strength, s_u

Sensitive fines	0.25 - 5.25m		
Clay		6.25 – 12.25m	16.5 – 37.5m
Variable	θ_V (m)	θ_V (m)	θ_V (m)
CP10	0.320	0.782	0.871
CP11	0.278	0.690	0.631
CP12	0.465	0.585	0.678
CP13	0.348	0.356	0.503
CP14	0.386	0.638	0.671
CP16	0.320	0.650	0.652
CP17	0.405	0.420	0.517
CP18	0.482	0.316	0.518
CP19	0.434	0.628	0.613
CP20	0.238	0.706	0.601
CP22	0.682	0.601	0.830
CP23	0.204	0.354	0.667
CP24	0.328	0.468	0.768
CP26	0.191	0.388	0.875
CP27	0.510	0.875	0.500
CP28	0.805	0.708	0.773
Mean (m)	0.400	0.573	0.667
Std Dev (m)	0.166	0.169	0.127
COV (%)	41.45	29.54	19.02

Table 9: COV of spatial variability with depth

Variability	Parameter	Coefficient of Variation COV (%) at depths				
		0.25-5.25	6.25-12.25	12.25-16.25	16.5-37.5	37.5-46.5
Vertical	q_c,	45.6	37.8		14.7	
	s_u	26.9	22.5		18.5	

Table 10: Vertical Correlation lengths of soil parameters compared with those reported in literature

Soil Property	**Vertical Correlation length (m)**			**Soil type**
	Study	**Literature**	**Source**	
$\boldsymbol{q_c}$,	0.14-0.25	2.00	Chiasson et al. (1995)	Sensitive clay
		0.80-1.80	Popescu et al. (1995)	Clay
		1.00	Vanmarcke (1977)	Clay
		0.1-2.2	Phoon and Kulhawy (1996)	Sand, Clay
	0.20-0.40	1.6	Kulatilake and Ghosh (1988)	Clean sand
$\boldsymbol{s_u}$	0.13-0.3	0.3-0.6	Keavenly et al (1989)	Offshore soils
		0.8-6.1	Phoon and Kulhawy (1996)	Clay
		0.06-0.24	Jaska (1999)	Clay

5 CONCLUSIONS

Spatial variability analyses of two geotechnical parameters commonly used in pile foundation analysis (cone tip resistance and undrained shear strength) were performed over the depth of penetration, taking each soil layer separately. The analyses estimated the vertical spatial variability of the soil parameters in terms of the correlation characteristics of the parameters within the domain.

- The generalized soil profile comprises of upper soft organic sensitive fines and alternating layers of clay and sand/gravely sand. The upper organic sensitive fines exhibit small correlation lengths for the parameters, indicative of a rough field up to a depth of about 5m. The geotechnical properties on this layer are largely unpredictable and changes over extremely small distances. In practice, such soil is usually adjudged unsuitable to support any structure and will be recommended for replacement and preloading with sandfills.
- The vertical correlation length for cone tip resistance and undrained shear strength for the clay and sandy soils of the site investigated are as shown in Table 11. Both parameters show strong spatial correlation as depth increases
- The estimated correlation lengths are recommended for use in planning and preliminary design for foundations in the area.

Table 11: Range of correlation lengths for Mangrove swamp site

Soil Type	Depth range of stratum (m)	Scale of Fluctuation,$\boldsymbol{\theta}$ (m)	
		Cone tip resistance	Undrained shear strength
Sensitive fines	0.25-5.25	0.135-0.250	0.191-0.805
Clay	6.25-12.25	0.135-0.188	0.316-0.875
sand	12.5-16.25	0.135-0.322	
Clayey silt to silty clay	16.25-37.5	0.176-0.216	0.500-0.875
Sand	37.75-46.5	0.125-0.254	

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