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Trans-dimensional Bayesian inversion of soil-structure interaction pressures on in-service underground structures

Inversion de la pression d'interaction sol - structure des structures souterraines en service par TANS vibayes

Zhiyao Tian, Quanmei Gong & Yu Zhao
Shanghai Key Laboratory of Rail Infrastructure Durability and System Safety and Key Laboratory of Road and Traffic Engineering of the Ministry of Education, Tongji University, China, Tianzy@tongji.edu.cn

ABSTRACT: Identification of the soil-structure interaction pressure is of great significance for analyzing, maintaining, and predicting the life-cycle performance of in-service underground structures, especially for the large-deformation ones whose load state is beyond expectation of design values. Direct measurement of the pressures by the sensors installed before the structure completed is rather costly. Inversion of the load pressure based on easily observed structural response, say deformation, is a more realistic way. Here, an adaptive Bayesian load inversion approach is proposed that do not require presumptions on the pressure distribution beforehand as most current methods do. Firstly, cubic spline with n unknown interpolation points ($\mathbf{x}_{(n)}=(x_1,x_2,\ldots,x_n)$) is adopted to parameterize the load inversion model space; Then, trans-dimensional Bayesian rule is used to build a relationship between observed structural response and unknown variable-dimension parameters (n, $\mathbf{x}_{(n)}$); Lastly, a sampler called Reversible Jump Markov Chain Monte Carlo (RJ-MCMC) algorithm is adopted to sample the variable-dimension model space to estimate the posterior probability density (PDF) of the unknowns. A numerical example is present for the necessary demonstration and verification.

RÉSUMÉ : L'identification de la pression d'interaction sol - structure est importante pour l'analyse, l'entretien et la prédiction de la performance du cycle de vie des structures souterraines en service, en particulier pour les structures souterraines à grande déformation dont l'état de charge dépasse la valeur de conception.Les capteurs installés avant l'achèvement de la structure pour mesurer directement la pression sont très coûteux.L'inversion de la pression de charge basée sur les données de déformation structurale est une méthode plus réaliste.Dans cet article, nous présentons une méthode adaptative d'inversion de la charge bayésienne, qui n'a pas besoin de présupposer la distribution de la pression comme les méthodes existantes.Tout d'abord, l'espace du modèle d'inversion de charge est paramétré par la fonction spline cubique avec n points d'interpolation inconnus ($\mathbf{x}_{(n)}=(x_1,x_2,\ldots,x_n)$);Ensuite, la relation entre les données d'observation de déformation et les paramètres de dimension variable inconnus (n, $\mathbf{x}_{(n)}$) est établie par la règle bayésienne transdimensionnelle.Enfin, l'algorithme de Monte Carlo de la chaîne de Markov à sauts réversibles (RJ - MCMC) est utilisé pour échantillonner l'espace du modèle de dimension variable et estimer la densité de probabilité postérieure (PDF) de l'inconnu.Un exemple numérique est donné pour démontrer et vérifier.

KEYWORDS: Inversion problem; trans-dimensional Bayesian; pressure identification; underground structures.

1 INTRODUCTION

The underground environment is complex. As a result of geotechnical condition uncertainty, geologic changes (e.g. landslide), or engineering activities, earth pressures on many in-service underground structures may exceed expectation of design values. For example, in soft soil areas of China, large-scale urban development activities brought unexpected additional load on many in-service shield tunnel structures, which have result in gross distortion of the tunnel rings with severe structural diseases. (Huang et al. 2016; Shi et al. 2016; Di et al. 2020; Tian et al. 2020)

In such a case, identification of the current earth pressures on these in-service structures is of great significance for safety monitoring and performance prediction of them. The pressures can be monitored with sensors that were installed in the structure before its completion (Han et al. 2016). However, for the in-service underground structures, there is no condition for the sensor installation. In addition, embedding the sensors before structure completion can be rather expensive and aimless (Li et al. 2020). Compared with direct measurement with sensors, inversion of the load pressures based on easily observed structural response, say deformation, is a more realistic way.

Some previous researches focus on inversion of design earth pressures on well-performing structures (e.g. Yan et al. 2018). Intuitively, in these studies, distribution of the pressures was presumed as design mode, and accordingly, the inversion load model space was parameterized into specified unknowns, e.g. $\mathbf{x}=(x_1,x_2,x_3)$ in Fig. 1 (a). An optimal solution of back analysis can be achieved by searching in the pre-defined model space. Nevertheless, for inversion of current earth pressures on the in-service underground structures, it is not feasible to presume a distribution of the pressures as the current load states of these structures have already exceeded expectation of design mode (Li et al. 2020; Mashimo and Ishimura 2003).

For the beyond-expectation pressure distribution, Gioda and Jurina (1981) used a series of polynomials to parameterize the inversion load model space. Similarly, Liu et al. (2019) and Liu et al. (2021) used a set of nodal loads on presumed superimposed loading meshes for the parameterization. Although their work did not require presumptions on the pressure distribution like a design mode, the number of parameters is fixed and needs to be presumed beforehand, which may lead to under-parameterization or over-parameterization. In addition, their work were all based on deterministic inversion, which means the a deterministic solution can be reached by minimization a pre-defined error function. Nevertheless, in the entire model space, many vastly different solutions can fit equally well the error function. That is to say, non-uniqueness can be encountered in the inversion problem but ignored by the previous studies.

In this paper, a statistical thinking is introduced, and accordingly, an adaptive Bayesian load inversion approach is proposed for the underground structures where non-uniqueness is quantified by probabilities. In this approach, a cubic spline is adopted to parameterize the load inversion model space that abandons presumptions on the pressure distribution beforehand; a trans-dimensional Bayesian inversion is introduced where the number (dimension) of parameters can also be seen as an unknown; a sampler called Reversible Jump Markov Chain Monte Carlo (RJ-MCMC) is adopted for estimation of posterior probability density function (PDF) of the unknown parameters. A numerical example is present for the necessary demonstration and verification.

2 METHODS

2.1 *Parameterization*

To avoid presumptions on the distribution of load on the underground structures, a concise and flexible parameterization method is proposed using cubic spline: parameterize the inversion load model space into a series of pressure interpolations on n evenly distributed knots. Take the diaphragm wall as an example (similar with the tunnel structures where interpolation in polar coordinates can be adopted), as seen in Fig. 1(b)−(c), the load inversion model space was parameterized into n interpolation parameters $\mathbf{x}_{(n)}=(x_1,x_2,\ldots,x_n)$ on the knots. That is to say, the entire pressures on the structures can be represented by unknow vector $\mathbf{x}_{(n)}$, according to the interpolation function:

$$\mathbf{p} = \mathbf{S_P}\mathbf{x}_{(n)} \tag{1}$$

Where $\mathbf{p}$ is the vector of entire pressures on the structure; $\mathbf{S_P}$ is the interpolating matrix of cubic spline and its detailed derivation can be seen in Press et al. (1993).

It is worth noting that as seen in Fig. 1(b)−(c), parameterization capacity of the cubic spline depends on the number of interpolation knots n. The more parameters, the stronger the parameterization capacity. As long as there are enough parameters, any load distributions can be covered in the model space. But in engineering practice, it is expected to make a tradeoff between model complexity and model fit. Thus, to choose a reasonable value of n, we regard the number (or dimension) of parameters n as an unknown as well, and try to infer the variable-dimension parameter (n, $\mathbf{x}_{(n)}$) based on the observation data (say deformation) itself.

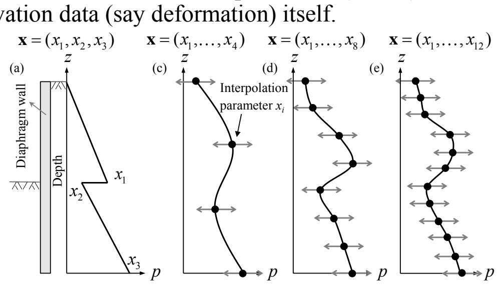

Figure 1. Parameterization the inversion model space: (a) as a design mode; (b) using spline with 4 parameters; (b) 8 parameters; (c) 12 parameters.

2.2 *Trans-dimensional Bayesian inversion*

As a result of the efforts in section 2.1, the load inversion model space has been parameterized into a variable-dimension parameter (n, $\mathbf{x}_{(n)}$). For a fixed-dimensional problem, inversion of parameters $\mathbf{x}$ can be realized with Bayes' rule given the "evidence" $\mathbf{d}$:

$$p(\mathbf{x}|\mathbf{d}) = \frac{p(\mathbf{d}|\mathbf{x})p(\mathbf{x})}{\int p(\mathbf{d}|\mathbf{x}')p(\mathbf{x}')d\mathbf{x}'} \tag{2}$$

Where $p(\mathbf{x}|\mathbf{d})$ is the posterior probability density function (PDF) of parameters $\mathbf{x}$ given the observed field data $\mathbf{d}$ (say the easily observed deformation data); $p(\mathbf{x})$ is the prior distribution of $\mathbf{x}$; $p(\mathbf{d}|\mathbf{x})$ is the likelihood function; the denominator in Eq. 1 is the probability of the observed data, and it is usually ignored in the inversion proves since it is a normalizing factor that is not contingent on any model.

Furtherly, considering the variable-dimension parameter that includes a hyperparameter n, i.e., the number of parameters is not yet fixed, Bayes' rule can be written for a trans-dimensional mode (Green 1995). The trans-dimensional Bayesian inversion is proved to have a property of "natural parsimony" (Malinverno 2002): among all models that fit the observed data, those with fewer parameters have higher posterior probabilities. It means we can adopt trans-dimensional inversion to infer the hyperparameter n with $\mathbf{x}_{(n)}$ simultaneously to make an adaptive tradeoff between model complexity and model fit. According to (Green 1995):

$$p(n,\mathbf{x}_{(n)}|\mathbf{d}) = \frac{p(\mathbf{d}|\mathbf{x}_{(n)},n)p(\mathbf{x}_{(n)}|n)p(n)}{\sum_{n'}\int(\mathbf{d}|\mathbf{x}'_{(n')},n)p(\mathbf{x}'_{(n')}|n')p(n')d\mathbf{x}'_{(n')}} \tag{3}$$

Where the posterior PDF $p(n,\mathbf{x}_{(n)}|\mathbf{d})$ is defined over the trans-dimensional model space; $p(\mathbf{x}_{(n)}|n)$ $p(n)$ is the prior distribution of the state (n, $\mathbf{x}_{(n)}$);

2.2.1 *The prior distribution*

The prior distribution reflects our prejudgment on the unknown parameters based on the prior information. It can be determined with our engineering judgment and any reasonable judgements can serve as the prior information.

It is often the case that we know little about the field condition. Thus, a minimal prior judgement can be assumed and a uniform distribution can be adopted indicating that each parameter has equal prior probability within the pre-defined bounds, e.g., x_i~Y($p_{\min}$,$p_{\max}$) (i=1,2,…,n) and n~Y($n_{\min}$,$n_{\max}$). Certainly, the bounds can be determined with the rough experience. For example, since the pressures on the underground structures are always positive, $p_{\min}$ can be set as zero. In short, the prior can be written as:

$$p(n) = \begin{cases} \frac{1}{\Delta n} & n_{min} \le n \le n_{max} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$p(\mathbf{x}_{(n)}|n) = \begin{cases} \prod_{i=1}^{n} p(x_i|n) = \prod_{i=1}^{n}\frac{1}{\Delta p} & p_{min} \le x_i \le p_{max} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Where $\Delta n=n_{max}-n_{min}$; $\Delta p=p_{max}-p_{min}$.

2.2.2 *The likelihood function*

The likelihood function is a quantitative measure of how well a given model with a particular set of parameter values can reproduce the observed data. It can be determined by the magnitude of the error vector:

$$\mathbf{e} = \mathbf{d} - \mathbf{g}(\mathbf{x}_{(n)},n) \tag{6}$$

Where $\mathbf{g}(\mathbf{x}_{(n)},n)$ is the forward modelling function which returns a vector of predicted data. The error vector $\mathbf{e}$ consists of model errors and measurement errors that can be assumed to be normally distributed. The likelihood function is then

$$p(\mathbf{d}|\mathbf{x}_{(n)},n) = \frac{1}{[(2\pi)^l \det\widehat{\mathbf{C}}_\mathbf{e}]}\exp\left(-\frac{1}{2}\mathbf{e}^\mathrm{T}\widehat{\mathbf{C}_\mathbf{e}}^{-1}\mathbf{e}\right) \tag{7}$$

Where $\hat{\mathbf{C}}_\mathbf{e}$ is the expected covariance of errors for the measured data, and the errors are typically assumed to be mutually independent and Gaussian distributed with constant variance (e.g., Sambridge et al. 2010); l is the length of vector $\mathbf{e}$.

2.2.3 *The forward model*

A forward model is required to generate predicted data given a particular set of parameter values. In geotechnical engineering, the load-structure model can be adopted that can predict deformation data of the structures under any given load parameters (n, $\mathbf{x}_{(n)}$). The well-known relationship in Finite Element Method is present here,

$$\mathbf{g}(\mathbf{x}_{(n)}, n) = \mathbf{K}^{-1}\mathbf{f} = \mathbf{K}^{-1}\mathbf{L_E}\mathbf{p} = \mathbf{K}^{-1}\mathbf{L_E}\mathbf{S_P}\mathbf{x}_{(n)} \quad (8)$$

Where the structure is discretized into a series of elements and $\mathbf{K}$ is the global stiffness matrix; $\mathbf{f}$ is the equivalent nodal forces that is equivalent to the pressures $\mathbf{p}$ with the transformation rules $\mathbf{L_E}$ based on virtual work; Certainly, $\mathbf{p}$ is uniquely determined by parameters (n, $\mathbf{x}_{(n)}$). Derivation of the $\mathbf{K}$ and $\mathbf{L_E}$ of a specific underground structure can be referred to Smith et al. (2015).

2.3 Reversible Jump Markov Chain Monte Carlo

In the absence of analytical solutions of Eq. 2, Markov Chain Monte Carlo (MCMC) can be a practical way to evaluate the posterior probabilities. MCMC is an iterative method, and at each iteration, the typical idea can be described as: (i) randomly perturb the current state $\mathbf{x}_c$ to produce the proposed sample $\mathbf{x}_p$ with proposal function q($\mathbf{x}_p|\mathbf{x}_c$); (ii) randomly accept or reject it with the acceptance ratio α($\mathbf{x}_p|\mathbf{x}_c$) based on Detail Balance (Metropolis et al. 1953):

$$\alpha(\mathbf{x}_p|\mathbf{x}_c) = \min\,[1, \frac{p(\mathbf{x}_p)}{p(\mathbf{x}_c)}\cdot\frac{p(\mathbf{d}|\mathbf{x}_p)}{p(\mathbf{d}|\mathbf{x}_c)}\cdot\frac{q(\mathbf{x}_c|\mathbf{x}_p)}{q(\mathbf{x}_p|\mathbf{x}_c)}\cdot|\mathbf{J}|] \quad (9)$$

Where $p(\mathbf{x})$ is the prior distribution of $\mathbf{x}$; $p(\mathbf{d}|\mathbf{x})$ is the likelihood function; q($\mathbf{x}_c|\mathbf{x}_p$) is the proposal function; matrix $\mathbf{J}$ is the Jacobian of the transformation from $\mathbf{x}_c$ to $\mathbf{x}_p$, referred to Bodin et al. (2012). After generating a number of samples, called "burn-in" period, the samples should converge to the target posterior PDFs.

For inversion of model parameters $\mathbf{x}_{(n)}$ and model dimensionality n simultaneously, Green (1995) extend MCMC to a trans-dimensional case, called Reversible Jump MCMC (RJ-MCMC). The idea of RJ-MCMC is introduced in our problem, and the iteration steps for joint state (n, $\mathbf{x}_{(n)}$) are designed as follows.

Three possible perturbation types with an equal probability 1/3 are considered:

i) Move. Dimension keeps unchanged. Thus, a Gaussian perturbation $\mathbf{p}_m$~N($\mathbf{0}$,$\mathbf{C}_m$) can be directly added on the current parameters $\mathbf{x}_c$ to generate $\mathbf{x}_p$;

ii) Birth. Seen in Fig. 2, Add a dimension at the current parameters n_p=n_c+1, generate n_p proposal parameters $\mathbf{x}_p$, evenly distributed, on the current spline fitted by current parameters, added by a Gaussian perturbation $\mathbf{p}_b$~ N($\mathbf{0}$,$\mathbf{C}_b$);

iii) Death. Similar with "Birth" step, delete a dimension n_p=n_c-1. generate n_p proposal parameters $\mathbf{x}_p$, evenly distributed, on the current spline fitted by current parameters, added by a Gaussian perturbation $\mathbf{p}_d$~N($\mathbf{0}$,$\mathbf{C}_d$);

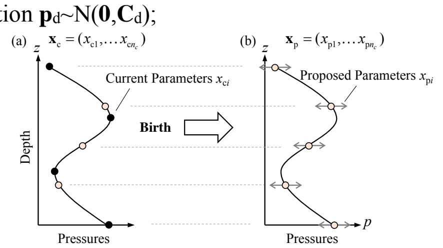

Figure 2. Diagram of the designed perturbation rule for birth step: (a) before perturbation; (b) after perturbation.

It is noted that the choice of proposal perturbation does not affect convergence of chains, but can strongly affect the iteration efficiency. Thus, the scale of perturbations $\mathbf{C}_m$, $\mathbf{C}_b$, and $\mathbf{C}_d$ are tunned to achieve a reasonable accept ratio (typically between 10%−60%) to avoid too many meaningless proposals. (Gallagher et al. 2011; Dosso and Wilmut 2008)

According to the above proposal scheme, the proposal function for the trans-dimensional case q(n_p,$\mathbf{x}_p$|n_c,$\mathbf{x}_c$) can be obtained. Then, the accept ratio α(n_p,$\mathbf{x}_p$|n_c,$\mathbf{x}_c$) can be computed with Eq. 9 to sample on the Markov chain. Take the Birth step as an example:

$$q(n_p, \mathbf{x}_p|n_c, \mathbf{x}_c) = q(n_p|n_c)q(\mathbf{x}_p|n_p, \mathbf{x}_c) \quad (10)$$

$$= \frac{1}{3}\frac{1}{[(2\pi)^{n_p}\det\mathbf{C}_b]}\exp\,(-\frac{1}{2}\mathbf{p}_b{}^{T}\mathbf{C}_b{}^{-1}\mathbf{p}_b) \quad (11)$$

Accordingly, the accept ratio α(n_p,$\mathbf{x}_p$|n_c,$\mathbf{x}_c$) can be computed with Eq. 9 with Eqs. 4,5,7, and 11.

2.4 Scheme of the Bayesian load inversion approach

Although some terms can be new to the field of geotechnical engineering (say, RJ-MCMC), the scheme of the approach is clear. As seen in Fig. 3, firstly, parameterize the inversion load model space into parameters (n,$\mathbf{x}_{(n)}$) using cubic spline; Secondly, our rough engineering judgement on the pressures can serve as prior information to determine the prior distribution; Simultaneously, the likelihood function can be built to measure the fitness between the observation data and the predicted values from the load-structure model given a particular set of parameter; Thirdly, a relationship between the posterior PDFs and prior distributions with likelihood function can be built based on Bayesian rule; Lastly, In the absence of analytical solutions, a numerical method called RJ-MCMC is adopted to estimate the posterior PDFs.

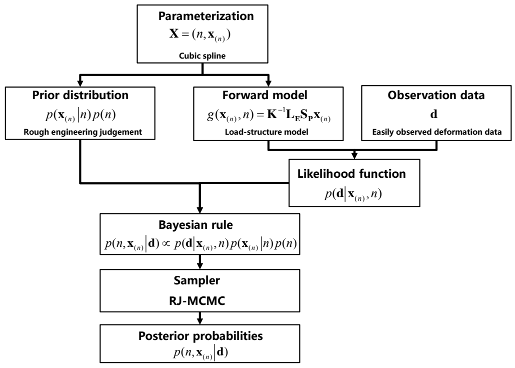

Figure 3. Scheme of the adaptive Bayesian load inversion approach.

3 NUMERICAL EXAMPLE

3.1 Preliminaries

A numerical example is present to demonstrate how to apply this approach. As seen in Fig. 4(a), under the earth pressures behind the cantilever wall, it deformed towards a pit. This process can be regarded as a "beam on elastic foundation" loaded by the active earth pressures. In the example, a set of casually determined pressures were assumed as seen in Fig. 4(b). Then, a set of synthetic deformation data (Fig. 4(c)) can be generated based on the forward model (Eq. 7). Here, inversion of the true pressures (assumed unknown now) acting on the wall merely with the "observed" deformation data is the objective. Note: basic setting of this numerical example can be seen in Fig. 4(a).

Firstly, the inversion load model space is parameterized into n evenly distributed pressure interpolations $\mathbf{x}_{(n)}$=(x_1,x_2,…,x_n) on the wall with a hyperparameter n, i.e. (n,$\mathbf{x}_{(n)}$).

Secondly, our rough engineering judgement can serve as the prior information for these parameters. As stated in section 2.2.1, uniform distributions x_i~Y(p_{min},p_{max}) (i=1,2,…,n) and n~Y(n_{min},n_{max}) can be adopted in this case. As for the bounds, from our engineering judgement, the lateral pressures on the wall must be positive and within a limited boundary, say the vertical pressure at bottom of the wall. Then, p_{min} is set as 0 while p_{min} is set as 400 kPa, i.e. 20 kN/m^3×20 m. Bounds of n can be set as n_{min}=3 (The minimum number to fit a cubic spline) and n_{max}=100 (a big enough number). It is noting that, under natural condition, the lateral pressure at the top of the wall (depth=0) is equal 0 and there is no need to set an unknown parameter at here. Thus, pressure at the ground remains to be 0 (for example, the diagram for 4 parameters can be seen in Fig. 4(d)).

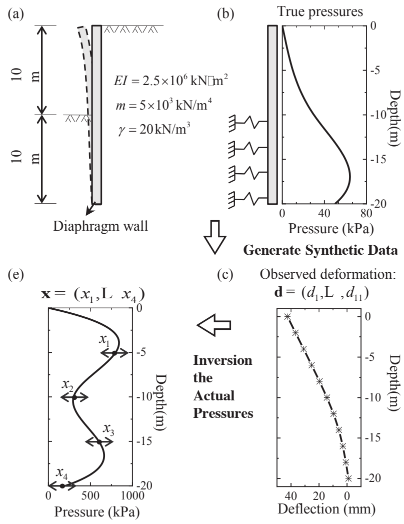

Figure. 4. The numerical example: (a) background of this example; (b) the assumed pressures acting on the wall; (c) synthetic deflection data generated by the assumed pressures; (d) diagram for 4 parameters situation.

The likelihood function can be built with the forward model with integration of the observation data (Fig. 4(c)). Then, based on the sampler introduced in section 2.3, combined with the prior distribution, likelihood function, and Bayesian rule, an ergodic Markov chain can be built, which will converge to the posterior distributions after "burn-in" period.

3.2 Results

We run the chain for 2×10^6 iterations. Fig. 5(a) shows the log likelihood (data fit) LL and number of parameters n during the whole sampling process. It is clearly that, during the first 200 steps, LL quickly converges to stable value (around -1); Simultaneously, number of parameters n, search for wide range from 12 to 4 for the initial steps; then converges to a stable range interval from 3 to 4. After a series stable sampling, the second half of the chain (step 1×10^6–2×10^6) is present in Fig. 5(b). Since there are no significant trends in the sampling but looks like white noise (indicating the end of "burn-in" period), it is concluded that sampling from the posterior PDF has been achieved. Thus, the second half chain is used to estimate the posterior PDFs.

Inversion results of the pressures can be seen in Fig. 6(a). Marginal PDFs of the pressures on the wall are present quantitatively with this colormap. Intuitively, light colours indicate high posterior density and dark colours lower density. It is found that the true pressures (white dashed line) are perfectly bound by the "hot" areas. Besides, the posterior mean pressures (Grey dotted line) fit perfectly with the true pressures also indicating effectiveness of Bayesian inversion. Fig. 6(b) shows the posterior distribution of the number of parameters n. It is shown 3 parameters has a lower posterior probability than 4 parameters. It can be interpreted that since the parameterization capacity of 4 parameters is higher than that of 3 parameters, the true pressures are covered in the model space of 4 parameters but not in 3 parameters. In addition, it is noted that 4 parameters has the highest posterior probability than the more potential parameters (e.g. 5, 6, and so on, equal zero). It indicates the "natural parsimony" of trans-dimensional Bayesian inversion that among all models that fit the observed data, those with fewer parameters have higher posterior probabilities.

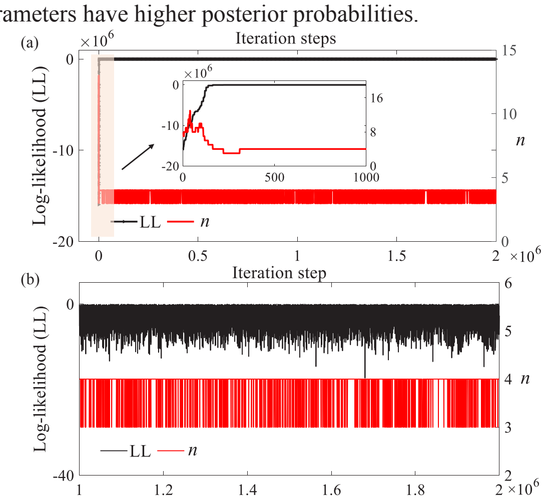

Figure 5. Samples on the Markov Chain: (a) the whole chain;(b) the second half of the chain

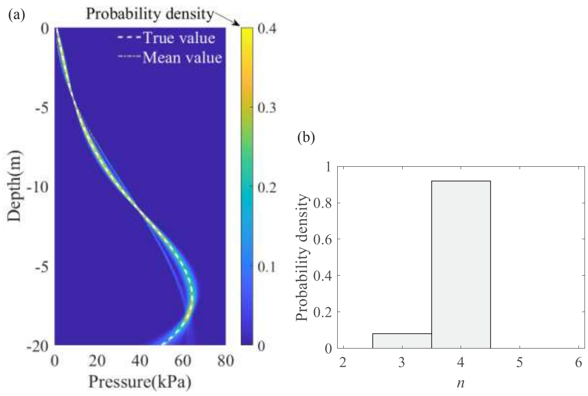

Figure 6. Posterior PDF of these parameters: (a) marginal PDF of the pressures distributed on the wall; (b) posterior distribution of the number of parameters n.

4 DISCUSSION AND CONCLUSIONS

An adaptive Bayesian load inversion approach for in-service underground structures is proposed in this paper where the inversion load model space can be parameterized into a variable-dimension parameter, indicating no presumptions on the potential load distribution is needed and the solutions can be inferred merely from the observed data.

In this approach, non-uniqueness of the inversion solution can be quantified with probabilities rather than being ignored in the previous studies. It means a statistical thinking is needed for further engineering decision. For example, the PDFs of internal force or damage degree of the in-service structures can be computed, and furtherly, statistics like expectation of maintenance cost, can be derived to make a more robust engineering decision.

We are eager to apply this approach into large-deformed shield tunnel structures. However, there is still a long way to go, since computation of the forward model of a shield tunnel structure can be time-consuming (strong nonlinearity) that is not suitable for quantities of iteration in RJ-MCMC. Thus, how to simplify the forward model or improve efficiency of RJ-MCMC can be the future work.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

Bodin T., Sambridge M., Tkalčić H., Arroucau P., Gallagher K., and Rawlinson N. (2012). Transdimensional inversion of receiver functions and surface wave dispersion. Journal of Geophysical Research: Solid Earth, 117, B02301.
Di H., Zhou S., Guo P., He C., Zhang X., and Huang S. (2020). Observed long-term differential settlement of metro structures built on soft deposits in the Yangtze river delta region of China. Canadian Geotechnical Journal, 57: 840–850.
Dosso S. E., and Wilmut M. J. (2008). Uncertainty estimation in simultaneous Bayesian tracking and environmental inversion. The Journal of the Acoustical Society of America, 124(1): 82–97.
Gallagher K., Bodin T., Sambridge M., Weiss D., Kylander M., and Large D. (2011). Inference of abrupt changes in noisy geochemical records using transdimensional changepoint models. Earth & Planetary Science Letters, 311(1–2): 182–194.
Gioda G.,and Jurina L. (1981). Numerical identification of soil-structure interaction pressures. International Journal for Numerical and Analytical Methods in Geomechanics: 5(1), 33–56.
Green P. J. (1995). Reversible jump Markov Chain Monte Carlo computation and Bayesian model determination. Biometrika, 82(4): 711–732.
Han L., Ye G. L., Li Y. H., Xia X. H., and Wang J. H. (2016). In-situ monitoring of frost heave pressure during cross passage construction using ground freezing method. Canadian Geotechnical Journal, 53: 530–539.
Huang H., Shao H., Zhang D., and Wang F. (2016). Deformational responses of operated shield tunnel to extreme surcharge: a case study. Structure and Infrastructure Engineering, 13(3): 345–360.
Li X., Zhou S., and Di H.. (2020). Observed Ground Pressure Acting on the Lining of a Large-Diameter Shield Tunnel in Sandy Stratum under High Water Pressure. Advances in Civil Engineering, 2020: 1–12.
Liu H., Liu Q., Liu B., Tang X., Ma H., Pan Y., and Fish J. (2021). An efficient and robust method for structural distributed load identification based on mesh superposition approach. Mechanical Systems and Signal Processing, 151: 107383.
Liu Q., Liu H., Huang X., Pan Y., Luo C., and Sang H. (2019). Inverse Analysis Approach to Identify the Loads on the External TBM Shield Surface and Its Application. Rock Mechanics and Rock Engineering, 52: 3241–3260.
Malinverno A. (2002). Parsimonious Bayesian Markov Chain Monte Carlo inversion in a nonlinear geophysical problem. Geophysical Journal International, 151(3) :675–688.
Mashimo H., and Ishimura T. (2003). Evaluation of the load on shield tunnel lining in gravel. Tunnelling and Underground Space Technology, 18: 233–241.
Metropolis N., Rosenbluth A. W., Rosenbluth M. N., Teller A. H., and Teller E. (1953). Equations of state calculations by fast computing machines. Journal of Chemical Physics, 21: 1087–1091.
Press W.H., Flannery B.P., Teukolsky S.A., and Vetterling W.T. (1992). Numerical Recipes in C: The Art of Scientific Computing, 2nd edition. Cambridge University Press, New York.
Sambridge M., Bodin T., Reading A., Gallagher K.. (2010). Inference from noisy data with an unknown number of discontinuities: ideas from outside the box. ASEG Extended Abstracts, 2010:1, 1–5.
Shi C., Cao C., Lei M., Peng L., and Ai H. (2016). Effects of lateral unloading on the mechanical and deformation performance of shield tunnel segment joints. Tunnelling and Underground Space Technology, 51: 175–188.
Smith I. M., Griffiths D. V., and Margetts L. (2015). Programming the Finite Element Method, 5th edn. Chichester: John Wiley & Sons.
Tian Z., Gong Q., and Di H., (2020). What causes the excessive metro tunnel settlement A case study. 99th Annual Meeting of the Transportation Research Board, Washington, D.C.
Yan Q., Zhang W., Zhang C., Chen H., Dai Y., and Zhou H. (2018). Back analysis of water and earth loads on shield tunnel and structure ultimate limit state assessment: a case study. Arabian Journal for Science & Engineering, 44: 4839–4853.