Proceedings of the 21st ICSMGE, Vienna, Austria, 14 – 19 June 2026. Pistrol, Adam & Schweiger (eds.)
Published by: ÖGG, Austrian Society for Geomechanics, Salzburg, Austria, ISBN 978-3-9503898-4-5
https://doi.org/10.53243/ICSMGE2026-1079

Quantification of energy dissipation in nonlinear time-history simulations

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ABSTRACT: A method is presented for quantification of dissipation in dynamic nonlinear finite element models. The method is illustrated on an academic test problem and then applied to a practical case involving an embankment dam. The method is intended to support the validation of physically based energy dissipation mechanisms that arise from properly modelling hysteretic stress-strain behavior in advanced material models. Quantifying hysteretic damping in such models is essential for calibrating model parameters. With this capability, it may be possible to move beyond Rayleigh damping, which has gained widespread use more for its simplicity than for its adequacy in representing dissipation in engineering applications.

KEYWORDS: Numerical modelling, dissipation, damping, finite elements.

1 INTRODUCTION

This paper focuses on energy dissipation in nonlinear numerical simulations of dynamic problems. Material dissipation arises both from plastic deformation at large strains and from nonlinear behavior within the small-strain range. Advanced material models enable the simulation of complex soil behavior, including hydromechanical coupling. Radiation of energy through model boundaries represents another significant form of dissipation. Accurately controlling and quantifying each type of dissipation is essential for validating numerical results.

For linear problems, quantification of energy dissipation is straightforward, as the system can be analyzed in the frequency domain by decomposition into a series of single-degree-of-freedom oscillators. However, for nonlinear systems, modal decomposition is not applicable because vibration properties evolve in response to effects such as plasticity, interface separation and slipping, or poromechanical coupling. Even in structures dominated by the first mode - such as wind turbines - modal analysis becomes challenging due to the dependency of the fundamental frequency on deformation amplitude. As a result, commonly used procedures in guidelines and building codes that are based on fixed damping ratios need to be reviewed.

In this paper, the method of logarithmic decrement for quantifying energy dissipation during free-vibration tests is discussed and evaluated. With regard to material behavior, we focus on energy dissipation mechanisms excluding plasticity, as the associated nonlinear effects are typically more localized in space and time and are therefore best analyzed separately.

2 CURRENT PRACTICE

Current practice in numerical analysis of nonlinear SSI-systems is largely based on work done for linear systems. The method of constructing damping matrices by linear combination of mass and stiffness matrices, known as Rayleigh damping, is commonly used. For the soil strain-dependent damping ratios based on experimental results are often used (Seed, et al., 1986). In linear elastic approaches this leads to an iterative approach adjusting damping as well as stiffnesses for the effective shear deformation. However, as multiple eigenmodes often interact in a single model the method of Rayleigh damping is not really suitable.

The use of advanced material models including hysteretic damping at small strains is more physically sound. However, the drawback of this approach is in the difficulty of validating the numerical damping with the target strain-dependent values.

A literature review of methods for quantification of dissipation shows some publications in the late 1990's, mainly focusing on experimental methods in the field of dam safety (Cantieni, 2001). The topic gained considerable interest in recent years due to the construction of offshore wind turbines (Malekjafarian, et al., 2021). In the context of numerical modelling, however, the criteria for a method to be suitable for damping estimation are slightly different from experimental applications. This mainly has to do with the effort required for numerical analyses, which is proportional to the number of time steps to be computed. Methods based on forced or ambient vibrations generally lead to large time spans that need to be looked at (Preisig, 2025). On the other hand, analysis of free-vibration tests is more suitable for numerical approaches, as it can be done in a single time-history analysis provided that a suitable initial configuration can be defined.

3 PROPOSED APPROACH

The proposed approach is based on free vibration analysis and the logarithmic decrement method. The method is intended to be applicable to any type of geotechnical system, from offshore wind turbines on monopiles with well-separated modes over buildings of any type to purely geotechnical structures such as embankments and earth dams.

The steps of the approach can be outlined as follows:

- Analysis of the geotechnical system and the numerical model in terms of stability, if applicable, optimization of the model
- Estimation of predominant mode shapes and frequencies
- Evaluation of load-deformation behavior for mode shapes of interest
- Free-vibration analysis by specification of mode shapes as initial condition, analysis of damping properties for modes of interest as a function of strain amplitudes

The approach is illustrated using a 1D soil column modelled by the Hardening Soil Small Strain material model (HSS). The column is fixed at its base and has a surface load of -50 kPa applied at the top. The following HSS stiffness moduli were used: $E_{50} = E_{oed} = 25\,MPa$, $E_{ur} = 50\,MPa$, $E_0 = 240\,MPa$, stiffness exponent $m = 0$, threshold shear strain $\gamma_{0.7} = 0.0001$. Strength was modelled by a friction angle of $\phi = 25°$ and a cohesion of $c = 10\,kPa$. A unit weight of $\gamma = 20\,kN/m^3$ was selected in the total stress analysis. The simulations are performed using the software ZSoil (ZACE Services Ltd; GeoDev Sarl, 2025).

3.1 *Analysis and optimization of the numerical model*

Before proceeding to complex nonlinear time history analyses a couple of standard checks should be performed on any numerical model. These include thorough investigation of static stability, influence of different modelling choices such as material models, structural elements and soil-structure interfaces. Verification of discretization in time and space also needs to be done carefully to make sure that the important aspects of model behavior can be captured. Finally, the model should be optimized such that the computational effort remains reasonable without affecting the results of interest.

3.2 *Estimation of vibration modes*

For geotechnical structures with simple vibration mode shapes, such as wind turbines or tall buildings, the first mode can often reasonably well be excited by application of a single displacement to a node at the top of the structure. For retaining walls or embankment dams, mode shapes have to be estimated beforehand. When doing this by means of eigenvalue analyses one has to keep in mind that stiffnesses are strain dependent and can therefore change during the course of an analysis. Nevertheless, in this approach we propose to apply the mode shapes obtained with constant stiffnesses as initial displacement for free vibration analyses.

Aspects to be considered include initial stress state, stress or depth-dependent stiffnesses and hydraulic conditions such as ground water and dynamic interaction with reservoirs. Figure 1 shows a comparison of the first eigenmode with stress-independent stiffness ($m = 0$) and a stiffness exponent of $m = 0.5$ (ZACE Services Ltd; GeoDev Sarl, 2025).

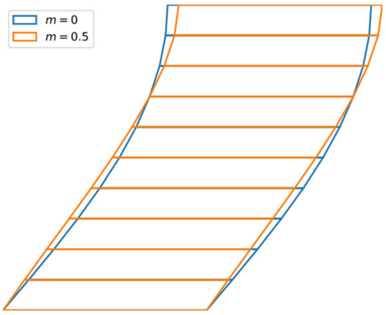

Figure 1. 1st mode for a 1D soil column. Blue: Stress-independent stiffness ($T_0 = 0.22$ s), Orange: Exponent m = 0.5 ($T_0 = 0.27$ s).

3.3 *Load-deformation behavior*

With the modes determined in the previously the static load-deformation behavior of the system can be calculated. By gradual application of the previously estimated mode shape a load-displacement curve is obtained, allowing determination of appropriate displacement amplitudes for which the behavior remains reversible and plastic deformation remains negligible.

Figure 2 shows a load-displacement curve for a push-over test of a 1D soil column. The mode shape was imposed on the horizontal displacements only, leaving the vertical displacements free.

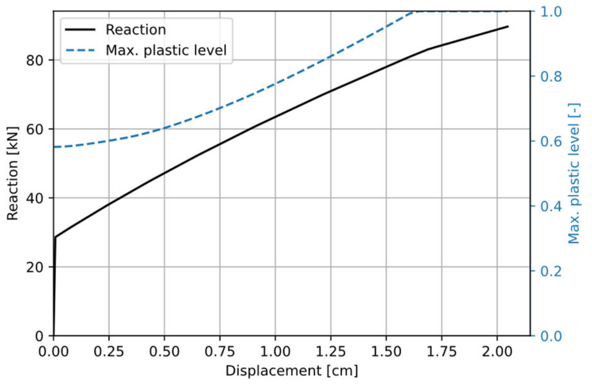

Figure 2. Load-displacement curve for push-over test of 1D soil column. Maximum plastic level over all elements of the column is shown by the dashed line.

For the free vibration analysis an initial displacement amplitude of 1.6 cm will be selected, which is just before the first element will fully plastify.

3.4 *Free-vibration analysis*

The free-vibration analysis is performed after imposing an initial displacement in the form of the first mode shape. Vertical displacements are left free in order to avoid over constraining the column in its initial configuration.

The damping behavior is analyzed using the velocity time history recorded at the top node of the column (black line in Figure 3). The figure shows that some distortion of the sinusoidal signal is occurring, probably stemming from the strain dependency of the small-strain stiffness of the HSS material model.

Ratios of critical damping ξ are computed as follows:

$$\xi_{\mathrm{k+1}} = \frac{\Lambda_{\mathrm{k+1}}}{\sqrt{4\pi^2 + \Lambda_{\mathrm{k+1}}^2}} \tag{1}$$

where the logarithmic decrement $\Lambda_{\mathrm{k+1}}$ is obtained from the velocity amplitudes of consecutive peaks of equal sign A_k and A_{k+1} (red dots in the figure):

$$\Lambda_{\mathrm{k+1}} = \ln \frac{A_k}{A_{k+1}} \tag{2}$$

In Figure 3 the ratios of critical damping are indicated for each pair of consecutive peaks. Also plotted are logarithmic curves fitted to variable numbers of peaks, starting at the first, the second etc. The damping ratios obtained illustrate the reduction of damping in an averaged way.

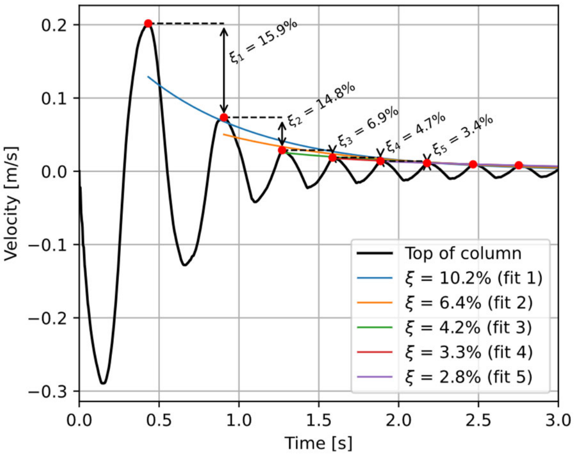

Figure 3. Free-vibration analysis of 1D soil column. Damping ratios are calculated from amplitude decay for consecutive cycles as well as by fitting a logarithmic curve starting from the first, second etc. peak.

The results highlight the strong amplitude dependence of the damping ratios. This observation is consistent with our expectations, as for higher amplitudes hysteresis loops enclose larger areas leading to larger amounts of dissipated energy.

The amplitude dependency of the damping ratio is further investigated by means of parameter variations of the small strain parameters of the HSS material model. The two parameters governing this behavior are the maximum shear modulus G_0 and the characteristic shear strain $\gamma_{0.7}$, value at which the secant shear modulus reduces to 70% of its initial value G_0. In this study G_0 is kept constant but the ratio of initial stiffness to unloading stiffness E_0/E_{ur} is varied. For each set of parameters, a free-vibration test is performed for which the damping ratios fitted through 3 consecutive peaks are plotted against the averaged maximum shear strains for these peaks. These values are plotted on the curves developed by (Vucetic & Dobry, 1991) in Figure 4. The results indicate that:

- Orders of magnitude of damping ratios and shear strains as well as their evolution are consistent with the data proposed for soils.
- The characteristic shear strain $\gamma_{0.7}$ is the main parameter of the HSS model governing the amount of damping in the small strain range.
- The ratio E_0/E_{ur} influences the slope of the damping curve, although to a lower extent.

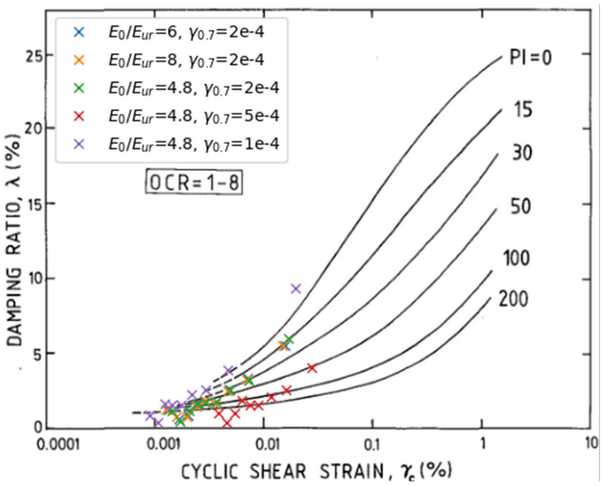

Figure 4. Strain-dependent damping ratios obtained from free-vibration tests on columns of soil modelled by HSS model, plotted against the data from (Vucetic & Dobry, 1991).

4 EXAMPLE

As a slightly more complex example the analysis of an embankment dam is presented. The dam is modelled in a 2D cross section through the deepest part of the river (Figure 5).

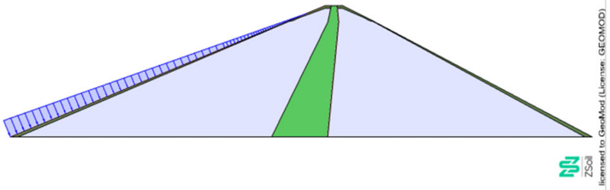

Figure 5. 2D FE-Model of the embankment dam.

Applying the exact shape of the first mode of vibration is generally possible, even with commercial FE software. It requires most likely some user developments, which is possible in most software for instance through a python API. Results of an eigenmode analysis can be extracted and be applied as an initial displacement field to the model, on which a free vibration analysis will be performed in time domain activating all relevant nonlinear aspects of the model.

However, such a procedure might not be feasible in some instances. We therefore investigate the possibility of applying an approximation of the first mode shape as initial condition.

The 160 m high dam is composed of a clay core and a coarser grained shell. The constitutive model is formulated for effective stresses, using a seepage flow analysis to determine the pressure field. The dynamic analysis is then performed in undrained conditions. The material behavior is modelled using the HSS model, which by default does not include increase in pore water pressure due to cyclic straining. Fixed boundary conditions are applied at the base, so no radiation damping is allowed to occur. The fundamental period of the dam resulting from the eigenmode analysis is 0.95 s.

No Rayleigh damping is present in these analyses. Dissipation is purely due to hysteretic dissipation in the dam body due to small strain nonlinearity. Some numeric dissipation is active at very high frequencies as a result of the use of the HHT time stepping algorithm (Hughes, 2012), but no influence on the calculated damping ratio is to be expected.

The simplified initial horizontal displacement is defined as a quadratic function of height, with the displacement at the crest being scaled to 0.1 m. This displacement has been selected based on the pushover analysis, aiming at an initial state with minimal plastic strain. The vertical displacement is left free. The influence of the initial displacement is evaluated by comparison with a second analysis in which the latter is obtained through an eigenmode analysis. The first eigenmode, also scaled to a displacement of 0.1 m, is applied. As in the first case, only horizontal displacements are imposed while vertical components of the eigenmode are ignored leaving these degrees of freedom free. Figure 6 shows the initial configurations, which are applied stepwise before the free vibration analysis.

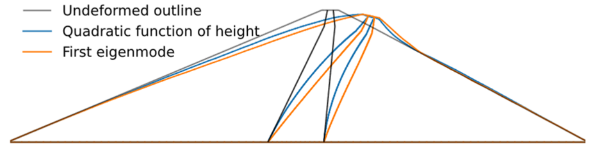

Figure 6. 2D FE-Model of the embankment dam: Initial displacements for free-vibration analyses (amplified by 500).

The results in terms of velocity time histories are shown in Figures 7 and 8. In order to reduce some high frequency oscillations, the velocity is averaged over nodes near the dam crest, at various depths and on the up- and downstream sides. Velocity is chosen as displacement shows some drift due to permanent deformations and acceleration has larger high frequency content, therefore making analysis more difficult.

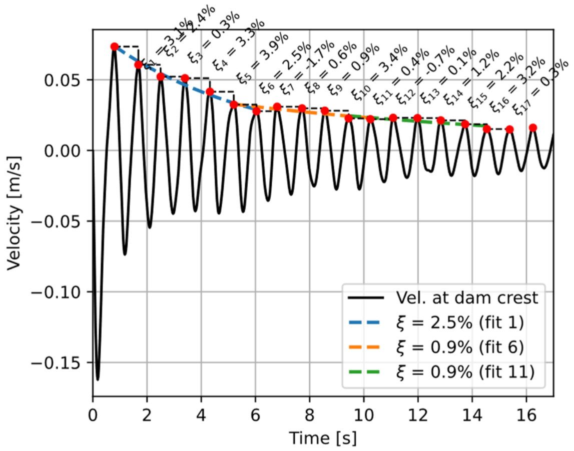

Figure 7. Velocity time history at the dam crest, quadratic function of height applied as initial displacement.

In the figures damping ratios are computed between each pair of consecutive positive peaks. Averaged damping ratios are obtained by fitting logarithmic functions to 7 consecutive peaks starting from the first, the 6th and the 11th peak.

It can be seen that the damping ratio decreases with velocity amplitude. While at the beginning oscillations exhibit some unsteady behavior, amplitudes tend to decay relatively smoothly later on. We also note that the form of the initial displacement has strong influence: the closer the shape is to the first eigenmode, the less impact the unsteady behavior has.

Overall, we note that pure material damping is relatively small. A preliminary analysis of averaged shear strains indicate that damping ratios are consistent with (Vucetic & Dobry, 1991). With such small material damping it is reasonable to assume that other forms of energy dissipation, such as radiation of energy away from the dam need to be looked at more closely.

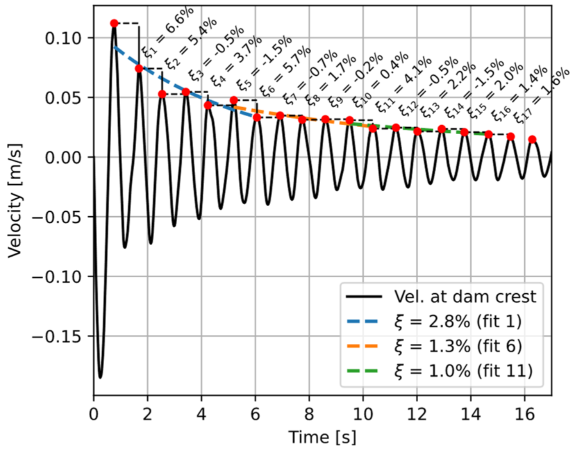

Figure 8. Velocity time history at the dam crest, first eigenmode applied as initial displacement.

In an attempt to quantify radiation damping we first add a stiff rock layer at the base of the dam. The rock is modelled elastically with a Young's modulus of about 30 times the small deformation modulus E0 at the base of the dam. Laterally the rock layer is extended on both sides by about 1.3 times the dam width at its base. Both lateral sides are linked by kinematic constraints, representing periodic boundary conditions. The depth of the rock layer in the FE-model is 200 m.

First, we perform a free-vibration analysis on this model having the base of the rock layer fixed in both horizontal and vertical directions. Then we apply radiating boundary conditions by adding Lysmer elements, which absorb outgoing shear waves. Results are shown in Figures 9 and 10.

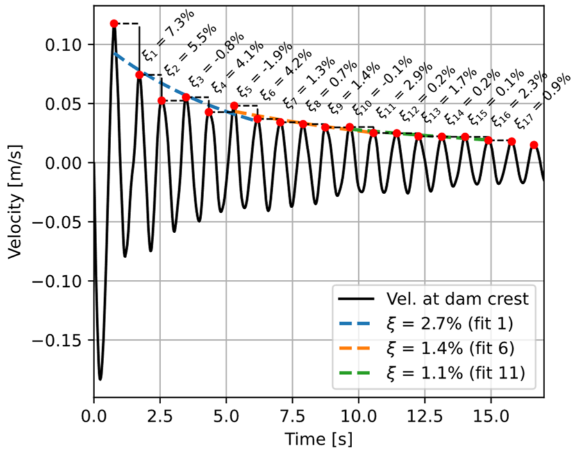

Figure 9. Velocity time history at the dam crest, dam on 200 m deep rock layer on fixed base.

Firstly, we note that the stiff rock base by itself causes almost no change to the vibration behavior at the dam crest. Closer inspection shows only a small elongation of the period. This is the intended outcome for such a high stiffness contrast between dam and rock foundation, and it seems that the rock base can safely be neglected without any change to the results.

However, this conclusion has to be revised when we look at the results of the model that features absorbing boundaries at its base. The velocity at the dam crest decays at much higher speed, exhibiting about twice as much dissipation in terms. This result may appear surprising, since most of the wave energy is reflected at the dam base. In the end it highlights the importance of properly modelling radiation damping, which sometimes might necessitate extending a model at greater depths even when stiff rock horizons are encountered.

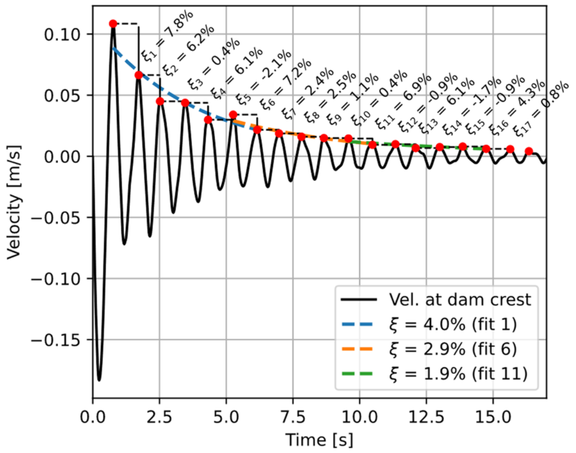

Figure 10. Velocity time history at the dam crest, dam on 200 m deep rock layer with absorbing boundaries at base.

5 CONCLUSIONS

An approach is presented allowing quantification of damping in dynamic finite element simulations. The approach is shown to be applicable to any type of model and provide damping ratios for complex constitutive models such as the Hardening Soil with small strain model (HSS). Furthermore, the approach can also quantify radiation damping.

Limitations of the method are mainly related to the difficulty of applying an initial displacement that results in a free-vibration in the form of the fundamental mode. For complex model geometries this can be challenging and requires some form of control of a numerical model that commercial FEM codes facilitate to different degrees. Furthermore, the method is limited to material behavior outside the plastic domain. Nonlinear behavior in the small strain range, as it is implemented in the HSS model available in many commercial software packages, can also present challenges due to the strain dependency of the stiffness.

For practical applications the method allows quantification of advanced sources of energy dissipation through physical concepts such as hysteretic material damping and radiation damping. This should allow users to move away from the concept of Rayleigh damping, which is difficult to justify from a physical standpoint.

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