

Geotechnical design: “when it all began”

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ABSTRACT: Probably the oldest geotechnical problem handled with mathematics was the stability of a gravity wall submitted to earth pressure. It all began on the eve of the 18th century in both civilian and military contexts. Many facts about this history are already well known but there is still room for comments based on a thorough reading of the original texts. These approximately 240 pages range from 1691 to 1729 and include the works of the five first authors of theories on earth pressure, all French: Bullet, Gautier, Buchotte, Couplet and Bélidor. None of them was cited by Coulomb in 1773. We show that, beyond some archaisms and a lack of knowledge on shear resistance, a number of their analyses and results were definitive.

KEYWORDS: history of geotechnics, soil mechanics, earth pressure, geotechnical model, gravity wall, 18th century.

1 INTRODUCTION

“Where it all began” is the ISSMGE 2026 Conference’s tagline. It is a reference to a historical fact, the birth of modern soil mechanics foremostly with Karl Terzaghi’s *Erdbaumechanik auf bodenphysikalischer Grundlage*, published in 1925 in Vienna. According to Skempton (1985, p. 95), the mid 1920’ were in fact the beginning of the *second phase of modern* soil mechanics, which is still ours. The *classical* soil mechanics had started with Coulomb’s essay, brought out in 1773 but with general agreement after 1800 thanks to Prony (Gillmor, 1969, p. 189). So, Skempton’s first period of soil mechanics, called *pre-classical*, coincided with the 18th century, without any definite beginning. This was a time “characterised by empirical earth pressure theories based on the ‘natural slope’ and unit weight of earth fill materials” (*ibid.*, 1985).

Asking “when it all began” can, by all means, be applied to this first period of soil mechanics in order to know to what extent we can still benefit from “the Fathers” concrete problems, observations, means and logic related to what is now called broadly speaking ‘geotechnical design’. The difficulty with such a question is accessing the original texts, not always easily available, understanding the, sometimes, outdated terms and concepts and even the poor handwriting in the case of manuscripts!

2 GUIDELINES OF THIS ARTICLE

A significant number of authors wrote about that subject: Feld (1948), Kerisel (1956, 1985 and 1993), Verdeyen *et al.* (1968), Skempton (1981 and 1985), Corradi (1995), Bordes (2000), Hettler and Kurrer (2020), to cite just a few. They all agree on the fact that scholar soil mechanics didn’t start all at once with the entire range of subjects we address nowadays but with only one: the stability of masoned gravity walls – it was in fact the very first geotechnical problem to be handled with mathematics in construction engineering as a whole. The problem depended on solving the theoretical as well as experimental phenomenon of lateral earth pressure.

One of the first authors who gave a broad overview on the 18th century attempts to solve the earth pressure problem, and so as to foster Coulomb’s results, was Mayniel (1808) (Feld, 1948). This French military engineer and others in the first half of the 19th century, such as Navier, have had a considerable influence on contemporaneous studies of *pre-classical* soil mechanics because their practical and scholarly summaries or commented reediting of older books made less necessary an access to primary sources, with all the aforementioned reading

difficulties. However, as part of a thesis in progress on earth pressure theories through time, carried out by this article’s first author and supervised by the four others, it has been considered as an opportunity for the chapter on the *pre-classical* period to collect exhaustively and work directly on the original texts, insofar it be possible.

In the early decades of the 18th century, presumably for defensive military reasons, building theories on earth pressure applied to the design of retaining walls was almost exclusively a French concern. Indeed, before Coulomb’s essay, to the best of our knowledge and relying on Feld (1948, pp. 7-8), Corradi (1995, pp. 362-365), and Hettler and Kurrer (2020, pp. 128-144), it appears that the only exceptions to that observation are the Italians Borra (1748) and Lorgna (1763), maybe the Austrian Kinsky (1763), and a Dutchman, Ypey (1765). This list may usefully be extended by professional historians.

Within the framework of the present article, we will concentrate on the first few decades of the *pre-classical* period of soil mechanics and geotechnical design and therefore focus on the first five authors known for having considered, after centuries of practice, the earth pressure problem as a theoretical topic. These authors are Bullet (1691), Gautier (1717), Buchotte (1718 and 1726), Couplet (1726, 1727 and 1728), and Bélidor (1729), all preceded by Vauban (1687) who – with Bullet – can be regarded as a forerunner.

An overview of the general mechanical concepts prior to these early decades is nevertheless necessary.

3 MECHANICAL CONCEPTS AROUND 1700

To take stock of the state of mechanics around 1700, we used Duhem (1903 and 1905), Dugas (1950 and 1954), Timoshenko (1953) and Benvenuto (1991), unless mentioned otherwise.

3.1 “Machines simples”

The first essay entirely dedicated to mechanics, and more particularly to of mix of kinematics and statics, was called the *Mechanical Problems*. It is attributed to Aristotle or someone of his time (4th/3rd century B.C.) and dedicated to the study of “machines”. The word could be translated by “basic mechanisms” and was referred to in French as “*machines simples*”. Among these were for instance the wheel, the pulley, the screw, the wedge, but the fundamental one was the lever – for instance materialised by a steelyard balance. The inclined plane, key for early soil mechanics, was to join the list in the late Antiquity. All *machines simples* had been of common use for practical purposes but explaining how they worked in (simple) mathematical terms has proven difficult – it took

centuries to succeed. Even though the *machines simples* gradually disappeared from the secondary school curricula in the second half of the 20th century, to let in the more abstract and general concept of the conservation of mechanical energy (Dupont, 2014, p. 150), they had been the central object of Statics ever before, and triumphantly after the 17th century once all the geometric demonstrations concerning them had been completed.

Some general properties of the *machines simples* are that they involve idealised (non-deformable) solids, friction is deliberately ignored wherever there are contacts between the solid bodies, any compound machine can be seen as a combination of *machines simples*, the way each one of them modifies a force can be reduced to the law of the lever. Thanks to the experimental property of solids that is their centre of gravity (introduced by Archimedes in the 3rd century B.C.), the action of all weights may be interpreted from those particular points for any geometric processing.

3.2 The inclined plane

The earliest attempt to explain the law of the inclined plane dates back to the 4th century A. D., in a version consisting in stabilising a ball on a slope with a force inclined as the slope is inclined. The corresponding interpretative theory was wrong because the reasoning it was built on didn't take the effect of friction at a particular stage of the analysis (the extreme case of the horizontal plane). With all the attention he had for friction's trickiness in any mechanical experience, Galileo (1564-1642), as an heir of Cardano (1501-1576) and more anonymous scholars of the late Middle Ages, found the correct principle in the 1630's and made it widely known.

3.3 The parallelogram of forces

The law of composition of forces was once designated in a geometrical manner as the parallelogram of forces. Roberval (1602-1675), helped by the works Stevin (ca. 1549-1620) had left, found how to demonstrate it in 1634. In 1640, Descartes (1596-1650) not only ignored the demonstration but even the result, as he believed that the algebraic sum of the two components of a weight is equal to the weight itself. The parallelogram of forces was popularised in France by Varignon (1654-1722) after 1687.

3.4 Friction and cohesion

As Galileo, those who discovered the mechanical laws governing the *machines simples* understood the critical importance of identifying the obscuring friction's effect on these laws in any experimentation and then of specifying the conditions under which friction could be disregarded in the formulation of these laws. Doing so, they may have influenced research in mechanics around 1700 with the idea that friction was a flaw and frictionless systems the condition of any new discovery.

Furthermore, the central law of dry friction, its proportionality to the normal load and not to the apparent contact surface, was rediscovered in 1699 (after da Vinci's unknown works) by Amontons (1663-1705), and checked and correctly interpreted at the end of the same year by La Hire (1640-1718), both fellows of the *Académie Royale des Sciences* of Paris. Amontons' experiment was carried out in the context of glass polishing (see Hutchings, 2021). Thus, the law was associated to kinetic friction for the purpose of manufacturing. It seems that no one before Coulomb dared establish a link between the two scientists' results and soils' internal behaviour.

As far as "cohesion" is concerned (or related words such as "adhesion" or "ténacité" in French), it seems that before 1700 the property was discussed as an intermolecular property

offering tensile strength rather than associated to granular media and proportional to a shear failure surface.

3.5 Hydrostatics

Particular soils, as sands in certain conditions, are fluids. They have some mechanical behaviours in common with liquids. So, regarding hydrostatics, after Archimedes' law on buoyancy it seems that nobody clearly stated the principles of pressure within a liquid before Stevin and later Pascal (1623-1662). The Antique law, depending on differential pressures around a body, was expressed in terms of force (thrust). All the subtleties in the definition of a pressure as a common phenomenon to liquids and fluids were hardly understood in 1700.

4 WHEN IT ALL BEGAN

4.1 Vauban, 1687

Sébastien Le Prestre de Vauban (1633-1707) is the most famous French military engineer ever, who was after 1678 in charge of the maintenance and the construction of all the kingdom's fortifications, and in particular their "*revêtements*" the name given to the ramparts' retaining walls, in particular when retaining "freshly turned earth" – that is, cohesionless soils. Having found a compromise between stability and economy, he established the "*Profil général des revêtements*", a kind of standard ahead of its time. It is indeed known to be the earliest rule for the design of retaining walls (Feld, 1948, p. 2). There have been doubts about the year the *Profil* was disseminated within the Army, but relying on Mayniel's (1808, p. 66) detailed mention about it, the *terminus ad quem* for Vauban's Rule is 1687. Thirty years after, the document was partially published by Gautier; in 1729, it was Bédidor's turn to do so, more extensively. There are today several versions of the document, but with only minor differences between them.

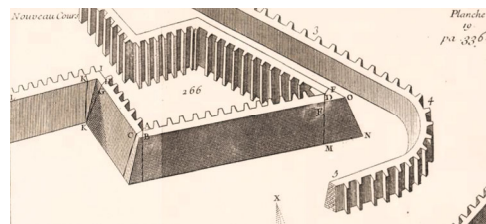


Figure 1. Vauban's type gravity walls, with 1/5 battered faces and trapezoidal buttresses. Bédidor, 1725, p. 336.

Except for Bullet, the first authors of earth pressure theories systematically mentioned, or even based their essay on a comparison of Vauban's *Profil général* with their own results, showing how influential it was on their theoretical works. Vauban hadn't given any detail about how he came to his conclusions, no physical nor mathematical justification was known, at a time when theoretical mechanics was beginning to gain ground in solving technical problems. What made matters difficult for the theorists referring to Vauban's archetypal retaining walls was that these had a relatively complex geometry, with 1/5 battered faces and non-rectangular buttresses (Figure 1). Their thicknesses, for heights defined from 10 to 80 ft (1 ft = 0.324m), were proportionally increasing at the bottom of the wall but fixed at the top.

4.2 Bullet, 1691

Pierre Bullet (1639-1716) was a royal civil architect. He published his *Architecture Pratique* in 1691, a book with a descriptive and quantitative approach to the profession (*le toisé*). However, it included – unique as such in the whole treatise – a chapter using mechanical concepts. It was entitled "*De la Construction des murs de Rempart & de Terrasse*" ("About the construction of bulwark and terrace walls"),

meaning that he was both interested in military and civilian applications of his new demonstrated principle, while he most probably hadn't heard about Vauban's *Profil* – supposedly kept secret at that time. Bullet's chapter was about twenty pages long, but the core of the earth pressure analysis and gravity wall design covered just about four pages (170-173).

4.3 Gautier, 1717

Henri Gautier (1660-1737) was an architect and a civil engineer. He had been one of the first *Inspecteur général des grands chemins, Ponts & Chaussées* of the French kingdom, at the time he brought out, in 1717, a *Dissertation* including an original and disconcerting theory on earth pressure. The chapter on the retaining walls comprised fifteen pages, not including the illustration plates. The first six pages were (explicitly) a partial but literal copy of Vauban's *Profil général* along with the Marshal's comments about it, and, after that, of Bullet's theory and new rule. In his own conclusions, he showed a great sense of diplomacy in commenting on the work of his predecessors, although he did not hesitate to point out a number of discrepancies between their results and his own.

4.4 Buchotte, 1718, 1726 (commonly said to be 1716)

The less well known of all, Nicolas Buchotte (1673-1757) was an infantry lieutenant and a self-educated engineer. He had become an *Ingénieur ordinaire du Roi* (Blanchard, 1981, p. 113) at the time he wrote the *Examen de la Règle* [du] *Sr Gautier* (1718) and later an *Examen du Profil général qu'on attribue à Mr. Le Mal. de Vauban* (1726, according to our analysis of the text). Buchotte was never printed but the Army carefully kept his writings. The 1718 document (three pages) is a critique of Gautier's 1717 theory and a pledge in favour of Vauban's *Profil général*. Ten years later, in an eight pages document, Buchotte unreservedly described what he considers to be the faults of the *Profil général* and, after having introduced a personal theory, built up a new *Profil* to replace Vauban's "so called rule". Buchotte never mentioned Bullet's theory.

4.5 Couplet, 1726, 1727, 1728

Pierre Couplet (1670–1744) was one of the leading scientists of his time, specialised in mechanics, and had become a full member of the Paris *Académie Royale des Sciences* in 1696. From 1726 to 1728, he read out three years running to his fellows of the Academy essays on the problem of retaining walls' stability. When published, the essays totalised 131 pages of text. He started all his work with a strong criticism of Bullet's assumptions and model. He mentioned twice the name of Gautier but did not give any comment on his theory and seems to have never heard of Buchotte's contribution. Couplet's 1728 *mémoire*, adding the case of buttressed backs to his design models, didn't amend his 1726-1727 earth pressure theories.

4.6 Bélidor, 1729

Bernard Forest de Bélidor (ca. 1698-1761), was appointed professor of mathematics in one of the five *Ecole Royale d'Artillerie* in 1720. Bélidor devoted to both the civil and military retaining walls' problem the first chapter of his once upon a time famous textbook, *La Science des Ingénieurs* (1729), an 80 pages long chapter. In this part of the book, he published almost in extenso Vauban's *Profil général* with its original own comments and frequently cited the late military engineer who he admired, although he had to question his Rule in certain respects. Apart from Vauban, Bélidor only explicitly cited Bullet, to distance himself from him, but had almost certainly read Gautier and Buchotte's works also. On the other hand, we believe that Couplet and Bélidor had worked completely independently from one another.

5 BRIEF DESCRIPTION OF THEORIES AND MODELS

5.1 *Bullet, 1691*

Bullet observes that sand is the more fluid of all soils so the most compromising. Theoretically, the grains should form piles with an angle of repose of 60° (Figure 2, on the left, limited to a 2D analysis) but 45° is observed when a vertical obstacle is suddenly removed. Therefore, the wedge of sand above the stable 45° slope is responsible of the thrust on walls. As Feld (1948) summarised it, Bullet then concluded with the inclined plane theory (Figure 2, centre): the weight of the wall must be to a weight of this wedge (CAB) as the length of the plane of rupture (CB) is to the height of the wall (AB). But this was using the traditional version of this theory, giving a coefficient equal to the sine of the angle of repose, here with a wall's upward reaction force inclined according to the angle of repose. This implicitly came to assign friction between the wall and the wedge, knowing that Bullet never mentioned any friction in the rest of his model.

To conclude, Bullet gave the same density to both the wall and the wedge and then considered that any wall shape with the correct weight would do (Figure 2, on the right).

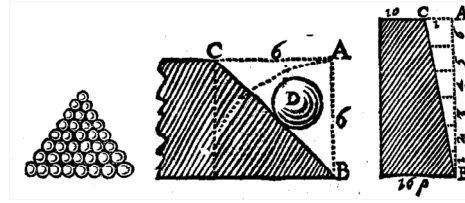


Figure 2. The three original figures in Bullet (1691).

As his theory - the first ever - was purely scalar, with no explicit analysis of the direction of forces, it can be summarised as shown in Figure 3. The angle of repose α controls the thrust efficiency of the wedge's weight through $\cos(\alpha)$ (see Table 1); the resulting $\cos(\alpha)$ could be interpreted today as a form of "K₀" insofar as the model wouldn't be fundamentally different if it was to represent earth pressure at rest – the wall being replaced by a symmetrical earth wedge with the same result.

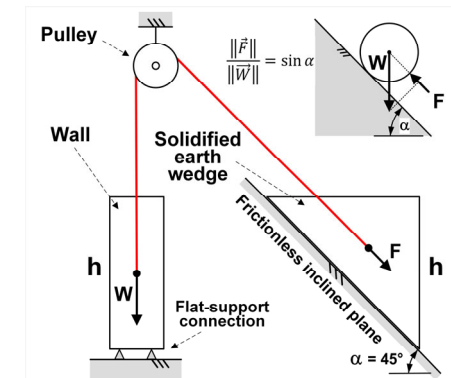


Figure 3. Our interpretation of Bullet's model with *machines simples*.

Bullet did not mention either against which type of movement the equilibrium had to be kept. It may correspond to the sliding of the wall on its base, the check of which only needs a balance of horizontal forces. But what is remarkable with Bullet is that, right from the beginning, he showed that when a wall is as high as the earth it has to retain, its thickness, or average thickness, has to be proportional to its height. Furthermore, in his model and as in all those after him, both the wedge of earth and the wall are *idealised solids in contact with each other*. But his simple scalar form, with a sort of disconnection between the wall and the wedge, avoided a sensitive theoretical problem Couplet and Bélidor were later going to be confronted to.

5.2 Gautier, 1717

Gautier started off his analysis considering a mound of ordinary earth, as one type of soil among several, and trying to go beyond Bullet's comment on the steady angle of repose. He thus noticed that, in the mound, EISF is stable thanks to EFG (Figure 4 bottom left); therefore, in a wedge and triangular wall system subdivided so as to show internal equilibria identical to those within the mound, GFBA (Figure 4, bottom right) is stable thanks to GAH and all the more thanks to GAHK, IMFG thanks to IGKL and so on. So... the wall is stable.

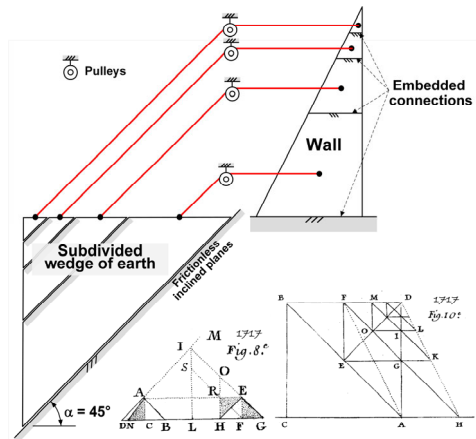


Figure 4. Two figures in Gautier (1717) and our interpretation.

This can be illustrated with *machines simples* (among several equivalent possibilities) as shown in the rest of Figure 4. Thus, the triangular wall's base thickness would be half of the height of the wall when the soil had an angle of repose of 45°; a battered wall would be as stable if its weight were the same as the triangular one.

This way, beyond the particular example Gautier gave in his essay, his theory could be used to prove that *any choice* of wall profile and size is stable, what Buchotte pointed out as early as 1718. The reason is a biased interpretation of the stability conditions of the mound of sand, with no consideration of the forces' direction at the base of the mound, where a horizontal component of shear resistance is necessary for the overall stability. Not considering it, Gautier unintentionally attributed an embedded connection between the mound and the ground, and such a connection was then reinserted within the model wherever he had placed interfaces.

5.3 Buchotte, 1726

Buchotte's main principle can be summarised as: "Oppose the volume of earth with a volume of masonry equal in weight". Implicitly, the volume of earth is assessed with a Bullet-type wedge with the same 45° angle (Figure 5) and a wall-to-earth relative density of 3 to 2 explicitly inspired by Gautier.

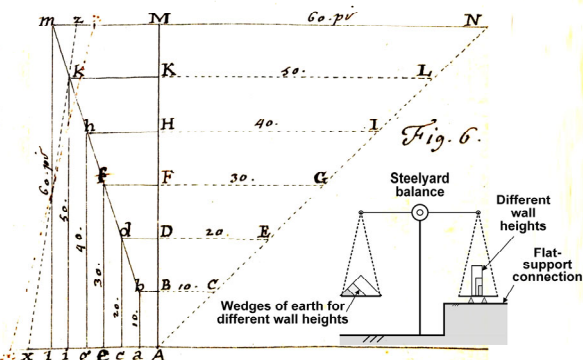


Figure 5. Original figure in Buchotte (1726) and our interpretation.

Buchotte, as Bullet and Gautier, doesn't have to look for the wedges' centres of gravity. The weight as a scalar quantity plays almost separately on both sides of the wall/earth interface. It then is a matter of relative density between the two media. There is no attempt to define a real interaction between the wall and the earth behind it. Figure 5 (bottom right) shows a way to express what Buchotte's oversimplified model came to.

5.4 Couplet, 1726 and 1727

Right from the beginning in 1726, Couplet's models were not made of a solid wedge of earth *sliding* on a frictionless inclined plane, in contrast with Bullet's basic assumption, but a solid wedge of earth directly *pushing* on the wall (Figure 6), with a direct soil-structure interaction, what was new.

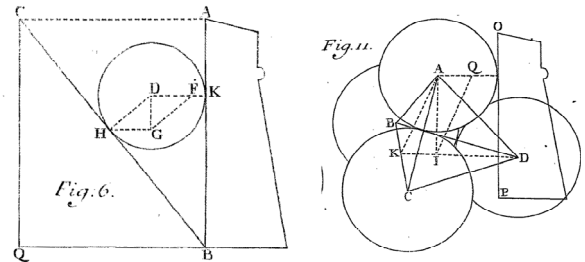


Figure 6. Two original figures in Couplet (1726).

Like Bullet, Couplet primarily considered the case of sand, however with a 3D basic pattern made of a stack of four spheres forming a tetrahedron (Figure 6, right). He defended the idea that a mound of sand should be like a pile of cannonballs, with slopes as steep as 55 to 70°. He introduced as his main analytical tool the parallelogram of forces. Then, he assessed some stabilising horizontal force in function of the upper sphere's weight. The horizontal force was obtained as the retaining wall's local vertical element reaction on a tetrahedron's upper sphere lacking the support of one or two lower spheres.

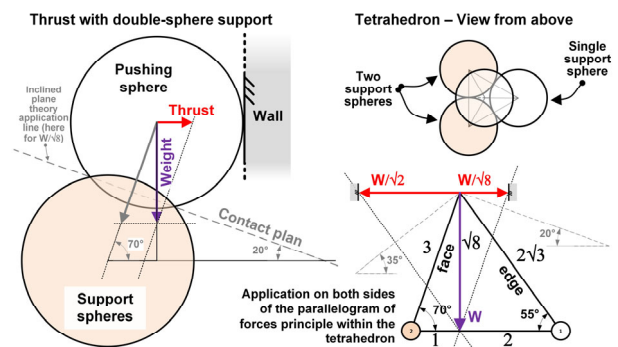


Figure 7. Transcription of some of Couplet's (1726) model details.

That way he found constant coefficients of $1/\sqrt{2}$ and $1/\sqrt{8}$ (Figure 7) that constitute part of Couplet's series of "K₀". He then combined these primary coefficients to the effect due to the choice of the wedge surface/weight, controlled by the very steep aforementioned slopes.

Another key aspect of Couplet's 1726 theory is his appreciation of the wall's failure mechanism, sound and explicit: a toppling around the wall's face toe. It entailed significant consequences, the first of which being the need to locate the solid wedge's resulting thrust application point in order to obtain the force's lever arm. Couplet, without any comment on this assumption, decided to position the resultant two thirds up the height of the wall. He then found the expression $h^3/12$ (h: height of wall) to express the moment of the wedge on the wall, leaving the relative wall/earth density ratio in the form "p/q" (two arbitrary scalars).

The rest of his 1726 *mémoire* introduces a series of ten typical shapes of walls, with for each a maximum reactive moment to be calculated and equated to the $h^3/12$ expression. Couplet has probably no equivalent in the history of geotechnics to have reached such refinements in trying to define in a general manner closed-form expressions for the design of all sorts of gravity wall geometries.

In his 1727 *mémoire*, Couplet reworked out from the beginning his entire theory to, among other important subjects, incorporate the case of friction between the wall and the wedge - with anticipated reduced levers -, and to add to the tetrahedron modified dual coefficients a third one derived from a square pyramid, the author leaving it up to his readers the choice of the right coefficient. He finally “accepted” to reduce the generality of his work by producing three tables, one with each resulting “ K_0 ” and $3/2$ as the relative wall/earth ratio of densities, certainly after Gautier.

5.5 Béliidor, 1729

The last author of the *pre-classical* soil mechanics’ initial period is Béliidor. For full details on his retaining walls design model, see e.g. Vernhes and Barakat (2016).

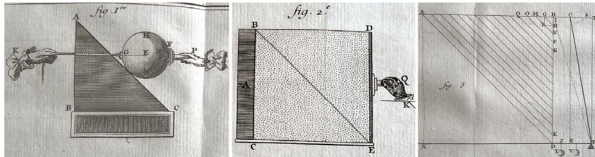


Figure 8. Three original figures in Béliidor (1729).

Béliidor borrows from Bullet some of the fundamental characteristics of his model: the inclined plane *machine simple* with a 45° angle of repose (Figure 8) - but in its horizontal $\tan(\alpha)$ version -, and the solid wedge. He assumed after Gautier a $3/2$ wall-to-earth density ratio. As Couplet and independently from him, his mechanical analysis of the interacting weights and forces leads him to consider very carefully what we would call today all the attributes of a vector. Béliidor’s key *machine simple* is the angled lever (Figure 9, bottom left) because he explicitly defined the loss of equilibrium of the wall as a rotational failure, more particularly through toppling for, according to his experience, the wall foundations were to be cleared of blame.

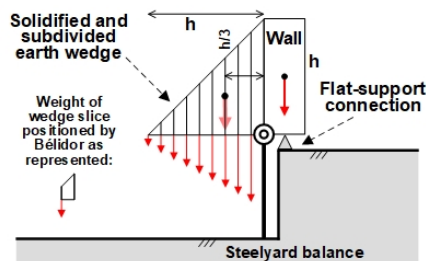


Figure 9. Original figure in Béliidor (1729) and our interpretation.

Where Béliidor proved to be unique was in his idea to subdivide the wedge of earth into one-foot slices (Figure 8 and Figure 8, right) having the same direction as the slope underneath the wedge. In doing so, he was allowing his model to generate a soil triangular pressure on the wall. Another pioneering aspect of his theory was that he had to find a way to reduce the earth pressure as his model was mathematically equivalent to a liquid-against-wall case. This is where he considered legitimate to introduce an explicit and arbitrary weight reduction coefficient, of 0,5, to take into account the earth’s “*ténacité*”, that is a function of depth and would therefore be translated by “friction”. The coefficient can be interpreted as the earliest explicit “ K_0 ” approach to handle the problem.

The wedge subdivision also reduced the effect of uncertainty in positioning the application point of the wedges’ resulting horizontal force. Béliidor consciously placed them at the top of each slice to be on the safe side, but after calculating the sum of the slices’ individual levers, his model “automatically” fixed close to one third of the wall height the global resulting lever arm (Figure 9). The professor seems never to have realised such an important result, contrary to what later commentators wrote. Béliidor, as Couplet but not as widely as him, worked out a wide range of different types of walls. His method had a strong echo in both civilian and military circles during the 18th century, and beyond the French borders.

6 A COMPARATIVE LOOK AT THE MODELS

The Figure 10 chart shows for walls from 10 to 80 ft high the slenderness ratios found by all the authors studied above and published in tables. This means that the shape of the recommended wall is specific to each author. It shows how the variety of different theories, added to technical criteria, led to a certain degree of dispersion in their operational results. But this dispersion has to be seen as independent of what could be expected in soil mechanics, a dependency to actual soils! In the *pre-classical* period, the variety of soils was spontaneously reduced to only one or almost one in any application of a theory, as the worse or the typical case, depending on the author.

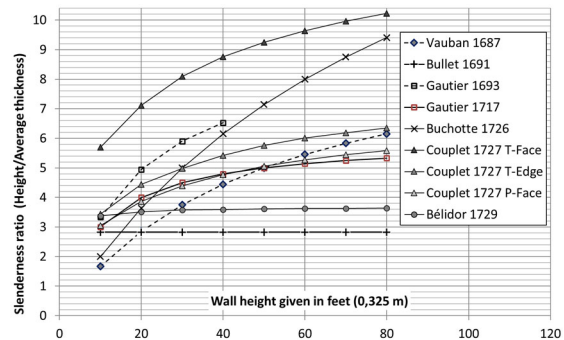


Figure 10. Walls’ slenderness ratios obtained from Vauban to Béliidor.

Fundamental differences between theoretical approaches, like the priority given to a simple balance of weights between the earth wedge and the wall or the more sophisticated equilibrium of moments around the wall’s exterior toe, make less significant their comparison through their results in terms of design. However, the first four theorists after Bullet systematically referred to Vauban’s Rule and this fact probably had a key influence on how they defined the numerical values controlling the quantitative results of their design model. Béliidor even wrote rather explicitly that he had tried to adjust his parameters to match the average values of the *Profil général*, while revealing its internal illogical trend of small, oversized walls and conversely undersized high walls (above 30 ft). It could be added: as long as earth pressure was to be considered as Vauban’s sole design criterion, what the Marshal never wrote.

Table 1. Summary of the authors theoretical and practical results.

Author	Equivalent “ K_0 ” (weight reduct. coef.)	Vertical walls average slenderness ratio
Vauban $h = 10/80$ ft	-	1,5 / 4,1
Bullet 1691	$\sim 5/7$ ($\cos 45^\circ = 1/\sqrt{2}$)	$\sim 36/13$ ($2\sqrt{2} \approx 2,8$)
Gautier 1717	1/2 (arbitrary)	4 (in fact: any)
Buchotte 1726	1	3
Couplet 1726	1/4	3
Couplet 1727	1/4 ; $1/2\sqrt{3}$; $\sqrt{3}/4$	$3+\sqrt{2}$; $2\sqrt{2}$; $\sqrt{3}+\sqrt{2}/2$
Béliidor 1729	1/2	3,6

In Table 1, the average slenderness ratio of Vauban's walls for the two extreme heights have been computed in the case of 15 ft distant buttresses. Their effect was included in the assessment. For the other authors, when the wall-to-earth bulk density ratio wasn't imposed by the text, the corresponding value was set to 3/2 in order to make the numerical results more comparable. On the contrary, more hidden aspects of the theories, as what can be considered as their "K₀", are much less sensitive to the authors uneven bouquets of hypotheses.

7 CONCLUSIONS

Geotechnical design started in the forty years extending from 1691 to 1729, corresponding to the beginning of Skempton's pre-classical period for soil mechanics. The five authors of that bout of time broke new ground and did so trying to explain how earth pushes on a wall without the concept of internal shear resistance. Beyond these difficulties stood the aporia of the transmission of forces between two idealised solids with a non-punctual contact, still discussed in the 20th century regarding Coulomb's wedge of earth. There, Bédidor showed how ingenious he was to slice the triangular wedge in order to clear off the theoretical problem, as Fellenius and Bishop's later did with a rounded solid wedge.

Even with severely flawed models, part of the first authors results could be considered as definite with regard to the wall toppling reaction as a function of its geometry, even complex. This helped the following generations to concentrate more on earth pressure itself rather than on the entire problem of gravity wall design - what is obvious in Coulomb's short 1773 essay.

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