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Reliability analysis of long slopes with spatially variable random properties

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ABSTRACT: In the assessment of landslide mechanics and stability, the infinite slope equations serve as a useful tool for initial estimates of the factor of safety. Similarly, the finite element method can provide insights into the failure mechanisms of long, shallow slopes. Both methods are amenable to probabilistic analyses by overlaying spatially variable random field structures that allow material properties to vary with location. While the infinite slope method solely considers one-dimensional variability in the vertical direction, finite element simulation of long slopes can incorporate spatial variability in multiple directions. This paper presents the behaviour of long slopes with anisotropic undrained random fields. The random field correlation lengths of long slopes required to produce failure mechanisms and safety factors resembling the infinite slope case are a particular focus, in addition to correlation lengths producing distinctly different failure mechanisms that fail neither through the toe nor the crest of the slope. The influence of slope angle and shear strength distributional parameters are further examined through parametric studies.

KEYWORDS: long slope, random finite element method.

1 INTRODUCTION

The behaviour of long slopes remains of continued importance for understanding shallow, translational landslide hazards (Griffiths et al., 2008, Ulmer et al., 2009, Zolfaghari and Heath, 2008). As a commonly used analytical equation for assessing stability, the infinite slope equation provides estimates of the factor of safety (FS) along a failure surface parallel to the ground surface. This framework has been adapted for a wide range of applications, from evaluating the stability of vegetated hillslopes (Cecconi et al., 2015, Chirico et al., 2013, Wu and Sidle, 1995) to assessing seismic slope performance (Dreyfus et al., 2013, Jibson et al., 2000). Comparisons between Finite Element (FE) simulations and the infinite slope (Griffiths et al., 2011a) showed strong agreement when the slope length-to-depth ratio exceeded approximately 16:1. The infinite slope model tends to produce conservative factors of safety for shorter slopes.

Although the closed form equation for the infinite slope provides a simple tool for landslide susceptibility, several important restrictions exist, as the material and geometric properties remain constant parallel to the slope. This also limits the spatial variability of soil properties to one-dimensional profiles. In reality, soil properties can vary in multiple dimensions. Griffiths et al. (2011b) introduced one-dimensional random fields into the model to account for spatial variability, finding that failures could initiate above the base of the soil column. This approach is effectively "infinitely anisotropic" due to an assumed infinite correlation length downslope. Zhu et al. (2021) incorporating linearly increasing, spatially variable undrained shear strengths, noting that critical slip surfaces typically occur at the slope base, though increased variability (higher coefficients of variation) produces failure forming at multiple depths.

Similar to the infinite slope model, FE approaches can incorporate spatially variable shear strengths but are not constrained to variation in a single direction. In the present study, the effects of random field correlation lengths in both downslope and cross-slope directions are evaluated, using the infinite slope as a reference case. A Monte Carlo framework employing the Random Finite Element Method (RFEM) is used to explore how lateral variability influences the extent of slope failure and the corresponding FS distribution.

2 PROBABILISTIC INFINITE SLOPE ANALYSIS

In its classical form, the infinite slope model considers a soil slice aligned parallel to the ground surface (Figure 1). The factor of safety (FS) for such a slice can be expressed analytically as

$$FS = \frac{c' + (H\gamma\cos^2\beta - u)\tan\phi'}{H\gamma\sin\beta\cos\beta} \qquad (1)$$

where H is the depth of the soil layer to the potential failure surface; β is the slope inclination; γ is the total unit weight of the soil; u is the pore pressure at the base of the slice; ϕ' and c' are the effective soil friction angle and cohesion, respectively, at the base of the slice.

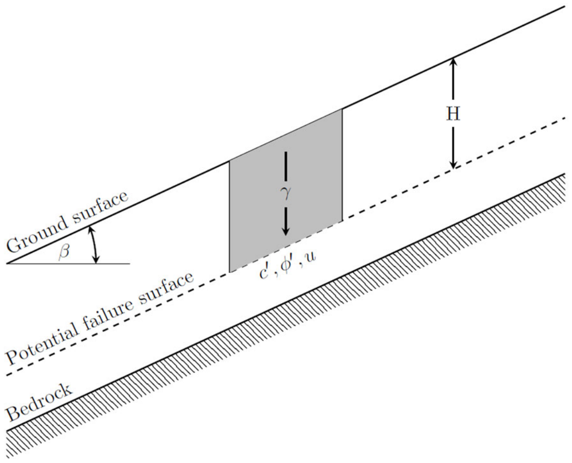

Figure 1. Typical infinite slope layout.

When a probabilistic approach is adopted for infinite slope analysis, random fields are generated to represent spatial variability in shear strength parameters (cohesion c and friction angle ϕ) in the vertical direction. These properties are typically assigned both a statistical distribution and a spatial correlation length (θ).

The spatial correlation length provides a measure of how rapidly soil properties vary with distance: larger values indicate smoother, more gradually changing profiles, while smaller values correspond to rapid fluctuations. In this study, a Markov exponential decay model is used for the correlation function, due to its simplicity and common usage (Fenton, 1999)

$$\rho = e^{-2\tau/\theta} \tag{2}$$

where ρ is the correlation coefficient; and τ is the vertical distance between points. A non-dimensional spatial correlation length Θ based on the depth of the failure surface is defined as by

$$\Theta = \frac{\theta}{H} \tag{3}$$

In the case of a probabilistic undrained clay slope, the infinite slope equation given in Eq. (1) reduces to the form

$$FS = \frac{s_u}{\gamma z \, \sin\beta \, \cos\beta} \tag{4}$$

where s_u denotes the undrained shear strength, and H is replaced with the depth component z.

3 PROBABILISTIC UNDRAINED CLAY SLOPE ANALYSIS

Table 1 outlines the FE model inputs for an undrained clay slope. In this study, the undrained shear strength is treated as a spatially random variable and modelled with a lognormal distribution, reflecting the need for all values to remain strictly greater than zero.

An indicative geometry and boundary conditions are given in Figure 2, simulated with a modified version of the freely available software package slope64 at the first author's website (Smith and Griffiths 2003). In the case presented *D* denotes the distance from the surface of the slope to the base of base, at right angles to β. The mesh base is constrained in all directions, whereas the far-left and far-right edges are restricted to vertical displacement only. For all simulations in this study, a discretisation of 400 elements in length by 20 elements in depth was considered sufficient to model long-slope behaviour.

As the infinite slope approach assumes a planar failure surface parallel to the ground, long slopes consistent with this mechanism are expected to develop shear bands spanning the full slope length. This condition can be expressed by normalising the failure surface length L_f by the total slope length (i.e. $L_f/L = 1$).

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Figure 4 provides the relationship between mean factor of safety and cross-slope correlation length (in this case, when the non-dimensional vertical correlation is equal to unity: $\Theta_y = 1$). For each angle considered, a common trend in the factor of safety with respect to the level of cross-slope correlation is observed, with higher safety factors observed for highly fluctuating correlation lengths where significant spatial averaging occurs. Further increase to Θ_x results in a reduction in *FS,* followed by a slight increase for large, smoothly varying correlation lengths which can be considered as akin to highly stratified soils.

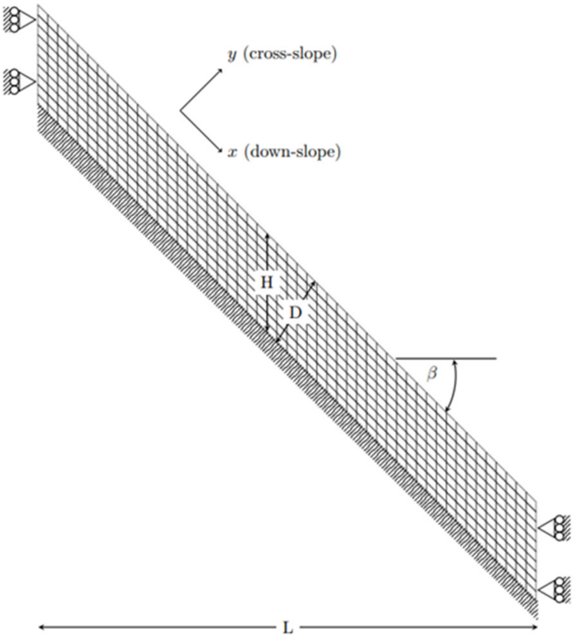

Figure 2. Example of a FE long slope geometry.

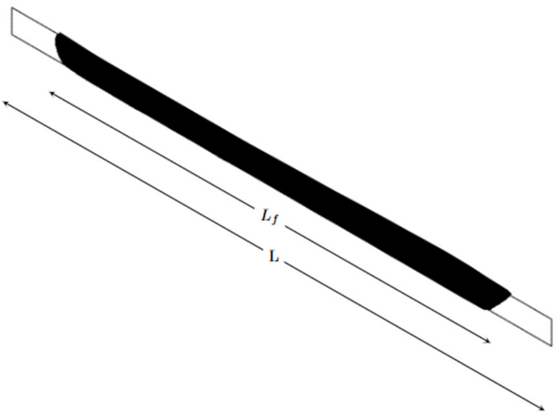

Figure 3. Example of a long slope at failure.

Table 1. FE model input parameters.

μ_{s_u}	25 kN/m^2
σ_{s_u}	2.5 kN/m^2
Distribution	lognormal
γ	20 kN/m^3
H	2.5 m
β	25 – 45°
E	50,000 kN/m^2
ν	0.3
Θ_x	1 – 200
Θ_y	1

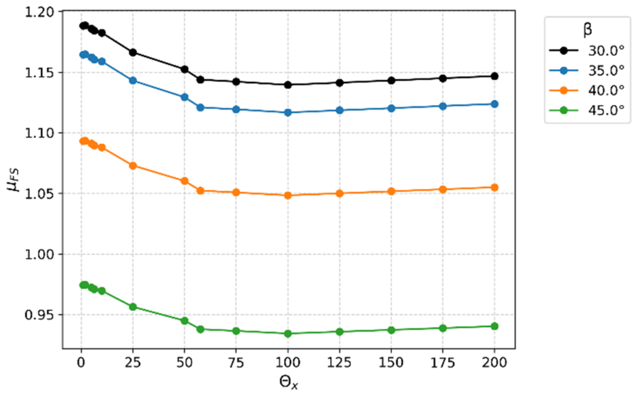

Figure 4. Relationship between factor of safety (mean) and increasing down-the-slope anisotropy of s_u for various slope angles.

While a reduction in mean safety factor is evident based on slope angle, minimal change to the variation in FS is observed, for steeper slopes up to a 1:1 gradient (Figure 6). Nevertheless, some deviation is observed for the shallowest slope considered. It is evident that small cross-slope correlation lengths provide minimal influence over safety factor distributions, regardless of the angle. As the cross-slope correlation increases, so does the variability in the safety factor. For each of the given angles, a "worst-case" is observed when μ_{FS} reaches a minimum when σ_{FS} achieves a maximum, signifying the highest levels of fluctuation in the factor of safety distribution, with the lowest mean. In each of the given values of β, this occurs when Θ_x is approximately 100 times the depth of the slope, corresponding to correlation lengths of 40 metres in this particular case (for a 2.5 metre slope).

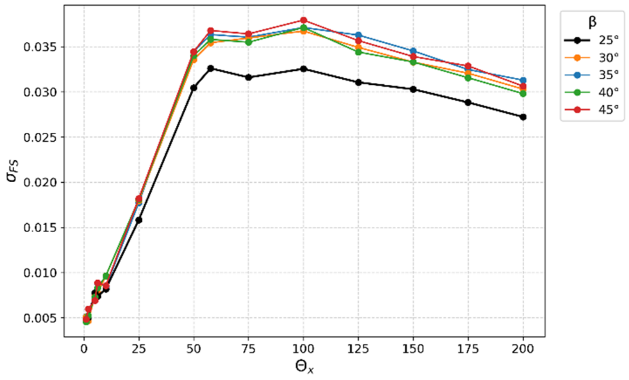

Figure 5. Relationship between factor of safety (standard deviation) and increasing down-the-slope anisotropy of s_u for various slope angles.

Although the primary objective of slope stability analysis is often to determine the factor of safety a given slope, FE long slope simulations provide the additional benefit of determining complex failure surfaces compared to the infinite slope method. Figure 6 provides several long slope failure mechanisms in the presence of spatially variable undrained shear strengths. It is evident that unlike the infinite slope, localised failure is possible, and in some cases, compound failure mechanisms.

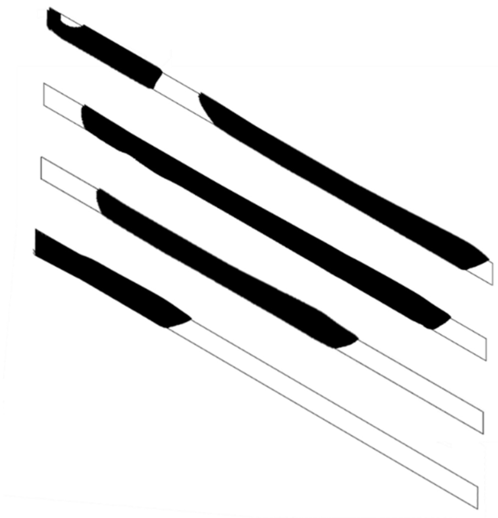

Figure 6. Influence of random field profiles: failure surfaces for various RFEM realisations.

Figure 7 presents the mean non-dimensional failure length with respect to various cross-slope correlation lengths. When Θ_x is small, slope failure approximates the infinite slope, with most of the length of the slope at failure. Thereafter, a significant drop is observed with an increasing correlation length. While minimal differences are noted for various slope angles, the 1:1 produces slightly longer failure mechanisms. This is reinforced in the Figure 8 heatmap, indicating that while highly fluctuating values of Θ_x can impact the shape of shallow, translational failure, the length of failure is robust to changes in slope angle and spatially variable undrained shear strengths.

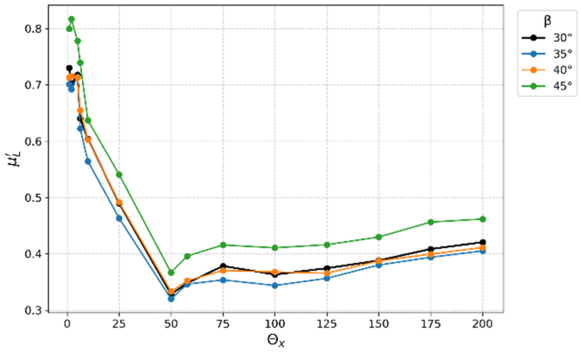

Figure 7. Relationship between normalised slope failure length (mean) and increasing down-the-slope anisotropy of s_u for various slope angles.

As with the distribution of safety factors, when considering the variation of slope failure lengths (Figure 9), a change in behaviour is observed for non-dimensionalised correlation lengths at $\Theta_x = 50$. This signifies a set of cases where localised failure mechanisms are prominent but with widely varying failure lengths. As cross-slope correlation lengths increase, so does the level of variation in the length of failure (Figure 10).

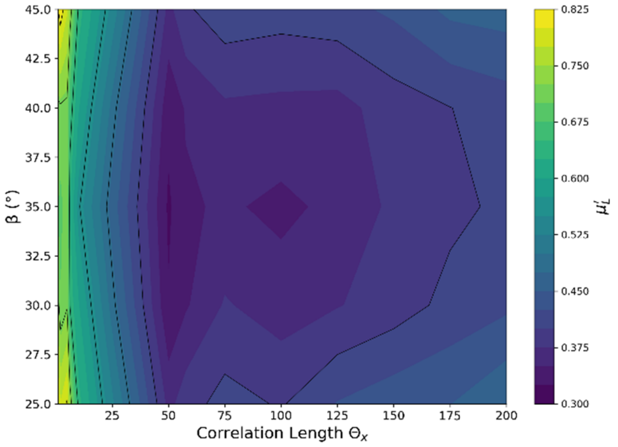

Figure 8. Heatmap of normalised slope failure length (mean) and increasing down-the-slope anisotropy of s_u.

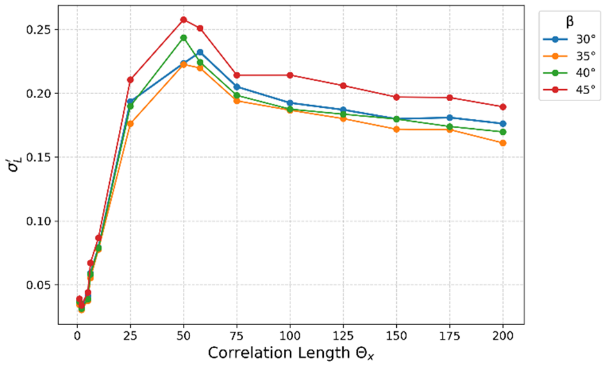

Figure 9. Relationship between normalised slope failure length (standard deviation) and increasing down-the-slope anisotropy of s_u for various slope angles.

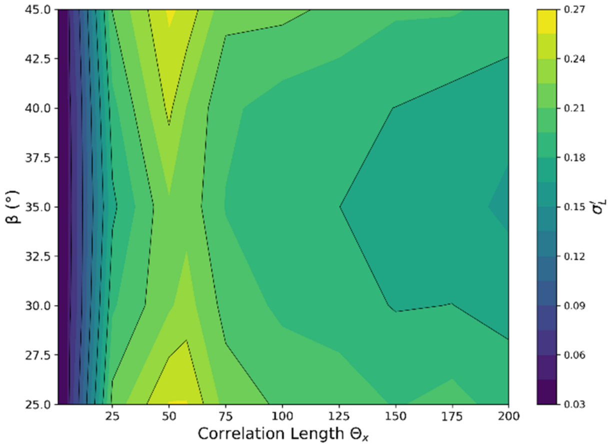

Figure 10. Relationship between normalised slope failure length (standard deviation) and increasing down-the-slope anisotropy of s_u for various slope angles.

4 CONCLUSIONS

This study has examined undrained long slopes using finite-element (FE) analysis with varying degrees of material anisotropy and slope angle, comparing results with the classical infinite-slope solution. While the infinite-slope method presumes a planar failure surface that spans the full slope length, this assumption breaks down once spatial variability is permitted in multiple directions when considering RFEM simulation. For sufficiently large downslope correlation lengths, failure becomes strongly localised. Because of localisation, the infinite slope can generally be considered as conservative in failure length compared to spatially variable RFEM models. In most cases, the distribution of safety factors and failure lengths are not significantly impacted by slope angle.

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