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Numerical Analysis of Stability of Embankments over Aggregate Columns-Improved Soft Ground under Linked, Undrained and Drained Conditions

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ABSTRACT: Aggregate columns have been successfully used to increase bearing capacity, reduce settlement, accelerate consolidation, and enhance stability of embankments over soft ground. The equivalent strength method converts the column-improved ground into an improved mass with a weighted average strength and has been conveniently used in practice. To assess the accuracy of this method, two-dimensional (2-D) finite difference-based numerical software was used in this study to compare the numerical results of this method with the column-wall method, in which column walls were placed at certain spacing based on an area replacement ratio of columns. The numerical models with embankments over column walls-improved soft ground were first verified against the numerical results from past studies under undrained and drained conditions, which were analyzed independently and could not be compared between them. This study linked the strength values of the soft ground under undrained and drained conditions using the Cam-Clay model. In addition, these past studies did not consider the groundwater table effect, which was investigated in this study. The numerical results show that the predicted factor of safety from the equivalent strength method matched with that from the column-wall method under drained conditions. Under undrained conditions, however, the equivalent strength method overestimated the factor of safety by approximately 5% when the groundwater was considered and 10% when the groundwater was not considered. In addition, the numerical results show that the shear bands under undrained conditions were much deeper than those under drained conditions. The factors of safety of the embankments over the improved ground under undrained conditions might be greater than those under drained conditions especially with a groundwater table and a great area replacement ratio. This study recommends that linked, undrained and drained analyses should be conducted for embankments over soft ground improved by aggregate columns to identify the critical condition.

KEYWORDS: Aggregate columns, embankments, factor of safety, soft soils, stability

1 INTRODUCTION

Slope instability including deep-seated failure presents challenges for design and construction of embankments over soft ground. Aggregate columns including stone columns and rammed aggregate peris have been commonly used to improve the soft ground and prevent this problem (Abusharar & Han 2011; Zhang et al. 2014). The limit equilibrium method (LEM), finite element method (FEM), and finite difference method (FDM) (Duncan, 1996; Han & Leshchinsky 2006) have been commonly used in practice to analyze slope stability. When the LEM is used to analyze the stability of embankments over column-improved soft ground, an equivalent strength method is often utilized due to its convenience, which is based on the weighted average of the properties of soils and columns. Abusharar & Han (2011) found numerically that the equivalent strength method overestimated the factor of safety (FS) as compared with the column-wall method under undrained conditions. Zhang et al. (2014) confirmed Abusharar & Han (2011)'s findings under undrained conditions but found that the equivalent strength and column-wall methods calculated similar FS under drained conditions. Abusharar & Han (2011) and Zhang et al. (2014) conducted independent studies without any linkage between the properties of soils under undrained and drained conditions; therefore, their results could not be compared for assessment of a critical condition for embankments over soft ground. In addition, Zhang et al. (2014) did not consider the existence of groundwater table, which is important for design of embankments over soft ground that often has a high groundwater table. Therefore, their analyses should be improved to better analyze the stability of embankments over soft ground improved by aggregate columns.

In practice, embankment construction is often considered as an undrained condition due to the generation of excess pore water pressure and its limited dissipation in soft soils within a short construction period. After the construction, however, the excess pore water pressure dissipates, leading to soil consolidation with time. The end of soil consolidation is often considered as a long-term or drained condition. For embankments on soft ground with normally and slightly over-consolidated soils, it is known that the undrained condition is more critical than the drained condition in terms of their stability (Han, 2015). However, it is not clear whether this knowledge is still true when embankments are constructed over an aggregate column-improved soft ground. To verify this knowledge, critical-state soil mechanics could be used because it provides a way to link the undrained and drained shear strength of a soft soil (Budhu, 2010).

This study adopted 2D finite difference-based numerical software (FLAC2D Version 8.1) to assess the accuracy of the equivalent strength method for slope stability analysis of embankments over aggregate column-improved ground. This study evaluated the FS against embankment slope failure under undrained versus drained conditions. The undrained and drained conditions of the soft ground were linked by the Cam-Clay model but analyzed separately. This study also investigated the effect of the groundwater table on the calculated FS. The above assessments were carried out through a parametric study with different area replacement ratios of aggregate columns in the soft ground. The paper examines the shear strain rates for potential slip surfaces or shear bands and compares the FS against slope failures under different conditions.

2 ANALYSIS METHODS

The numerical software used a shear strength reduction technique to obtain the FS of an earth structure. Dawson et al. (1999) exhibited the use of this technique and verified the calculated FS with the LEM results for simple slopes. This technique uses a series of trial FS to adjust the shear strength of soil. For the Mohr-Coulomb soil model, the cohesion, c and the friction angle, ϕ, of a soil are adjusted as follows:

$$c_{trial} = \frac{1}{FS_{trial}} c \qquad (1)$$

$$\phi_{trial} = arc\tan(\frac{1}{FS_{trial}}\tan\phi) \tag{2}$$

To estimate the FS, iterations are needed by adjusting the shear strength of the soil until the model reaches a limit equilibrium state (i.e., from unstable to stable or stable to unstable). This FS_{trial} is considered as the FS sought for the analysed earth structure.

2.1 *Column-wall method (CW)*

For a plane-strain analysis, individual aggregate columns can be converted to walls by matching the geometry of the columns or the properties of the columns (Zhang et al., 2014; Tan et al., 2008). The method presented in this study assumes the effective width of the column wall to be the same as the diameter of individual columns. The properties of the individual columns and their surround soils are used to determine the properties of the column wall based on the area-weighted average of the properties of the aggregate columns and the soils by the following equations:

$$E_w = E_c a_r + E_s(1 - a_r) \tag{3}$$

$$c_w = c_s(1 - a_r) \tag{4}$$

$$\phi_w = arc\tan(a_r \tan\phi_c + (1 - a_r)\tan\phi_s) \tag{5}$$

$$a_r = \frac{A_c}{A_w} = \frac{\pi d^2}{4ds} \tag{6}$$

where a_r is the area ratio of the individual columns to the column wall; E_c, E_w and E_s are the elastic moduli of the individual columns, column wall, and soft soil; c_w and c_s are the cohesion of the column wall and soft soil; ϕ_c, ϕ_w and ϕ_s are the frictional angles of the individual columns, column wall, and soft soil; A_c and A_w are the areas of the individual columns and column wall; d is the diameter of aggregate columns and effective width of the column-wall; and s is the center to center spacing of the aggregate columns.

Zhang et al. (2014) verified this method for a 2D plane-strain analysis as compared with a 3D analysis. Figure 1 shows the geometry of the CW model in this study. The individual columns were converted into column walls in the CW model. Considering its symmetry, half of the embankment was built in the model. Roller supports were applied to the left and right side of the model to restrict horizontal displacements but allow vertical displacements. The bottom of the sand layer was fixed. In the case considering groundwater, the water table was set at the ground level. The material properties of the model for the following study are provided in the figure.

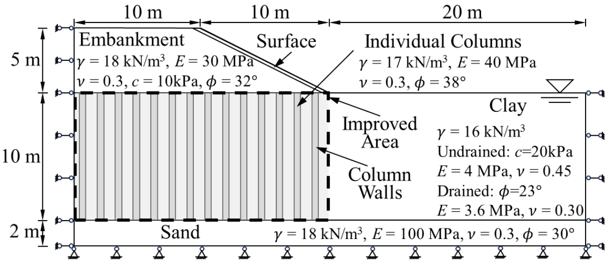

Figure 1. Geometry and material properties for the model.

2.2 *Equivalent strength method (ES)*

The dash-line block in Figure 1 shows the ES approach that considers the improved ground as a homogeneous composite mass with equivalent properties. This approach is commonly used in practice (Cooper & Rose 1999; Abusharar & Han 2011; Zhang et al. 2014). The equivalent properties of the improved area can be calculated using the following equations:

$$E_{eq} = E_c a_s + E_s(1 - a_s) \tag{7}$$

$$c_{eq} = c_s(1 - a_s) \tag{8}$$

$$\phi_{eq} = arc\tan(a_s \tan\phi_c + (1 - a_s)\tan\phi_s) \tag{9}$$

$$a_s = \frac{A_c}{A_s} = \frac{\pi d^2}{4s^2} \tag{10}$$

where a_s is the area replacement ratio of the improved ground; E_{eq}, c_{eq}, and ϕ_{eq} are the equivalent elastic modulus, cohesion, and frictional angle of the improved area; and A_c and A_s are the areas of the column and the soil, respectively.

3 DRAINED AND UNDRAINED SHEAR STRENGTHS OF SOFT SOIL

The critical-state soil mechanics provides a way to link the undrained and drained shear strength of a soft soil (Budhu, 2010). Table 1 lists the parameters of the soil using the Cam-Clay soil model and its undrained and drained shear strength can be derived by the following equations:

$$(S_u)_{cs} = \frac{M}{2} e^{\frac{e_\Gamma - e_o}{\lambda}} \tag{11}$$

$$e_\Gamma = e_o + (\lambda - \kappa)\ln\frac{p'_c}{2} + \kappa \ln p'_o \tag{12}$$

$$p'_o \approx \sigma'_{zo} = (\gamma - \gamma_w)\frac{h}{2} \tag{13}$$

$$p'_c = R_o p'_o \tag{14}$$

$$p'_{cs} = \frac{3p'_o}{3 - M} \tag{15}$$

$$q_{cs} = Mp'_{cs} \tag{16}$$

$$\sigma'_3 = p'_{cs} - \frac{q_{cs}}{3} \tag{17}$$

$$\sigma'_1 = q_{cs} + \sigma'_3 \tag{18}$$

$$\phi'_{cs} = \sin^{-1}\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} \tag{19}$$

where $(S_u)_{cs}$ and ϕ'_{cs} are the undrained and drained shear strength at the critical state; λ, κ, and M are the compression index, the unloading/reloading index, and the critical state friction constant; e_Γ is the void ratio on the critical state line when $p' = 1$; e_o is the initial void ratio; p'_c is the preconsolidation stress; p'_o is the initial stress; γ is the saturated unit weight of the soil; γ_w is the unit weight of water; h is the half depth of the soft clay layer; R_o is the overconsolidation ratio; p'_{cs} and q_{cs} are the mean effective stress and deviatoric stress in the soil; σ'_1 and σ'_3 are the effective axial stress and effective confining stress in the soil.

The derived undrained and drained shear strength of the soft clay were $(S_u)_{cs} = 20$ kPa and $\phi'_{cs} = 23°$ respectively.

Table 1. Properties of the soft clay in Cam-Clay model.

γ (kN/m³)	e_o	λ	κ	M	R_o
16	0.9	0.3	0.06	0.9	3.15

4 VERIFICATION OF NUMERICAL MODELS

The CW method was verified against the results from Zhang et al. (2014) by using the same model geometry and material properties for undrained and drained conditions in Figure 1. No water table was set in the verification model as was done by Zhang et al. (2014).

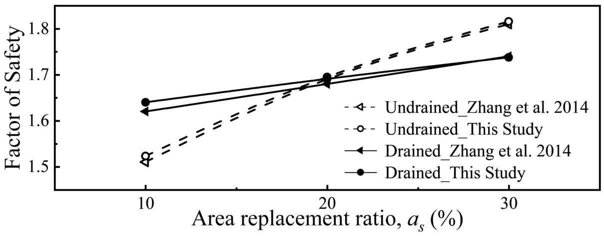

Figure 2. FS comparison of the CW model in this study vs. Zhang et al. (2014)

5 CASES FOR ANALYSIS

To assess the accuracy of the ES method for calculating the FS for embankments on aggregate column-improved soft ground, a parametric study was conducted, which included the soft clay improved by aggregate columns at the area replacement ratios of 10%, 20%, and 30%. For comparison purposes, the case without any improvement (i.e., area replacement ratio = 0%) was also analyzed. Considering the column diameter of 0.5 m in a 3D layout, these area replacement ratios correspond to the center to center spacing of the columns at 1.4, 1.0, and 0.8 m respectively in a square pattern. The material properties used in this study are provided in Figure 1. The shear strength of the clay under undrained and drained conditions were linked by the Cam-Clay model. In the case considering the groundwater, the water table was set at the ground surface.

Table 2 lists the properties for column walls, which were converted based on the weighted average properties of the individual columns and their surrounding soil in between. Due to the difference in the shear strength of the clay under undrained and drained conditions, the properties of the column walls were different.

Table 3 provides the properties of the improved zone as a homogeneous mass for the ES method.

Table 2. Material properties for column walls.

Condition	a_s (%)	γ (kN/m^3)	E (MPa)	ν	c (kPa)	ϕ (°)
Undrained	10	16.3	14.1	0.41	14.4	12.4
	20	16.4	18.3	0.39	12.1	17.2
	30	16.5	21.5	0.38	10.3	20.8
Drained	10	16.3	13.8	0.30	/	27.7
	20	16.4	18.0	0.30	/	29.5
	30	16.5	21.3	0.30	/	30.9

Table 3. Material properties for the improved area.

Condition	a_s (%)	γ (kN/m^3)	E (MPa)	ν	c (kPa)	ϕ (°)
Undrained	10	16.1	7.6	0.44	18.0	4.5
	20	16.2	11.2	0.42	16.0	8.9
	30	16.3	14.8	0.41	14.0	13.2
Drained	10	16.1	7.2	0.30	/	24.7
	20	16.2	10.9	0.30	/	26.4
	30	16.3	14.5	0.30	/	28.0

6 RESULTS AND DISCUSSION

6.1 *Strain rate contours under undrained conditions*

Figure 3 to Figure 7 show the shear strain rate contours for the embankment over the unimproved ground and column-improved ground calculated using the CW and ES models under undrained conditions.

Figure 3 illustrates that, under an undrained condition, the cases with and without groundwater had the same shear band in the unimproved ground, as the existence of groundwater did not affect the soil strength. Figure 4 and Figure 5 show the discontinuities along the circular shear bands in the CW models. With the increase of the area replacement ratio, the spacing of column walls decreased, leading to more significant shear strains happening in the soil between the column walls, like a squeezing action. At the same time, the improved ground tended to become a stronger composite block. The movement of the composite block under the embankment slope pushed the soil in front of the toe to rotate along the slip surface. Figure 4 and Figure 6 show that the models with groundwater had narrower shear bands and lower FS than those without groundwater. These results can be attributed to the reduction of shear strengths of aggregate columns by water pressures, leading to lower column resistance, less soil mobilized to resist, and a lower FS.

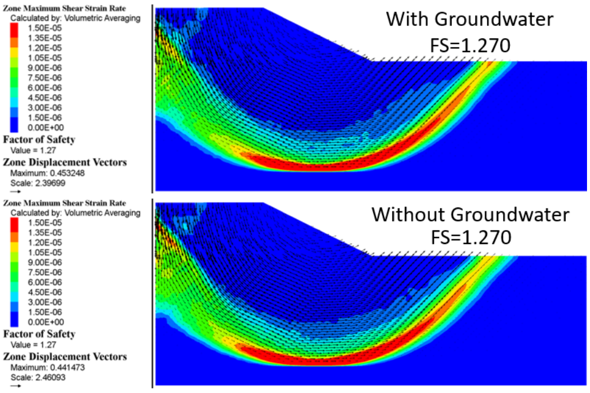

Figure 3. Strain rate of the model without improvement under an undrained condition.

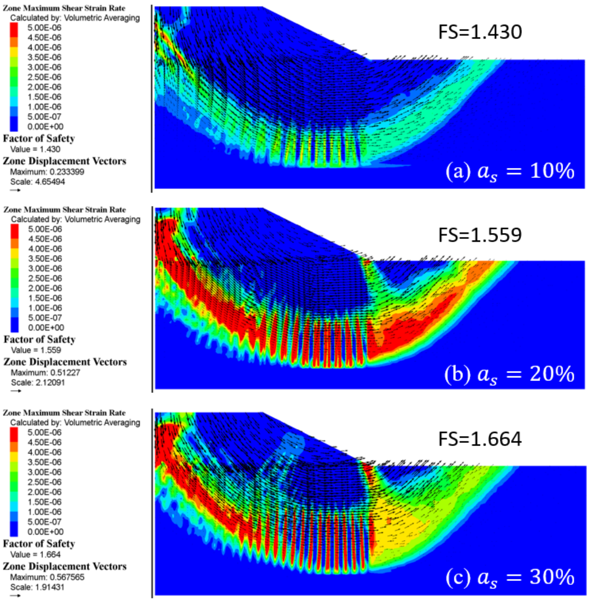

Figure 4. Strain rate of the CW model under an undrained condition with groundwater.

Figures 6 and 7 show that the shear bands in the ES model were mostly continuous and different from those in the CW model. Similar to the effect of the improved zone in the CW model at a large area replacement ratio, large shear strains developed along the interface between the improved area and the unimproved area, indicating the rotation of the soil in front of the toe. When the area replacement ratio was small, the lower boundary of the shear band surface reached to the top of the sand layer. With an increase of the area replacement ratio, the shear band moved upwards due to the increase of the shear strength provided by the aggregate columns. This phenomenon is even more obvious when groundwater was not considered as shown in Figure 7. This is because water pressure reduced more shear strength of columns at greater depths in Figure 6 but not in Figure 7. This shear strength reduction with the depth allowed the shear band to develop deeply as well as reduced the FS as compared with the case without groundwater.

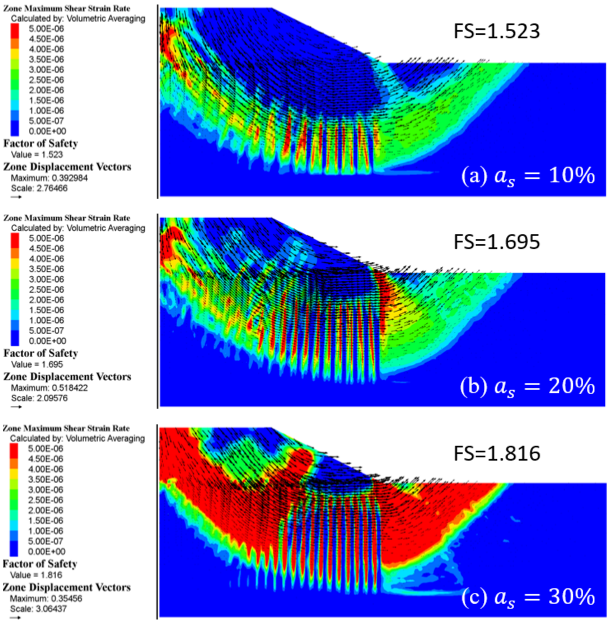

Figure 5. Strain rate of the CW model under an undrained condition without groundwater.

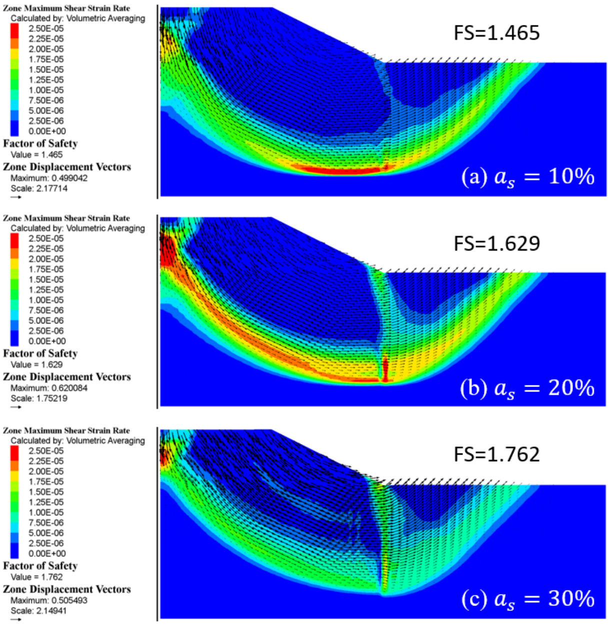

Figure 6. Strain rate of the ES model under an undrained condition with groundwater.

6.2 *Strain rate contour for drained conditions*

Figure 8 to Figure 12 show the shear strain rate in the embankment over unimproved ground and column-improved ground computed by the CW and ES method under drained conditions.

Since the soil strength increased with the confining stress under a drained condition, an increase of the depth increased the overburden stress and strength of the soil, making shear failure happen at greater depths more difficult. As a result, the shear band under a drained condition was shallower as shown in Figure 8 to Figure 12. Figure 8 illustrates that, under a drained condition, the cases with and without groundwater had different shear bands in the unimproved ground, which is different from the cases under an undrained condition. This is because the existence of groundwater reduced effective stresses and soil strength.

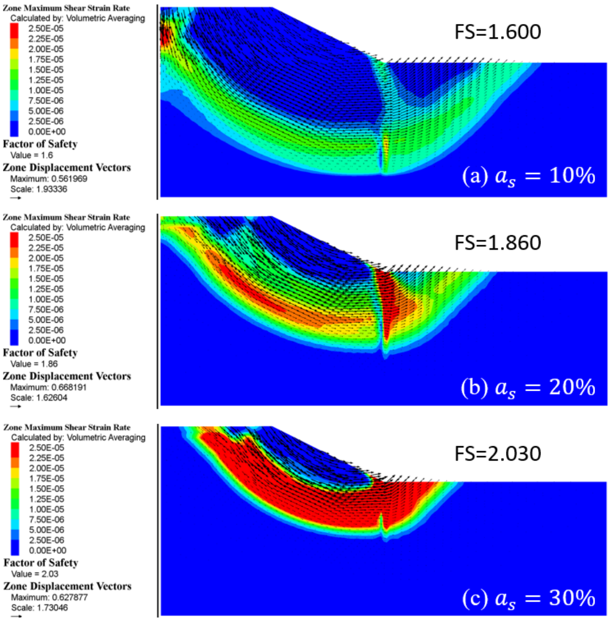

Figure 7. Strain rate of the ES model under an undrained condition without groundwater.

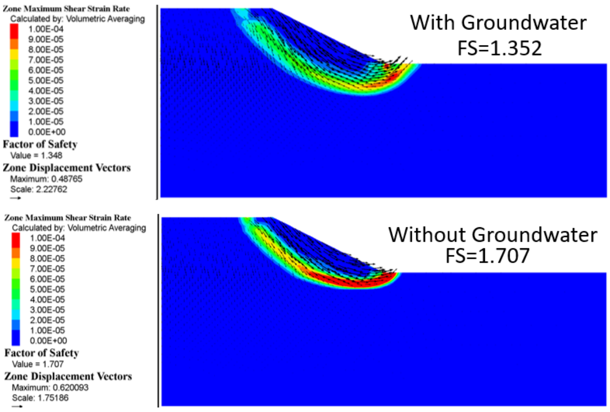

Figure 8. Strain rate of the model without improvement under a drained condition.

Figure 9 and Figure 11 show that the ES method resulted in a slightly higher factor of safety than the CW model, which is different from what Zhang et al. (2014) found under drained conditions. The major difference is that this study considered groundwater while Zhang et al. (2014) did not consider groundwater. Figure 9 and Figure 10 show that the model considering groundwater had a lower FS and a slightly deeper shear band than that without groundwater because the groundwater reduced the soil strength and allowed the shear failure to happen at greater depths.

Figure 10 and Figure 12 show that the CW and ES model resulted in almost the same result under a drained condition without groundwater as found by Zhang et al. (2014).

Figure 11 and Figure 12 show that the ES model with groundwater had a lower FS and deeper shear band than that without groundwater. This is because the groundwater reduced the soil and column strengths as discussed earlier.

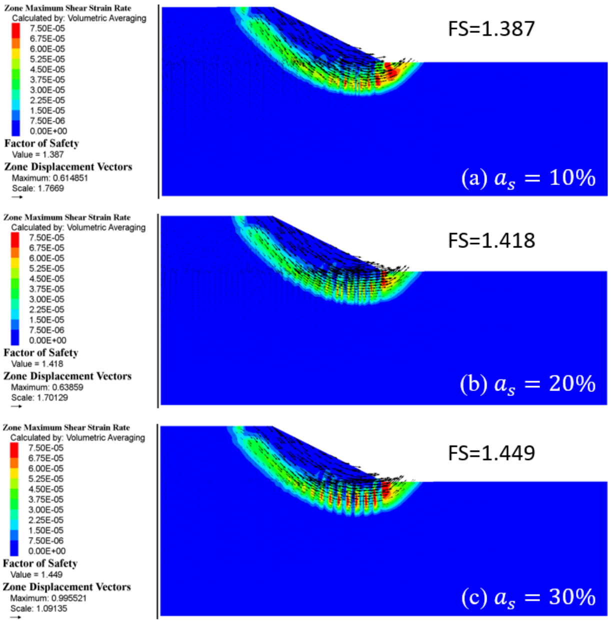

Figure 9. Strain rate of the CW method model under a drained condition with groundwater.

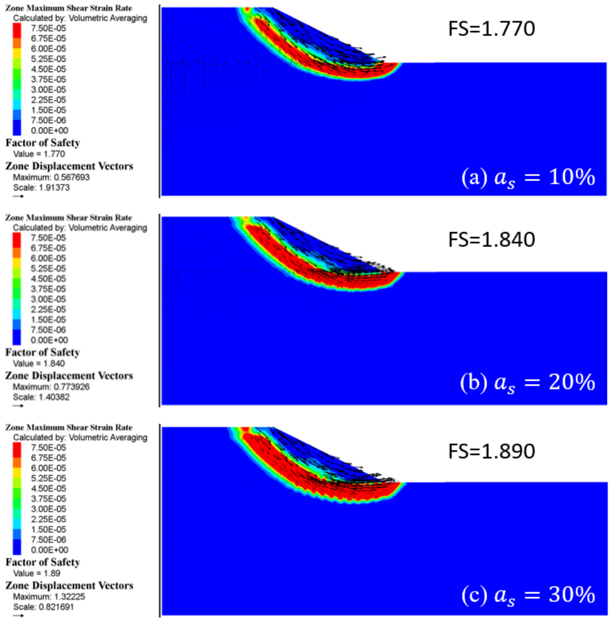

Figure 10. Strain rate of the CW model under a drained condition without groundwater.

6.3 *Factor of safety comparison*

Figure 13 and Figure 14 show the comparison of the FS of embankments over aggregate column-improved ground under undrained and drained conditions with and without groundwater computed using the CW and ES method. Under undrained conditions, Figure 13 shows that the FS increased with an increase of the area replacement ratio, the ES method computed a higher factor of safety than the CW method, and groundwater reduced the FS.

Figure 13Figure 14 shows that under drained conditions, the FS also increased with an increase of the area replacement ratio, but at a smaller rate than that under undrained conditions as shown in Figure 13. Figure 14 also shows that without considering groundwater, the ES model computed the same FS as the CW model, while with considering groundwater, the ES model calculated a slightly higher FS than the CW model, especially at a large area replacement ratio.

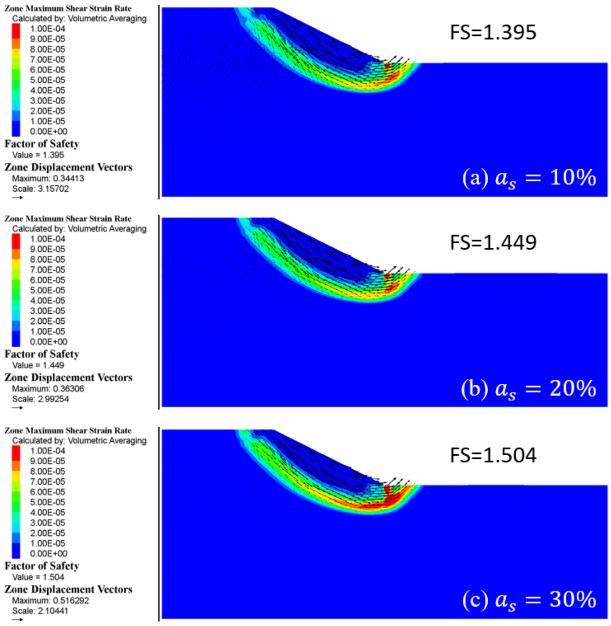

Figure 11. Strain rate of ES model under drained condition with ground water.

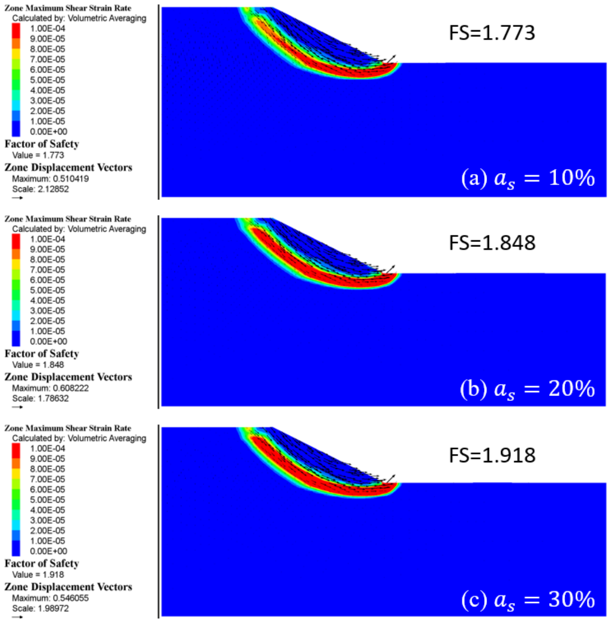

Figure 12. Strain rate of the ES method model under a drained condition without groundwater.

Figure 13 and Figure 14 show that the aggregate columns were more effective in increasing the FS under undrained conditions than under drained conditions. This is because the shear bands were much shallower under drained conditions than under undrained conditions so that less contributions were provided by the columns under drained conditions, especially for the cases with a greater area replacement ratio. Because of this fact, the FS under drained conditions could be lower than that under undrained conditions, especially considering groundwater and having a greater area replacement ratio. This finding is important and different from the common knowledge that an embankment on soft soils under an undrained condition is typically more critical than that under a drained condition as demonstrated in the cases with $a_s = 0\%$. This finding indicates that linked, undrained and drained analyses should be conducted for embankments over soft ground improved by aggregate columns to identify a critical condition.

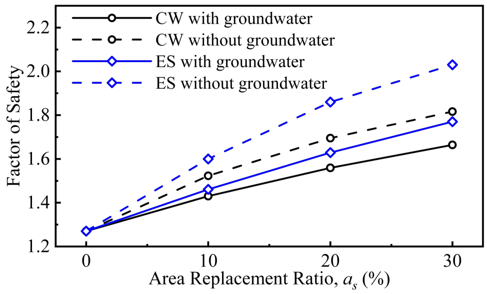

Figure 13. FS under undrained conditions.

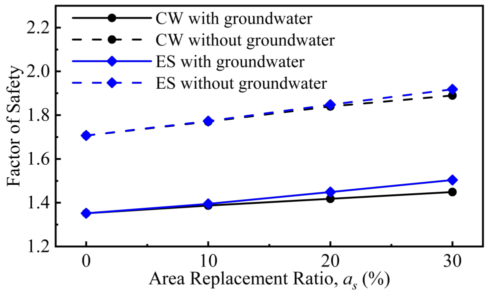

Figure 14. FS under drained conditions.

To further assess these methods and conditions, a factor of safety ratio (FS ratio) is defined herein as the ratio of the FS by the ES method to that by the CW method. Table 4 and Table 5 presents the FS ratios under undrained and drained conditions, respectively, with and without groundwater at different area replacement ratios.

Table 4 shows that under undrained conditions, the FS ratio increased with an increase of the area replacement ratio. By average, the ES method overestimated the FS by approximately 5% when the groundwater was considered and by approximately 10% when the groundwater was not considered, as compared with the CW method. Under drained conditions, these two methods yielded almost the same FS with and without considering groundwater at different area replacement ratios (with considering groundwater, the ES method slightly overestimated the FS as compared with the CW method).

Table 4. FS ratios under undrained conditions

Area replacement ratio (%)	10	20	30
With ground water	1.02	1.04	1.06
Without ground water	1.05	1.10	1.12

Table 5. FS ratios under drained conditions

Area replacement ratio (%)	10	20	30
With ground water	1.01	1.02	1.04
Without ground water	1.00	1.00	1.01

7 CONCLUSIONS

This study evaluated the equivalent strength method for the slope stability analysis of embankments over aggregate columns-improved soft ground under undrained and drained conditions with and without groundwater. Undrained and drained conditions of the soft clay foundation were linked by the critical state soil mechanics and analyzed separately. This study leads to the following conclusions:

(1) The embankment on uniform soft ground was susceptible to deep-seated failure under undrained conditions but shallow failure under drained conditions.

(2) The factor of safety of the embankment increased with the increase of the area replacement ratio. Under drained conditions with and without groundwater, the equivalent strength method computed the FS close to the column-wall method. Under undrained conditions, the equivalent strength method overestimated the FS by approximately 5% when the groundwater was considered and overestimated the FS by approximately 10% when the groundwater was not considered. A correction factor is suggested if the equivalent strength method is used for the analysis of stability of embankments on aggregate columns-improved soft ground under undrained conditions.

(3) This study recommends that linked, undrained and drained analyses should be conducted for embankments over soft ground improved by aggregate columns to identify a critical condition.

It should be noted that this study assumed a uniform soft clay layer with constant cohesion under undrained conditions. However, some soft clays (e.g., Gibson soils) may have their cohesion values varying with depth. The above conclusions should be verified for these soils. In addition, this paper presents a 2D numerical analysis, which does not simulate some 3D behavior, for example, soft soils may flow around columns. Aggregate columns can provide drainage to soft soils. Since the numerical analysis in this paper considered only undrained and drained conditions, the drainage process and effect of the aggregate columns were not considered.

8 ACKNOWLEDGEMENTS

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