

Representative geotechnical parameters of Chilean copper tailings and the calibration of PM4Sand and NorSand constitutive models

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ABSTRACT: Chile’s mining industry generates large volume of tailings, which must be stored safely in engineered facilities. Designing and analyzing these facilities require reliable geotechnical properties of the tailings under both monotonic and cyclic loading conditions. However, for new mining projects, tailings have not yet been produced, impeding direct geotechnical testing and introducing significant uncertainty when estimating parameters. This study presents an extensive review of published articles, theses, and engineering reports, comprising 1283 laboratory tests results on Chilean copper tailings, including both total tailings and coarse fraction (sand). Key geotechnical parameters – such as grain size distribution, Atterberg limits, specific gravity, minimum and maximum void ratio, internal friction angle, undrained strength and cyclic resistance curves – are compiled. The maximum, minimum and average values of these parameters are identified to establish a reference for design. Additionally, available stress-strain tests are used to calibrate PM4Sand (for cyclic response) and NorSand (for monotonic response) constitutive models in FLAC2D. The outcome of these calibrations provides a practical reference for the preliminary analysis and design of new tailings storage facilities, as well as a benchmark for comparing results from future laboratory tests on tailings samples.

KEYWORDS: Tailings, geotechnical properties, internal friction angle, cyclic resistance, calibration, PM4Sand, NorSand.

1 INTRODUCTION

This article presents and analyzes the results of a literature review of published articles, theses, and engineering reports, compiled in Ulacia (2022), which include results from 1030 laboratory tests on copper tailings sands (coarse fraction) and 253 tests on total copper tailings, totaling 1283 tests, with the aim of identifying representative ranges or values for the geotechnical parameters characterizing these materials. The fine fraction (slimes) of tailings is excluded from this article due to insufficient data in the reviewed documents to consider them representative.

Results include tailings from 23 of the most significant copper tailings storage facilities in Chile. These results are compiled in charts using a nomenclature where the uppercase letter “A” denotes tailings sands (cyclone underflow from tailings dams), and the uppercase letter “I” denotes total tailings, followed by a number to identify different facilities, a lowercase letter to identify different samples, and a sequential number in parentheses to indicate different specimen preparation methods: undisturbed or remolded with wet or dry compaction.

It should be noted that the total tailings included in this study originate from six conventional tailings storage facilities (I1 to I6), three thickened tailings facilities (I7 to I9), one filtered tailings facility (I10), and one paste tailings facility (I11).

Additionally, the resulting representative ranges or values are used to calibrate the PM4Sand and NorSand constitutive models in FLAC2D for cyclic and monotonic responses, respectively, for both tailings sands and total tailings. This calibration allows the provided parameters to serve as a reference for the preliminary analysis and design of new copper tailings storage facilities, as well as a benchmark for comparing results from future laboratory tests on copper tailings samples.

2 TESTS AND TRENDS

This section presents the results, analyzing and comparing trends and variability for both copper tailings sands and total copper tailings.

The typical ranges of specific gravity (G_s), minimum void ratios (e_{min}), maximum void ratios (e_{max}), and internal friction angle (ϕ) were obtained from frequency histograms and the corresponding cumulative frequency curves developed using

values from tailings sands and total tailings included in this study.

2.1 Basic tests and index properties

2.1.1 Grain size distribution, Atterberg limits, and classification of tailings

Figure 1 shows the average, minimum, and maximum granulometric curves.

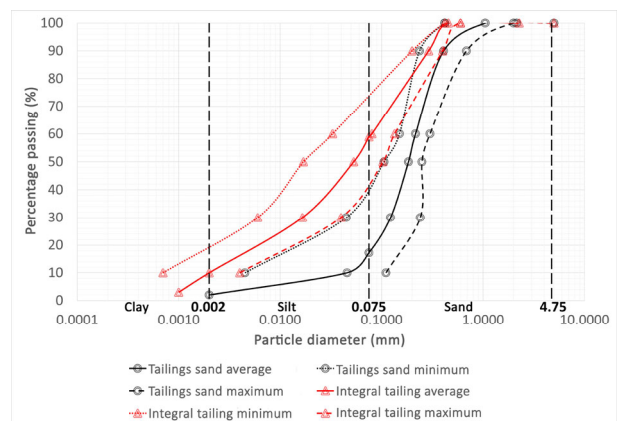


Figure 1. Average, minimum, and maximum granulometric curves for copper tailings sands and total copper tailings.

Figure 2 shows the plasticity chart obtained for total copper tailings (ASTM D2487, 2017). There are 13 data points for plasticity index (PI) and liquid limit (LL), with the PI value indicated for each point.

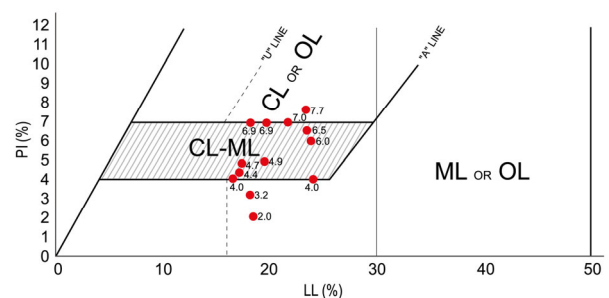


Figure 2. Plasticity chart for total copper tailings.

Seventy-seven percent (77%) of total tailings have a PI ranging from 4% to 7%.

All the copper tailings sands samples included in this study are non-plastic (NP).

Based on the grain size distribution and Atterberg limits obtained, both tailings sands and total tailings are classified according to the Unified Soil Classification System (USCS) per the ASTM D2487 (2017) standard.

For copper tailings sands, 94% of the total is classified as silty sand (SM). The remaining 6% is classified as poorly graded sand with silt (SP-SM) or poorly graded sand (SP).

For total copper tailings, 67% of the total is classified as sandy silt-clay (CL-ML), while 27% is classified as low-plasticity sandy silt (ML), and the remaining 6% as sandy lean clay (CL).

2.1.2 Specific gravity

The specific gravity (G_s) of copper tailings sands primarily ranges from 2.73 to 2.79, corresponding to 58% of the samples. The highest frequency is within the range of 2.75 to 2.77. This is complemented by the obtained average value and standard deviation, corresponding to 2.76 and 0.07, respectively.

Additionally, the G_s of 21% of the tailings sands ranges from 2.67 to 2.71, which may be due to the presence of quartz. Conversely, three samples exhibit a G_s greater than 2.85, corresponding to 2.86, 2.90 and 3.07, likely attributable to the presence of iron.

The G_s of total copper tailings primarily ranges from 2.75 to 2.85, corresponding to 67% of the samples. The highest frequency is within the range of 2.75 to 2.78, followed by the range of 2.83 to 2.85. This is complemented by the obtained average value and standard deviation, corresponding to 2.79 and 0.10, respectively.

Additionally, only one total tailings sample has a G_s greater than 2.85, corresponding to 3.12, which may also be due to the presence of iron.

2.1.3 Minimum and maximum void ratios

Based on maximum dry density ($\gamma_{d,max}$) data obtained using ASTM D2049 (1969), Japanese (JSSMFE), Standard Proctor (ASTM D698, 2021), and Modified Proctor (ASTM D1557, 2021) methods, for both copper tailings sands and total copper tailings, Equation 1 is used to calculate the corresponding minimum void ratios (e_{min}) for these soils, using the G_s values obtained for each material.

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} \quad (1)$$

In Equation 1, γ_w represents the unit weight of water, equal to 9.81 kN/m³.

The e_{min} of tailings sands typically ranges from 0.45 to 0.65, corresponding to 90% of the samples, with the highest frequency within the range of 0.50 to 0.55. This is complemented by the obtained average value and standard deviation, corresponding to 0.53 and 0.06, respectively.

The e_{min} of total tailings typically ranges from 0.40 to 0.60, corresponding to 64% of the samples. The highest frequency is within the range of 0.50 to 0.60. This is complemented by the obtained average value and standard deviation, corresponding to 0.49 and 0.13, respectively.

Based on minimum dry density ($\gamma_{d,min}$) data obtained using ASTM D2049 (1969) method, for both copper tailings sands and total copper tailings, Equation 1 is used to calculate the corresponding maximum void ratios (e_{max}) for these soils, using the G_s values obtained for each material.

The e_{max} of tailings sands typically ranges from 1.00 to 1.15, corresponding to 56% of the samples, with the highest

frequency within the range of 1.05 to 1.10. This is complemented by the obtained average value and standard deviation, corresponding to 1.09 and 0.10, respectively.

The e_{max} of total tailings typically ranges from 1.40 to 1.80, corresponding to 62% of the samples. The highest frequency is within the range of 1.40 to 1.60. This is complemented by the obtained average value and standard deviation, corresponding to 1.59 and 0.23, respectively.

2.2 Isotropically consolidated monotonic triaxial tests

A compilation of deviator stress at failure ($q_{failure}$) and mean effective stress at failure ($p'_{failure}$) results were conducted from CID and CIU tests under compression, performed on copper tailings sands and total copper tailings included in this study, to obtain the critical state line (CSL) and, consequently, the corresponding internal friction angle (ϕ) for each of these tailings. It should be noted that the materials analyzed exhibit zero or near-zero cohesion (c).

Equation 2 and Equation 3 were used to calculate the deviator stress (q) and the mean effective stress (p'), respectively.

$$q = \frac{\sigma_1' - \sigma_3'}{2} \quad (2)$$

$$p' = \frac{\sigma_1' + 2\sigma_3'}{3} \quad (3)$$

The following sections provide triaxial tests results, indicating the internal friction angle (ϕ) values obtained in each case. It should be noted that the provided ϕ values correspond to those adopted by the authors of the respective reviewed research studies, based on test results. In some cases, it was unclear whether the adopted value was the peak or remolded value.

Additionally, in both CID and CIU tests, it is considered that the materials are likely to exhibit dilatant behavior at high relative densities (DR), making it infeasible to reach the critical state.

2.2.1 Internal friction angle from CID tests

Results from CID tests conducted on tailings sands samples are compiled, with effective confining pressures (σ_3') ranging from 50 to 5000 kPa, fines content (CF) between 1% and 21%, and relative densities (DR) between 40% and 82%. Figure 3 shows the CSLs obtained from CID tests on copper tailings sands.

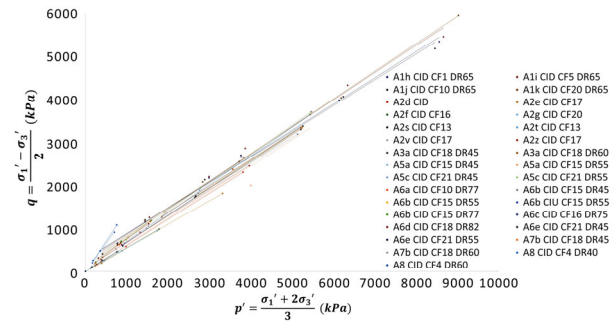


Figure 3. Critical state lines (CSL), from CID tests on copper tailings sands.

The ϕ of tailings sands under drained conditions typically ranges from 31° to 34°, corresponding to 62% of the samples. The most frequent range is between 32° and 33°. This is complemented by the obtained average value and standard deviation, corresponding to 33.7° and 1.9°, respectively.

Additionally, tailings sands samples with a ϕ of up to 32° account for 25% of the total data, while those with a ϕ of up to 33° and 34° represent 50% and 70%, respectively.

It is established that the ϕ of total tailings tends to be lower than that of tailings sands, as the latter is a coarse-grained soil, while fine grains predominate in total tailings.

The ϕ is a soil property, so similar results should be obtained regardless of whether the triaxial test is conducted under drained (CID) or undrained (CIU) conditions. The differences observed may be attributed to the criteria used by the authors of the respective research studies to conduct and analyze the test results, the characteristics of the materials used as samples for preparing the test specimens, or other factors that may have influenced the outcomes.

3 CALIBRATION IN FLAC2D OF THE NORSAND CONSTITUTIVE MODEL FOR MONOTONIC RESPONSE

The details of the theory and formulation of the NorSand constitutive model, as well as the physical meaning of each of its parameters, are found in the FLAC 8.1 Manual (Itasca, 2019).

Figure 6, Figure 7 and Figure 8 plot three stress paths from CID and CIU tests on samples of copper tailings sands and total copper tailings, with geotechnical parameters close to the average and mode values presented in Table 1 and Table 2. In all mentioned figures, the effective confining pressures (σ'_c) are 200, 500, and 1000 kPa, except in Figure 7, which corresponds to tailings sands under undrained conditions, where the second effective confining pressure is 400 kPa.

Additionally, the CSLs from Figure 3, Figure 4 and Figure 5, from CID and CIU tests, are used to define a band in Figure 6, Figure 7 and Figure 8, respectively. In these same figures, the stress paths from the calibration in FLAC2D of the NorSand constitutive model for monotonic response in CID or CIU tests, as applicable, are plotted.

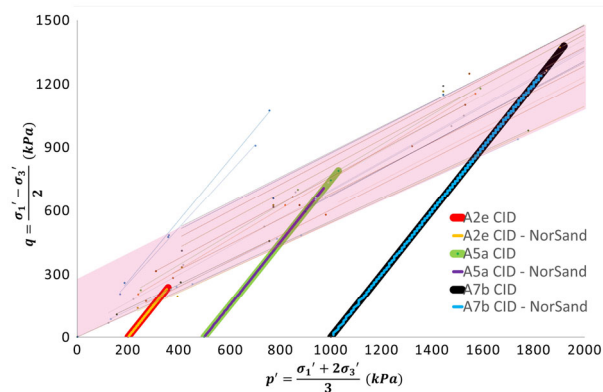


Figure 6. Stress paths from CID tests on copper tailings sands and calibration of the NorSand constitutive model in FLAC2D. Band defined by CSLs of the Figure 3 corresponding to CID tests conducted on copper tailings sands.

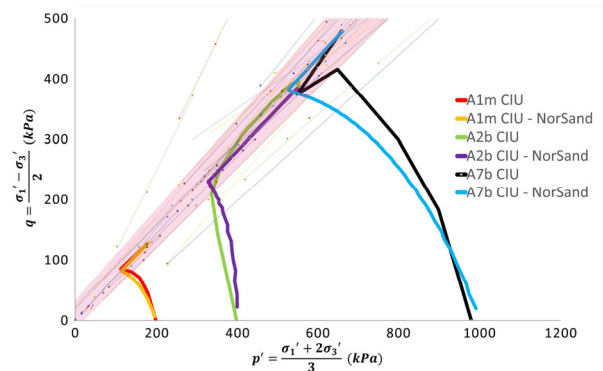


Figure 7. Stress paths from CIU tests on copper tailings sands and calibration of the NorSand constitutive model in FLAC2D. Band defined by CSLs of the Figure 4 corresponding to CIU tests conducted on copper tailings sands.

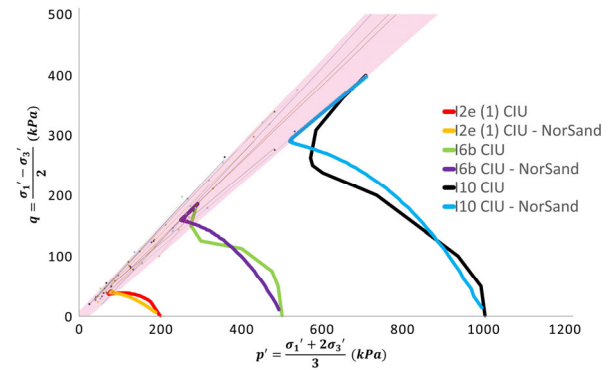


Figure 8. Stress paths from CIU tests on total copper tailings and calibration of the NorSand constitutive model in FLAC2D. Band defined by CSLs of the Figure 5 corresponding to CIU tests conducted on total copper tailings.

Table 3 provides a summary of the input parameter values, obtained iteratively for each studied case for the calibration in FLAC2D of the NorSand constitutive model. Parameters not mentioned were defined according to the model's default values.

Table 3. Input parameters of the NorSand constitutive model for its calibration in FLAC2D.

Parameter	Symbol	A-CID Value	A-CIU Value	I-CIU Value
Initial state parameter	ψ_0	-0.05-0.05	-0.05	-0.05
Critical-state-1	Γ	0.9	0.9	0.9
Critical-state-2	λ	0.02	0.02	0.02
Ratio-critical	M_{ic}	0.99-1.53	1.40-1.45	1.01-1.27
Factor-dilatancy	X_{ic}	3.5-20.0	0.2-0.4	0.3-0.7
Volumetric coupling factor	N	0.3	0.3	0.3
Hardening-0	H_0	300	100-1000	10-100
Constant shear rigidity	I_r	300	300-500	300-1000
Effective confining pressure	p_0	200-1000	200-1000	200-1000

Note: p_0 in kPa.

The stress paths in FLAC2D using the NorSand constitutive model were obtained iteratively by varying the parameters indicated in Table 3, considering the characteristics of the tested tailings and the representative values provided in Table 1 and Table 2. In cases where the material behavior was dilatant or contractive, the parameter defining this behavior, ψ_0 , was modified to achieve the corresponding calibration for each case. These iterations were performed while respecting the physics of each case and understanding the ranges in which the parameters vary, considering the representative values obtained in Table 1 and Table 2. It should be noted that the input parameter values used are within the range reported by Panes (2021) and Rousé & Panes (2024).

From the results obtained, it is observed that all stress paths generated in FLAC2D using the NorSand constitutive model are close to those of the tested tailings, and the q - p' stresses at failure fall within the defined bands.

4 ISOTROPICALLY CONSOLIDATED UNDRAINED CYCLIC TRIAXIAL TESTS

A compilation of results for cyclic stress ratio (CSR) and the number of load cycles to reach the liquefaction failure state (N) was conducted from CIU cyclic triaxial (TCUCI) tests performed on copper tailings sands and total copper tailings included in this study, to obtain the cyclic resistance curve (CRR) corresponding to each of these tailings.

To define the liquefaction failure state, the criterion of generating an excess pore pressure (PP) of 100% of the effective confining pressure (σ'_c) is considered.

The effective stress ratio (Kc) is obtained from Equation 4 and is equal to 1.0 for TCUCI tests, as the major (σ'_1) and minor (σ'_3) principal effective stresses of consolidation are equal.

$$Kc = \frac{\sigma'_1}{\sigma'_3} \quad (4)$$

Figure 9 and Figure 10 show the cyclic resistance curves (CRR) obtained from the results of TCUCI tests conducted on copper tailings sands and total copper tailings included in this study, respectively.

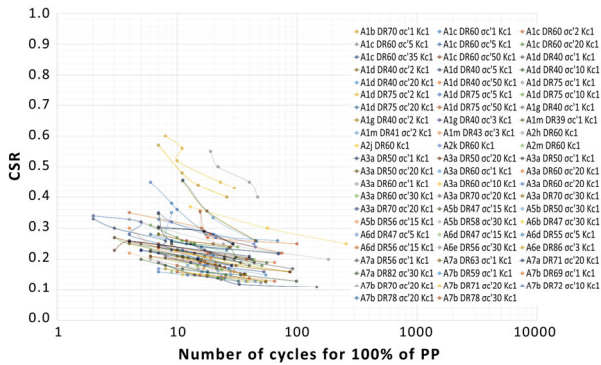


Figure 9. Cyclic resistance curves for copper tailings sands, considering the criterion of generating an excess PP of 100% of σ'_c , from TCUCI tests.

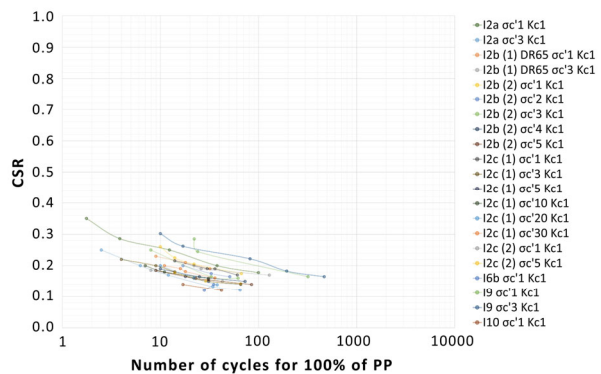


Figure 10. Cyclic resistance curves for total copper tailings, considering the criterion of generating an excess PP of 100% of σ'_c , from TCUCI tests.

It is observed that the cyclic resistances of total tailings tend to be in the lower part of the range of resistances of tailings sands.

5 CALIBRATION IN FLAC2D OF THE PM4SAND CONSTITUTIVE MODEL FOR CYCLIC RESPONSE

The details of the theory and formulation of the PM4Sand constitutive model, as well as the physical significance of each of its parameters, are found in Boulanger & Ziotopoulou (2023).

Figure 11 and Figure 12 plot three cyclic resistance curves obtained from TCUCI tests on three different samples of copper tailings sands and three different samples of total copper tailings, respectively.

Additionally, the curves from Figure 9 and Figure 10, obtained from TCUCI tests on copper tailings sands and total copper tailings, respectively, are used to define a band in Figure 11 and Figure 12. In these same figures, the cyclic resistance curves obtained from the calibration in FLAC2D of the PM4Sand constitutive model for cyclic response in Direct Simple Shear Test (DSS) are plotted, using the representative values for tailings sands and total tailings presented in Table 1 and Table 2, respectively.

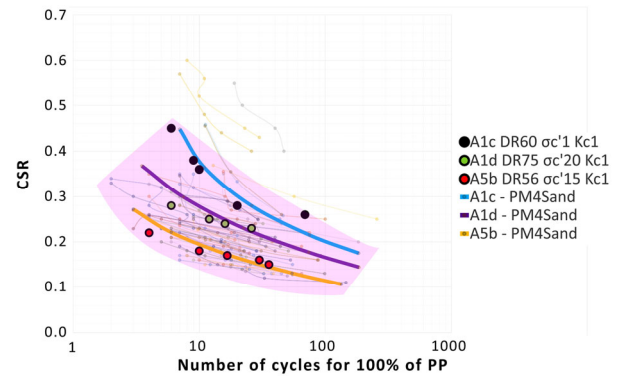


Figure 11. Cyclic resistance curves from TCUCI tests on copper tailings sands and calibration of the PM4Sand constitutive model in FLAC2D. Band defined by the total of cyclic resistance curves of the Figure 9 corresponding to TCUCI tests conducted on copper tailings sands.

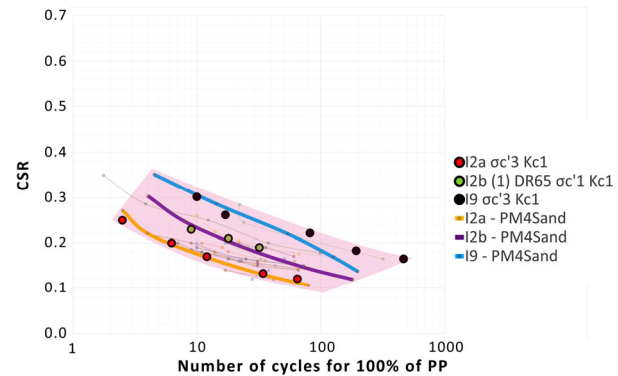


Figure 12. Cyclic resistance curves from TCUCI tests on total copper tailings and calibration of the PM4Sand constitutive model in FLAC2D. Band defined by the total of cyclic resistance curves of the Figure 10 corresponding to TCUCI tests conducted on total copper tailings.

Table 4 provides the values of the input parameters varied in each of the studied cases for the calibration in FLAC2D of the PM4Sand constitutive model. Parameters not mentioned were defined according to the model's default values.

Table 4. Input parameters of the PM4Sand constitutive model for its calibration in FLAC2D.

Parameter	Symbol	A-TCUCI Value	I-TCUCI Value	Unit
Relative density	DR	56-75	60-65	%
Contraction rate parameter	h_{po}	7	0.7-7	-
Minimum void ratio	e_{min}	0.53	0.49	-
Maximum void ratio	e_{max}	1.09	1.59	-

Specific gravity	G_s	2.76	2.79	-
Shear wave velocity	V_{SI}	170-200	100	m/s
Effective confining pressure	p_0	100-2000	100-300	kPa

Note: The values of parameters n^b and n^d were set to the default values of the constitutive model.

To obtain the cyclic resistance curves in FLAC2D, the parameters indicated in Table 4 were varied, considering the characteristics of the tested tailings and the representative values presented in Table 1 and Table 2. These iterations were performed while respecting the physics of each case and understanding the ranges in which the parameters vary, considering the representative values obtained in Table 1 and Table 2. It should be noted that the values used for the input h_{p0} are within the range reported by Salam et al. (2021) and Macedo et al. (2022).

From the results obtained, it is observed that all cyclic resistance curves generated in FLAC2D using the PM4Sand constitutive model adequately represent the behavior of the tailings tested in the laboratory.

6 CONCLUSIONS

This article presents various geotechnical parameters measured in Chilean copper tailings, including both total tailings and the coarse fraction (sand), based on an extensive literature review.

The geotechnical parameters studied – grain size distribution, Atterberg limits, specific gravity, minimum and maximum void ratios, internal friction angle, and cyclic resistance – show a narrow range of variation, indicating consistent properties in tailings produced at different copper mines throughout Chile.

Additionally, these representative ranges and values are used to calibrate the NorSand and PM4Sand constitutive models in FLAC2D for monotonic and cyclic response, respectively, for both tailings sands and total tailings.

Based on these results, the representative values and constitutive models are validated for use as a reference in the preliminary analysis and conceptual or profile design of new copper tailings storage facilities without available tailings samples, as well as a benchmark for comparing future laboratory test results on copper tailings samples.

It should be noted that a scarcity of geotechnical test data prevented the inclusion of certain results in this article, such as triaxial tests on undisturbed specimens or cyclic tests necessary to determine shear modulus degradation and damping curves.

The geotechnical community is encouraged to share test results to build a database that will benefit the development of this engineering field.

7 ACKNOWLEDGEMENTS

The engineering reports from which some data presented in this article were extracted were prepared by the authors or by colleagues who confidentially provided information to support this publication for the benefit of the geotechnical community. To maintain this confidentiality, no references are made to the engineering reports or to the names of the 23 Chilean tailings storage facilities that contributed data to this article. The authors thank the many geotechnical engineers who provided information for the development of this article.

8 REFERENCES

- American Society for Testing and Materials, 1969. *D2049 Test Method for Relative Density of Cohesionless Soils*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2007. *D422 Standard Test Method for Particle-Size Analysis of Soils*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2013. *D5311 Standard Test Method for Load Controlled Cyclic Triaxial Strength of Soil (Withdrawn 2022)*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2017. *D4318 Standard Test Methods for Liquid Limit, Plastic Limit, and Plasticity Index of Soils*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2017. *D2487 Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification System)*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2020. *D7181 Standard Test Method for Consolidated Drained Triaxial Compression Test for Soils*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2020. *D4767 Standard Test Method for Consolidated Undrained Triaxial Compression Test for Cohesive Soils*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2021. *D698 Standard Test Methods for Laboratory Compaction Characteristics of Soil Using Standard Effort (12,400 ft-lbf/ft³ (600 kN-m/m³))*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2021. *D1557 Standard Test Methods for Laboratory Compaction Characteristics of Soil Using Modified Effort (56,000 ft-lbf/ft³ (2,700 kN-m/m³))*. West Conshohocken, PA: ASTM International.
- American Society for Testing and Materials, 2022. *C128 Standard Test Method for Relative Density (Specific Gravity) and Absorption of Fine Aggregate*. West Conshohocken, PA: ASTM International.
- Boulanger, R. W., and Ziotopoulou, K., 2023. *Report No. UCSD/CGM-23/01 PM4Sand (Version 3.3): A sand plasticity model for earthquake engineering applications*. Davis, CA: Center for Geotechnical Modeling, Department of Civil and Environmental Engineering, University of California.
- Itasca Consulting Group, Inc., 2019. *Fast Lagrangian Analysis of Continua Version 8.1 Manual*. Minneapolis, Minnesota: Itasca Consulting Group, Inc.
- Macedo, J., Torres, P., Vergaray, L., Paihua, S., and Arnold, C., 2022. Dynamic effective stress analysis of a centreline tailings dam under subduction earthquakes. *Proceedings of the Institution of Civil Engineers - Geotechnical Engineering*, Volume 175 (2), 224-246.
- Panes, A. 2021. *Comparison of the NorSand and P2PSand constitutive models for the seismic modeling of a tailings dam in Chile using FLAC3D*.
- Rousé, P., and Panes, A. 2024. Comparative analysis of P2PSand and NorSand models for static liquefaction assessment in tailings dams. *Proceedings of the 17th Pan-American Conference on Soil Mechanics and Geotechnical Engineering (XVII PCSMGE), and 2nd Latin-American Regional Conference of the International Association for Engineering Geology and the Environment (IAEG)*, La Serena, Chile.
- Salam, S., Xiao, M., Khosravifar, A., and Ziotopoulou, K., 2021. Seismic stability of coal tailings dams with spatially variable and liquefiable coal tailings using pore pressure plasticity models. *Computers and geotechnics*, Volume 132.
- Ulacia, J. 2022. *Characteristic geotechnical parameters of tailings in Chile*.