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A novel method to evaluate soil parameter influences on the transient thermal behaviour of soils

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ABSTRACT: Understanding soil behaviour under thermal loading is crucial due to growing thermal stresses from climate change phenomena such as global warming and urban heat islands, and geothermal applications involving heat injection or extraction within soil deposits. The thermal response of soil is governed by factors including thermal conductivity, specific heat capacity, density (or void ratio), and temperature gradients. This study introduces a novel approach to investigate the combined effect of these parameters on soil thermal behaviour through coupled analytical modelling and experimental validations. Using the governing partial differential equations for one-dimensional transient heat transfer, an analytical solution was developed and calibrated against experimental data obtained from a newly designed laboratory thermal cell. The validated solution was then extended using MATLAB to develop an open framework for evaluating the effects of parameter variations on thermal distribution within soils. This integrated approach provides a reliable path for a comprehensive understanding of soil behaviour under diverse thermal scenarios, contributing to advancements in energy geotechnics and thermal resilience of subsurface infrastructure.

KEYWORDS: Heat transfer, Thermal conductivity, Specific heat capacity, Thermal diffusivity, Numerical framework

1 INTRODUCTION

It is undeniable that soil deposits worldwide are increasingly subjected to thermal loading. These loads induce temperature gradients and, consequently, drive heat transfer within soil layers. The heat transfer process is primarily governed by two fundamental thermal properties: thermal conductivity (k) and specific heat capacity (C). These parameters play a critical role in determining the thermal response of soils under both heating and cooling conditions (Abuel-Naga et al., 2007; Coccia & McCartney, 2016).

Despite the importance of thermal properties, many geotechnical studies and practical applications tend to rely solely on thermal diffusivity (i.e., $a = k/\rho C$) to interpret temperature distributions in soils subjected to thermal loading. However, it is essential to examine the individual components of thermal diffusivity to gain a more accurate and comprehensive understanding of heat transfer behaviour.

This study presents an open-access numerical framework that integrates a core theoretical formulation to analyse the heat transfer process in soil layers. The framework is built upon a tailored analytical solution for one-dimensional heat conduction in a cylindrical soil sample subjected to a constant peripheral thermal load. This tailored analytical model has been validated against experimental results obtained from a newly developed thermal cell device (Pirjalili et al., 2025).

2 THERMAL CELL CONFIGURATIONS

The experimental setup comprised a medium-length steel cylindrical mould with a height of 500 mm and a diameter of 200 mm. It was equipped with a peripheral heating system consisting of a spiral copper pipe surrounding the outer wall of the cylinder, along with a graphite sheet placed between the copper pipe and the cylinder wall to ensure uniform heat transfer. Hot water circulating inside the copper pipe provided the desired heat source at the wall of the cylinder. The temperature of the heat source (i.e., T_{out}) was maintained at a constant level with the aid of a relatively large thermal bath, which provided a constant closed-loop circulation inside the copper pipe. Thermal losses were minimised due to the complete insulation applied to the system. A schematic of the thermal cell is shown in Figure 1.

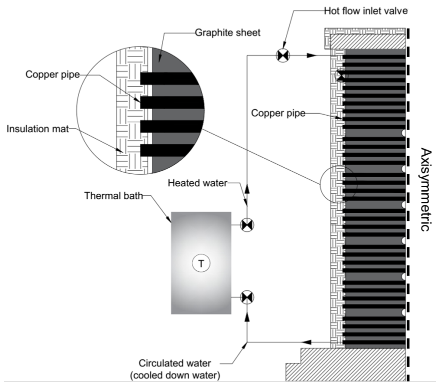

Figure 1. Schematic of the thermal cell

When the system was running, one-dimensional radial heat transfer occurred inside the soil sample, propagating from the cylinder wall toward the centre of the sample. To monitor the temperature evolution and heat transfer process within the sample, a series of precision thermocouples (accuracy ±0.01°C) was embedded at various locations inside the soil. The main measurements were taken on a horizontal plane located at mid-height (250 mm) of the sample. This position was chosen to minimise potential boundary effects from the top and bottom surfaces of the cylinder.

A range of target temperatures (approximately 35–75 °C, with 10 °C increments for each test) was applied to the sample. For each experimental run, it took about six hours for the sample to reach thermal equilibrium, during which the temperature became relatively uniform throughout the sample. During this period, temperatures at various locations were recorded for subsequent analysis, which required the use of a tailored analytical solution specific to the experimental setup. Further details can be found in Pirjalili et al. (2025).

3 ANALYTICAL FRAMEWORK

In general, the heat transfer phenomenon within a material primarily occurs through conduction and convection. In the absence of any form of liquid flow, heat conduction becomes the dominant mechanism. Accordingly, the partial differential equation governing heat transfer in a cylindrical soil sample subjected to a constant peripheral temperature is based solely on conduction and can be expressed by the following equation (Fourier, 1878):

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \emptyset^2}+\frac{\partial^2 T}{\partial z^2}+q_0=\frac{\rho C}{k}\frac{\partial T}{\partial t} \quad (1)$$

where T, t, q_0, ρ, C, and k are temperature (°C), time (s), internal heat generation (°C/m^2), density (kg/m^3), specific heat capacity (J/kg/°C), and thermal conductivity (W/m/°C), respectively. $\emptyset$, r, and z are cylindrical coordinates. According to the experimental setup, heat transfer occurs only in one direction (radially) within the sample. Additionally, it is assumed that there is no internal heat generation source. Therefore, the governing equation simplifies to the form shown in Equation (2).

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{\rho C}{k}\frac{\partial T}{\partial t} \quad (2)$$

It should be noted that, for this solution, the thermal properties of the soil were assumed to be independent of the applied temperature and dependent solely on the degree of moisture content. As no moisture variation occurs during the test, these thermal properties were considered constant.

To solve Equation (2), the dimensionless forms of the parameters were introduced and employed. Accordingly, Equation (2) was reformulated in terms of these dimensionless variables, as shown in Equation (3).

$$\frac{\partial^2 \Theta}{\partial R^2}+\frac{1}{R}\frac{\partial \Theta}{\partial R}=\frac{\partial \Theta}{\partial F_o} \quad (3)$$

where Θ, R, and F_0 are dimensionless temperature ($T - T_{out}/T_o - T_{out}$), dimensionless location (r/r_0), Fourier's number ($kt/\rho C r_0^2$), respectively. $\Theta(R, F_o)$ when F_o=0 and F_o=∞ is 1 and 0, respectively. Considering the constant temperature at the boundary of the problem, the other corresponding boundary condition of the problem could be shown as follows:

$$\frac{\partial \Theta}{\partial R}=-B_{\mathrm{i}}\Theta \quad (4)$$

where B_{i} is a crucial parameter which depends on the properties of both the thermal cell and the soil sample materials (i.e., B_{i}= $r_0 h/k$ where h (W.°C/m^2) is heat exchange efficiency factor). An analytical solution for this equation has been developed by Cebula (2014).

$$\Theta=2\sum_{i=1}^{\infty}\frac{1}{\lambda_{\mathrm{i}}}\frac{J_0(R\lambda_{\mathrm{i}})J_1(\lambda_{\mathrm{i}})}{{J_0}^2(\lambda_{\mathrm{i}})+{J_1}^2(\lambda_{\mathrm{i}})}e^{-{\lambda_{\mathrm{i}}}^2 F_{\mathrm{o}}} \quad (5)$$

where J_0 and J_1 are zeroth and the first order of the first kind of Bessel function, respectively. The variable λ_i represents the roots of ($B_{\mathrm{i}}=\lambda J_1(\lambda)/J_0(\lambda)$). For most applications, particularly for any time spot after $F_o > 0.2$, the first term of the solution in Equation (3) is typically sufficient to approximate the actual data (Cengel & Ghajar, 2020). Further details regarding the analytical solution could be found in Pirjalili et al. (2025).

Once the analytical solution was adapted to the experimental setup, the results of the laboratory experiments were interpreted alongside the analytically predicted data to derive the thermal properties of the soil sample. Accordingly, the reliability of the thermal cell and the accuracy of the tailored analytical solution were confirmed and comprehensively documented by Pirjalili et al. (2025).

4 NUMERICAL FRAMEWORK

Upon confirmation of the proposed method, the tailored analytical solution was implemented within an open-access framework developed in MATLAB. This framework enables users to evaluate the effects of various parameters on heat transfer within the context of the defined system, i.e., one-dimensional radial heat transfer. Given the consistency in thermal loading and boundary conditions, the influence of each parameter can be isolated, allowing for a focused analysis of its individual impact. For instance, to assess the effect of a specific parameter (such as thermal conductivity, k), that parameter is varied while all others are held constant. In this way, the framework facilitates efficient parametric studies for the defined problem.

In this study, thermal properties were examined under two distinct conditions, based on the definition of thermal diffusivity. These scenarios involved varying either the specific heat capacity or thermal conductivity, while keeping all other factors constant. Although both properties are known to be influenced by moisture content, only the dry and fully saturated states were considered. This decision was made based on the understanding that, in unsaturated conditions, the thermal properties of the soil vary over the course of the test, violating the underlying assumption of constant properties in the analytical solution (as stated in the introduction to Section 3).

4.1 *Material and sample features*

In this study, a silty sand, called Lyell Dam soil, was selected to be studied as the pilot soil material. According to the "Unified Soil Classification" system, it is classified as SM (silty sand) with about 27% of fine content, specific gravity of 2.61, and dry density of 1864 (kg/m^3). According to Pirjalili et al. (2025), the thermal properties of the tested soil at a dry state are as follows: k=0.455 (W/m/°C), C=590 (J/kg/°C), and α=4.14 × 10^{-7} (m^2/s).

Tested samples were prepared at an initial void ratio of 0.4 and 14 layers, when each layer having a thickness of 30 mm. Further information about sample preparation could be found in Pirjalili et al. (2025).

4.2 *MATLAB code verification*

To confirm the validity of the proposed method, the results generated by the MATLAB code were compared with those obtained from the analytical solution. Pirjalili et al. (2025) reported a strong agreement between the tailored analytical solution and the experimental results. Therefore, the observed agreement between the MATLAB-based results and the analytical solution indirectly confirms consistency with the experimental findings as well.

Figure 2 presents a comparison between the data generated by the developed MATLAB code and the analytical solution. For the sake of brevity, the comparison is shown for only one of the target temperatures (T=75°C). As illustrated in Figure 2, this agreement was observed not only across different spatial locations, but also throughout the entire duration of the experiment. This indicates that the model is capable of accurately replicating data across all three phases of heat transfer: initiation, transient, and equilibrium. Based on this verification, the framework is suitable for conducting further parametric studies.

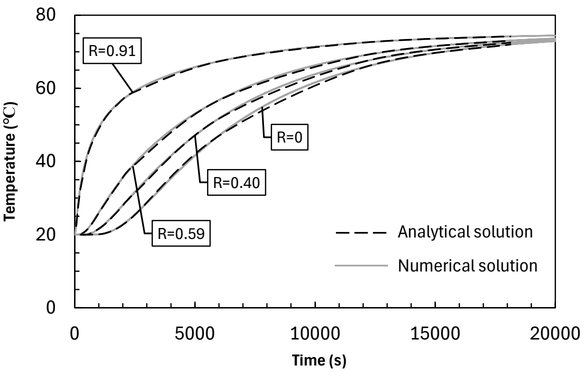

Figure 2. Validation of the proposed numerical solution against the analytical solution

5 RESULTS AND DISCUSSION

5.1 *Different Thermal Properties, Same Diffusivity: Implications for Heat Transfer*

In the context of constant thermal diffusivity, different combinations of thermal properties can be defined. Based on the definition of thermal diffusivity, the same value can be preserved by maintaining the ratio of thermal conductivity to specific heat capacity (i.e., k/C), assuming constant density (i.e., $\rho = cte$). Accordingly, given α_1=$k/\rho C$, it is evident that α_1=α_2=α_4 if α_2=$2k/2\rho C$ and α_4=$4k/4\rho C$. Using these thermal properties as inputs, the developed MATLAB framework (Section 4) was utilised to replicate the thermal cell experiments (Section 2). Accordingly, Figure 3 compares the temperature evolution of three samples with distinct thermal properties but identical thermal diffusivity (i.e., α_1, α_2 and α_4).

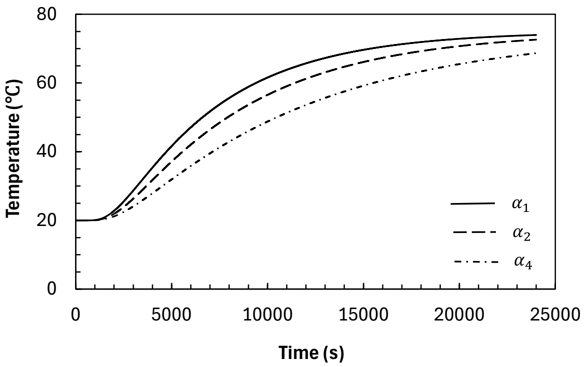

Figure 3. Temperature evolution inside the soil samples with the same thermal diffusivity but different k and C

As shown in Figure 3, even though all three cases have the same thermal diffusivity, the temperature evolution inside the samples differs significantly for each case. For instance, at 10,000 s after applying the peripheral temperature (i.e., the start of the test), the temperature corresponding to the α_1 case is approximately 62 °C—about 9% and 26% higher than the temperatures observed in the α_2 and α_4 cases, respectively. These discrepancies could be attributed to the influence of the thermal conductivity (k) in the boundary condition of the problem (Equation 4), or to the governing effect of specific heat capacity (C) within the system. Further analysis and comparison are provided in the next section.

In most research studies and practical applications, only thermal diffusivity is considered in computational analyses. However, the trends observed in this study highlight the importance of evaluating the individual components of thermal diffusivity separately, which is discussed in the following sections.

5.2 *Different thermal diffusivity*

In the previous section, thermal diffusivity was kept constant by maintaining a fixed ratio between thermal conductivity and specific heat capacity. In contrast, this section examines scenarios where thermal diffusivity is intentionally altered by either modifying k while keeping the C constant, or vice versa. These changes are applied in such a way that the resulting thermal diffusivity reaches a specific target value, allowing for controlled comparisons. For example, to reduce thermal diffusivity by 50%, one could either multiply k by 0.5 or multiply C by 2—both resulting in the same diffusivity reduction, assuming constant density. Accordingly, α represents the thermal diffusivity defined as $k/\rho C$, while $n\alpha$ refers to a modified version of α, achieved by either multiplying k by n (denoted as $n\alpha_{(nk)}$) or dividing C by n (denoted as $n\alpha_{(c/n)}$). In this study, three values of n were considered: 0.5 (Figure 4a), 0.33 (Figure 4b), and 0.25 (Figure 4c). The corresponding results for these cases—each representing the same thermal diffusivity—are compared and discussed.

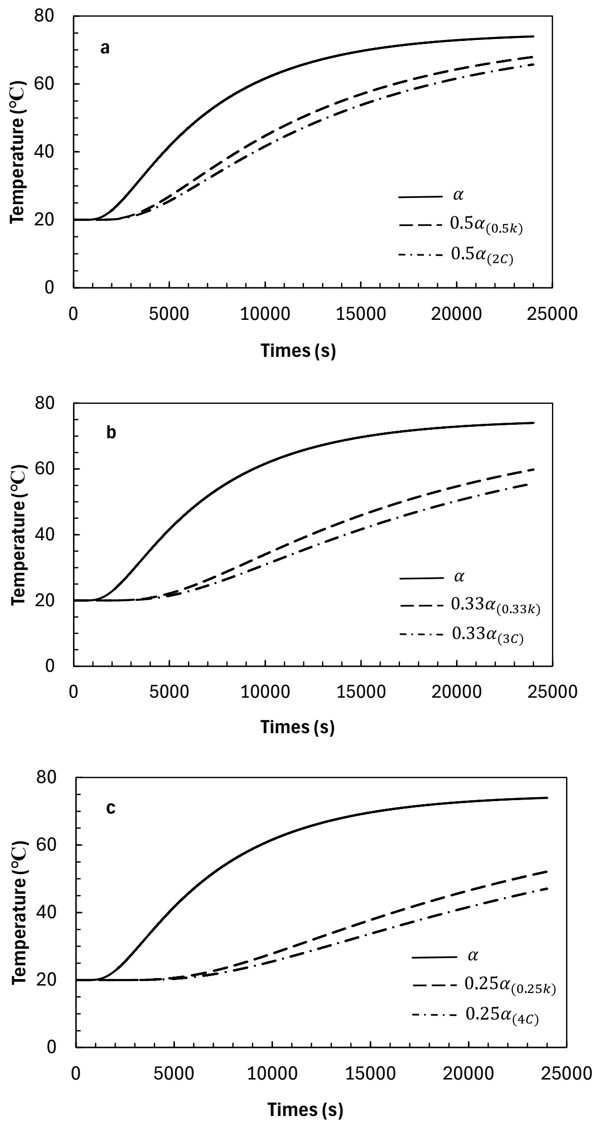

Figure 4. Effects of thermal diffusivity components on temperature distribution

By comparing the results of the reference thermal diffusivity (α) with modified cases ($0.5\alpha, 0.33\alpha$ and 0.25α), the plots in Figure 4 clearly demonstrate that decreasing thermal diffusivity results in a significant delay in temperature evolution. For instance, when thermal diffusivity is reduced by 50%, the sample fails to reach thermal equilibrium even 20,000 s after the start of the test (Figure 4a). This delay becomes more pronounced as further reductions in thermal diffusivity are applied. Thus, the transient phase of heat transfer is extended considerably as diffusivity decreases. This primary observation is aligned with both theory and the published literature.

In addition to this primary observation, the plots in Figure 4 reveal an important insight. For a given thermal diffusivity, a noticeable difference exists between cases where the diffusivity is achieved through higher specific heat capacity ($n\alpha_{(c/n)}$) versus lower thermal conductivity ($n\alpha_{(nk)}$), e.g., up to about 10%. In all scenarios, samples with greater specific heat capacity exhibit a slower heat transfer response. Interestingly, this difference is not constant over time. During the initial phase, the two curves nearly coincide; however, once the transient phase begins, the gap widens steadily. As the system approaches equilibrium, the gap narrows again, likely becoming negligible at longer time scales.

This behaviour suggests that specific heat capacity is the dominant parameter during the transient phase of heat transfer. In contrast, thermal conductivity (k)—despite its role in the boundary condition—is not the governing factor during the transient phase. To elaborate, the heat transfer process can be conceptually divided into three stages: initiation, transient, and equilibrium. Since the boundary condition remained constant throughout the experiments, any gap between two samples with the same thermal diffusivity but different thermal properties (i.e., k and C) should have persisted over the entire experiment duration. However, as shown in Figure 4, this gap is pronounced only during the transient phase and is negligible during the initial phase and, likely, in the equilibrium stage (i.e., after the test time cap of 24,000 s).

Based on the findings of this study, it is evident that relying solely on thermal diffusivity is insufficient for analysing and interpreting heat transfer processes in soil layers. Instead, the individual components of thermal diffusivity—thermal conductivity and specific heat capacity—must be accurately measured and evaluated. This underscores the importance of direct measurement of thermal properties. Furthermore, it should be emphasised that separate measurements of k and C may not fully capture the coupled thermal behaviour. Therefore, the ideal approach is the simultaneous measurement of both properties within a single experimental setup—a methodology already introduced by Pirjalili et al. (2025).

6 CONCLUSIONS

This study has presented a novel numerical framework developed in MATLAB and validated through comparisons with both analytical solutions and laboratory experiments. The purpose of this framework is to facilitate the evaluation of key parameters influencing heat conduction in soils. Following the validation of the proposed method, several case studies were conducted, leading to the following key findings:

- To evaluate the heat transfer process accurately, it is insufficient to rely solely on thermal diffusivity. Instead, its individual components—thermal conductivity and specific heat capacity—must be independently measured and considered.
- For a given thermal diffusivity, samples with higher specific heat capacity exhibit slower temperature evolution compared to those with lower thermal conductivity. This finding suggests that specific heat capacity governs the thermal response more strongly than thermal conductivity under identical diffusivity conditions.
- For the same thermal diffusivity, the difference in temperature response between samples with different thermal properties is only significant during the transient phase. During the initiation and equilibrium phases, this gap diminishes and eventually disappears.

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