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Accelerating Monte Carlo Simulations in Reliability Analysis of Slope Stability Problems

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ABSTRACT: A persistent computational challenge in assessing the reliability of slope stability in geotechnical engineering arises from integrating Monte Carlo Simulation (MCS) with rigorous numerical methods, such as the Limit Equilibrium (LE) Method or advanced models. This integration can result in simulations that extend to days or weeks, compromising efficiency of the reliability analysis. This paper investigates the performance of the MCS with the Dynamic Bounds (DB) method as an efficient alternative to accelerate reliability analysis of slope stability problems, comparing its efficacy against classical approaches. The DB approach significantly reduces the time required for reliability assessment. It achieves this by iteratively assessing the failure conditions – the transition between stable and failure states –, reducing the actual number of numerical simulations needed within the MCS to determine the reliability. The comparison is conducted for cohesive slopes with both two-layered and three-layered configurations, where the undrained shear strength is treated as a random variable. Specifically, the study compares the performance of the DB approach, which uses Discontinuity Layout Optimization (DLO) in LimitState:GEO, with the 'classical approach', which uses MCS with the Generalised Limit Equilibrium (GLE) method in RocScience software. The findings indicate that while the classical MCS with LE method demands days for the analysis, the proposed MCS with DB and direct MCS approaches drastically reduce the computation time to minutes. It is important to note that in this study, the faster method yielded estimates of the probability of failure, P_f, in the same order of magnitude, but differed up to a factor of 3. However, observed differences in P_f may stem from both the failure state approximation and fundamental disparities between the underlying numerical models (e.g. LE vs. DLO) and corresponding numerical algorithms.

KEYWORDS: Slope Stability, Reliability Analysis, Monte-Carlo Simulations, Limit Analysis, Discontinuity Layout Optimization, Limit Equilibrium Method, Dynamic Bounds.

1 INTRODUCTION

Despite its critical importance in geotechnical engineering, the assessment of slope stability is persistently challenged by the inherent uncertainties of soil properties and loading conditions. While conventional semi-probabilistic methods, employing partial safety factors, have long been standard, they lack the ability to fully quantify these uncertainties. Reliability-based design and verification methods, which are now supported by the next generation Eurocodes, effectively address these uncertainties, resulting in more robust and cost-effective designs.

Combining Monte Carlo simulation (MCS) with rigorous numerical calculation methods, such as the limit equilibrium method (LEM) or more advanced numerical models, often poses a considerable computational challenge (e.g., JRC, 2024). With this “classical” approach, each MCS sample requires an independent numerical simulation to determine whether a failure condition is met. This results in unreasonably long computation times, extending to several days or weeks for complex problems, depending on available computing power.

In the work presented here, the performance of an alternative to this “classical” approach is presented: the combination of MCS with the Dynamic Bounds (DB) method (cf. Rajabalinejad et al. 2011). This technique significantly accelerates the MCS, reducing the time required to assess reliability from days or weeks down to hours. To demonstrate its efficacy, this method is applied to a two-layered slope. In this application example, the undrained shear strength is considered as a random variable. Two distinct parameter combinations have been selected to compare the methods, representing different failure conditions where the failure mechanism occurs either in the top layer or through both layers.

For the modelling, two numerical analysis tools are utilized and compared: Limit Analysis using Discontinuity Layout Optimization (DLO), as implemented in the software LimitState:GEO, and the Generalized Limit Equilibrium (GLE) method, specifically the Morgenstern-Price method, as implemented in the RocScience software. The analysis is then extended to a three-layered slope to further demonstrate the applicability of the method as the system complexity increases.

Finally, a promising approach is outlined utilizing numerical optimization coupled with the DB method to further enhance the technique for multi-parameter problems

2 SLOPE STABILITY ASSESSMENT

2.1 *Semi-probabilistic vs probabilistic approach*

To verify the overall stability of natural or engineered slopes, the second generation of Eurocode 7 (EN 1997) officially permits using reliability-based methods as an alternative to the traditional partial safety factor concept (semi-probabilistic approach). Within this semi-probabilistic framework, the Material Factor Approach (MFA) is commonly applied to assess overall stability. This approach involves using partial factors on actions (γ_F), such as $\gamma_G = 1.0$, and partial factors on ground properties (γ_M), such as $\gamma_{cu} = 1.4$, which apply to undrained shear strength (c_u) in undrained ground conditions. A semi-probabilistic calculation then uses design values, for instance, $c_{u,d} = c_{u,k} / \gamma_{cu}$.

In contrast, the fully probabilistic approach explicitly acknowledges the inherent uncertainties in geotechnical parameters by treating key properties as random variables, incorporating their variability through suitable statistical distributions. For undrained slope stability problems in layered cohesive soils, undrained shear strength (c_u) is a critical property due to its significant influence on the failure mechanism (e.g., Duncan et al., 2014). If all soil layers are cohesive and considered undrained, c_u can be chosen as a random variable i and represented by a log-normal distribution, a suitable choice for this property (e.g., Jiang et al., 2018).

2.2 *Application example*

A two-layer slope (top layer 1: clay, c_{u1}, and bottom layer 2: silt, c_{u2}, both of unit weight 20 kN/m3), was selected for the investigations outlined in this article (see Figure 1).

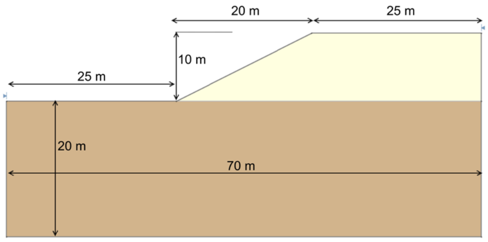

Figure 1. Application example: Two-layered cohesive slope.

To assess the stability of slopes, the Factor of Safety (FS) is commonly employed and generally defined as the ratio of the available shear strength (τ_f) to the shear stress required (or mobilized) for equilibrium (τ_{mob}):

$$FS = \frac{\tau_f}{\tau_{mob}} \quad (1)$$

For undrained conditions of cohesive soils, the available shear strength results from the undrained shear strength, thus (Duncan et al., 2014.

$$\tau_f = c_u \quad (2)$$

where cu is the undrained shear strength in [kPa].

For the probabilistic analysis, the mean values $\mu_{cu,mean}$ for the random variables c_{u1} and c_{u2} were selected such that, when converted into characteristic values ($c_{u,k}$) – as a cautious estimate of the mean following Annex A of Eurocode 7, part1 (FprEN 1997-1) – and used in a semi-probabilistic stability analysis, the resulting design values employing partial material factors led to a stable slope.

To address both potential failure mechanisms, two parameter combinations [μ_{cu1}, μ_{cu2}] are employed, Combination1: [70, 70] kPa and Combination 2: [70, 90] kPa. The corresponding FS calculated using the mean shear strength values for the two investigated parameter combinations were 2.0 and 2.5, respectively. It is assumed that the two soil layers and thus the parameters cu1 and cu2 are independent and that no correlation between the parameters needs to be assumed. In addition, it is assumed that the upper soil layer has a variability, expressed by the coefficient of variation V, defined as the ratio of its standard deviation (σ) to its mean (μ) ($V = \sigma/\mu$). The coefficient of variation for the upper soil layer is $V_{cu1} = 30\%$, and for the bottom soil layer, it is $V_{cu2} = 20\%$. The resulting log-normal distributions are shown in Figure 2.

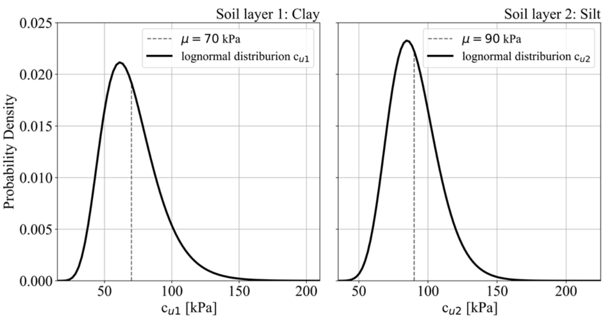

Figure 2. Lognormal distributions of random variables c_{u1} and c_{u2} for parameter combination 2.

A range of parameter combinations for the undrained shear strengths (c_{u1} and c_{u2}) was investigated for the presented two-layer slope. Depending on the strength ratio of these two layers, two main failure mechanisms were identified: Figure 3a illustrates a failure mechanism confined solely to the top layer, while Figure 3b shows a failure mechanism where both layers contribute to the failure state. The specific failure surface observed, therefore, depends directly on the combination of these soil strength parameters.

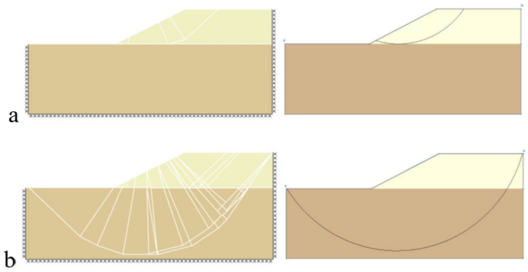

Figure 3. Failure modes: a) only the top layer is involved in the failure mechanism, and b) both soil layers contribute to the overall failure state. DLO results are presented on the left and LE results on the right.

It is important to note that the maximum size of the slip circles tested by the numerical models is limited by the model boundaries. Overall, the defined model area in both models (DLO and LE) is considered sufficient for identifying adequately representative failure mechanisms.

3 MONTE CARLO SIMULATION

3.1 *General remarks*

Monte Carlo simulation (MCS) is a sampling method that involves drawing random samples from the probability distribution of random variables and evaluating a limit state function for those samples which separates failure from non-failure. The probability of failure is estimated by the ratio of realizations leading to failure (n_{fail}) to the total realizations (n) as follows:

$$P_f \approx \frac{n_{fail}}{n} \quad (3)$$

For slope stability analysis, the limit state function g(X) is typically defined in terms of FS:

$$g(X) = FS(X) - 1 \quad (4)$$

where X represents the vector of random variables (here: c_{u1} and c_{u2}). A state of failure is defined when $g(X) \leq 0$, which corresponds to $FS \leq 1.0$. This function is central to reliability analysis, as it allows classification of each sample as either stable or failed. This condition is then used to calculate n_{fail} to obtain P_f according to Eq. 3.

The precision of the estimate is influenced by the number of (failed) realizations, which can be represented by the coefficient of variation of the estimated probability of failure (V_{Pf}):

$$V_{Pf} = \sqrt{\frac{1 - P_f}{n \cdot P_f}} \quad (5)$$

Where n is the total number of realizations, n_{fail} is the number of failed realizations, and P_f represents the failure probability.

A coefficient of variation (V_{Pf}) of 5 to 10% is generally accepted as a sufficient level of accuracy for estimating structural reliability. It can also serve as a convergence criterion for a crude Monte Carlo simulation. This V_{Pf} can further be used for calculating uncertainty bounds for estimated values, allowing for a priori estimation of the required sample size (n_{req}) based on the failure probability as follows (JRC, 2024):

$$n_{req} \approx \frac{1}{V_{P_f}^2 \cdot P_f} \quad (6)$$

For instance, for a structure classified as Consequence Class 2 (CC2), a target reliability index of $\beta_t = 3.8$ (corresponding to a target failure probability of $P_{f,t} \approx 7.2 \cdot 10^{-5}$) is typically associated with a 50-year reference period. Based on Eq. 6, achieving an estimated V_{Pf} of 5% would require approximately n = 4,000,000 total realizations in a crude MCS. These target reliability values, a prescribed requirement for reliability-based design and verification, are typically found in Eurocode 0 (EN 1990).

3.2 *MCS and Limit Equilibrium*

In the application example, the MCS process repeatedly samples values from the defined distributions for cu1 and cu2. For each set of sampled values, a numerical simulation using the Limit Equilibrium (LE) method is performed to determine FS. For this study, the Generalized Limit Equilibrium (GLE) method, specifically the Morgenstern-Price method, serves as the classical approach for LE calculations. All LE calculations and the MCS are performed using the inbuilt tool within RocScience software.

3.3 *MCS with Dynamic Bounds method*

The number of numerical simulations required within the MCS process can be significantly reduced using the Dynamic Bounds (DB) method. The method is applicable where the limit state function g(X) is monotonic with respect to X. This will be the case for parameters c_{u1} and c_{u2}. For this study, the Limit Analysis method Discontinuity Layout Optimization (DLO) as implemented in the software LimitState:GEO was utilized, together with the command line feature in the software to automate the DB process.

A simple DB algorithm involves generating a bounding list of pairs of values of (c_{u1}, c_{u2}) for which g(X) = 0. This was done using the Factor Strength analysis mode in LimitState:GEO (like strength reduction in an FE analysis). Let these values be denoted ($\hat{c}_{u1}$, $\hat{c}_{u2}$) and be stored in a list of length n and indexed by j. This list will be built up progressively during the MC process.

Next MC pairs (c_{u1}, c_{u2}) are generated as normal. For any pair, the bounding list is looped, starting from j = 1:

- If $c_{u1} \geq \hat{c}_{u1,j}$ and $c_{u2} \geq \hat{c}_{u2,j}$ then it is implicit $g(X) \geq 0$ and no numerical simulation is required.
- If $c_{u1} \leq \hat{c}_{u1,j}$ and $c_{u2} \leq \hat{c}_{u2,j}$ then it is implicit $g(X) \leq 0$ and again no numerical simulation is required.
- Otherwise, increment j and repeat.

If the loop is exited with j > n, then a numerical simulation is required for the MC pair (c_{u1}, c_{u2}), and this can be used to generate a new entry in the bounding list. Typically, a two parameter MC simulation with millions of sample values can be run with around 100 numerical simulations for the problem types studied in this paper, depending on the parameters' statistical distributions. The core method described above can be enhanced through various additional strategies. However, discussion of these is beyond the scope of this paper.

4 CALCULATIONS AND RESULTS

4.1 *The "classical" approach*

The probability of failure (P_f) and its corresponding variability (V_{Pf}) were determined using the classical approach, combining MCS with the LE method. Calculations were performed for both parameter combinations, each using a sample size of $n = 4 \cdot 10^6$ realizations. Table 1 summarizes the detailed results: the first combination shows a higher probability of failure and a longer computation time of about a week, while the second combination yields a lower P_f with a slightly reduced computation time.

Figure 4 illustrates distinct failure boundaries for the two parameter combinations. In Combination 2 (right-hand plot), the transition from stable to failed states is sharply defined by a vertical threshold in c_{u1}, indicating that the failure is primarily governed by the upper layer. In contrast, Combination 1 (left-hand plot) shows a more complex boundary defined by two intersecting lines, suggesting that failure depends on a combination of both c_{u1} and c_{u2}, reflecting a more complex mechanism.

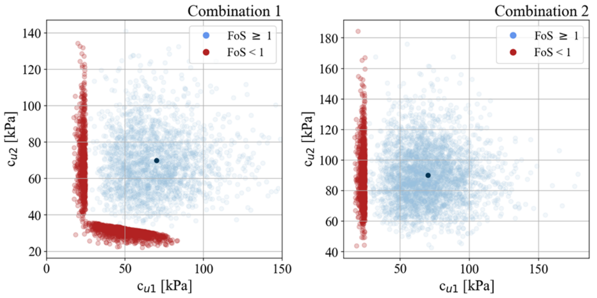

Figure 4. Scatter plots illustrate the distinction between stable and failure states based on samples generated from MCS with LE for Combination 1 (left) and Combination 2 (right). Samples with a Factor of Safety (FS) ≥ 1 are depicted in blue, representing stable conditions, whereas samples with FS < 1 are shown in red, indicating failure.

4.2 *MCS with Dynamic Bounds method*

Adopting the Dynamic Bounds (DB) approach within Monte Carlo Simulation (MCS) yielded results that were largely similar to those determined by the classical method, but with significantly reduced timescales. These results were generated using 2000 nodes in each of the Discontinuity Layout Optimization (DLO) analyses for the problem domain size depicted in Figure 3a. The specific estimated probability of failure (P_f), its variability (V_{Pf}), and computation times are detailed in Table 1, with the simulation results visualized in Figure 5.

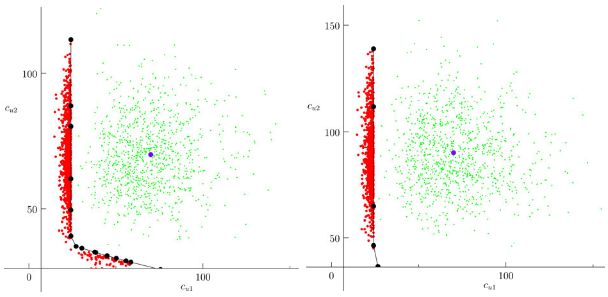

Figure 5. Scatter plots illustrate the distinction between stable and failure states based on samples generated from MCS with DB and DLO for Combination 1 (left) and Combination 2 (right). Samples with a Factor of Safety (FS) ≥ 1 are depicted in green, representing stable conditions, whereas samples with FS < 1 are shown in red, indicating failure. Samples representing results of numerical simulations are shown in black. For clarity not all pass solutions (green dots) are plotted due to the large numbers involved.

4.3 *Direct MCS*

The direct MCS refers to the evaluation of a large number of samples based solely on a predefined failure criterion (e.g., a set of linear inequalities), without requiring additional numerical simulations. This is made possible by first identifying the failure boundary through a limited number of simulations (e.g., via the DB method), after which each Monte

Carlo sample can be quickly classified as failed or stable by checking it against these boundary conditions.

Based on DLO analyses, the boundary between failure and stability appears to be well approximated by two straight lines, defined by the following failure state equations:

$$c_{u1} < 24.1382 \quad (7)$$

$$c_{u2} < 40.8127 - 0.1789 \cdot c_{u1} \quad (8)$$

These are both visible for Combination 1. However, for Combination 2, the statistical distribution of the parameters means that only Eq. 7 is required. In this case, only cu1 is involved in defining the critical failure surface, indicating that layer 2 is strong enough to confine the failure mechanism to the top layer for all MC pairs investigated. A single form of mechanism and an exact straight-line boundary are, therefore, as expected.

Generally, these boundaries would not necessarily manifest as straight lines, and the form of Eq. 8 is at first sight unexpected. However, an insight can be derived from examining the failure mechanisms in Figure 2 and considering the classic work of Taylor (1948). Examination of Taylor's slope stability charts indicates that for slope angles in uniform undrained soils of less than 53°, the failure mechanism will form a deep slip circle that will expand to be as large as possible. Supposing that the two-layer mechanism involves the second layer; it is expected that likewise, the mechanism will generally seek to be as large as possible, and this will be largely independent of the separate values of c_{u1} and c_{u2}. Thus, given an almost consistently single failure mechanism, the failure boundary would be expected to be a nearly linear combination of c_{u1} and c_{u2}.

Once the Dynamic Bounds (DB) approach approximates the failure boundary by selectively simulating samples (see Figure 6, Combination 1), a subsequent direct MCS can efficiently estimate the probability of failure. Each generated sample is checked against the predefined failure conditions, enabling a rapid calculation of the failure probability without the high computational cost of repeated numerical simulations.

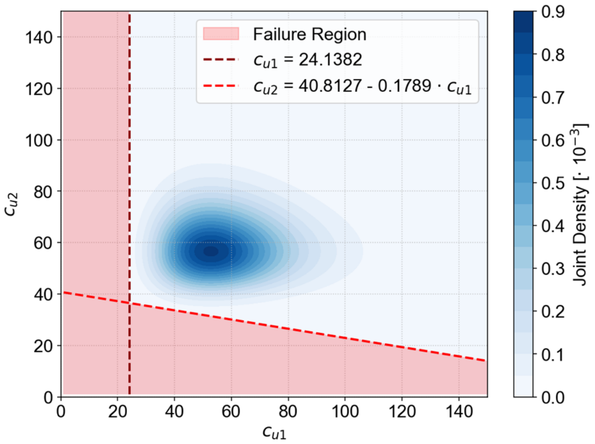

Figure 6. Figure 6. Failure region separated from stable state by two straight lines (failure conditions) for parameter combination 1.

Building on the failure boundary approximation from the DB analysis, a second MCS is performed using a custom Python script to efficiently estimate the probability of failure for both parameter combinations. Since the random variable distributions and failure conditions are known, this step requires no further numerical simulation. For each combination, $1 \cdot 10^9$ samples are evaluated against the predefined criteria, and P_f is calculated as the ratio of failed to total samples, The entire process completes in about a minute per combination, demonstrating substantial efficiency gains over traditional MCS with numerical modelling.

For Combination 1, failure is defined by two linear conditions (Eq. 7 and Eq. 8); a sample fails if either condition is met. For Combination 2, the failure is primarily governed by a single threshold in c_{u1}, corresponding to Eq. 7, resulting in a simpler, vertically bounded failure region (see Figure 4b and 5b). Table 1 summarizes the results, including P_f, the associated reliability index (β), and the variability of the P_f estimate (V_{Pf}) for both combinations

It is important to acknowledge that while this method provides rapid P_f estimation, it remains an approximation based on the pre-defined failure surface. However, the ability to clearly distinguish the failure state from the stable state through these analytical conditions allows for a robust and extremely fast quantification of P_f. Note that actual computation times are dependent on the computational hardware.

While in the example above, the failure boundaries were defined by simple straight lines, it would be reasonably straightforward to build up a curved boundary surface for the more general case through a selective sampling approach, which would allow for a reduced number of numerical simulations. While this is straightforward for a 2-dimensional 2-layer problem, extension to an n-layer n-dimensional problem poses additional challenges.

4.4 *Comparison*

Table 1 shows significant differences in computational efficiency and the precision of failure probability estimates across the approaches to assess the reliability of slope stability using MCS.

Table 1. Overview of Monte Carlo Simulation Results for Different Analysis Methods.

Method		Combination 1	Combination 2
MCS + LE	P_f =	$8.98 \cdot 10^{-4}$	$3.23 \cdot 10^{-4}$
	β =	3.12	3.41
	V_{Pf} =	1.67%	2.78%
	t =	6.94 days	4.28 days
MCS with DB + DLO	P_f =	$2.70 \cdot 10^{-4}$	$2.43 \cdot 10^{-4}$
	β =	3.46	3.49
	V_{Pf} =	3.04%	3.21%
	t =	23.5 min	12.6 min
Direct MCS	P_f =	$2.79 \cdot 10^{-4}$	$2.51 \cdot 10^{-4}$
	β =	3.45	3.48
	V_{Pf} =	0.19%	0.20%
	t =	~1 min	~1 min

The MCS with LE method represents a traditional approach where each simulated sample requires a full numerical stability analysis. This method stands out due to its significantly longer computation times, often extending to days or even nearly a week (cf. Table 1), making it impractical for rapid assessments or iterative design processes. In contrast, alternative approaches like MCS with DB and direct MCS substantially improve computational efficiency. These methods leverage a pre-approximated failure condition, drastically reducing the time required for analysis to mere minutes or even about one minute, representing orders of magnitude quicker execution than the classical MCS with LE approach. This efficiency gain underscores the considerable advantage of approximating the limit state prior to the full MCS run. Importantly, this pre-approximation, especially when based on more rigorous numerical models, can lead to a more precise estimate of the probability of failure and, consequently, higher reliability (closer to the target reliability).

The observed differences in the estimated P_f and V_{Pf} values between MCS with LE and the alternative approaches are likely to stem from fundamental differences in the underlying numerical stability analysis methods (e.g. between LE and other advanced numerical models such as DLO). This results in slight variations in the limit state conditions, which in turn lead to different failure probabilities, despite the use of the same distribution functions for the random variables and a comparable number of MCS.

5 APPLICATION TO 3-LAYERED SLOPE

To demonstrate the applicability and robustness of the presented methodology, the analysis is extended in the following to a more complex system: a three-layered slope with three distinct cohesive layers (see Figure 7). This expanded configuration introduces additional variability, necessitating the consideration of three undrained shear strength random variables (c_{u1}, c_{u2}, c_{u3}), and allows for the investigation of more complex failure mechanisms.

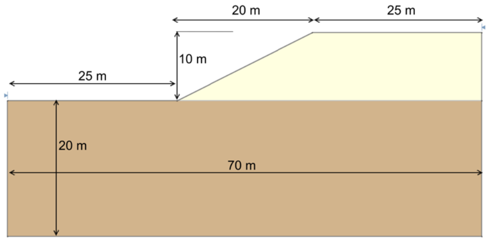

Figure 7. Second application example: Three-layered cohesive slope.

For this three-layered slope, three distinct failure modes were identified, which can occur depending on the combination of parameters: (a) failure confined solely to the top layer, (b) failure involving both the top and middle layers, and (c) a deep-seated failure surface passing through all three layers (see Figures 8).

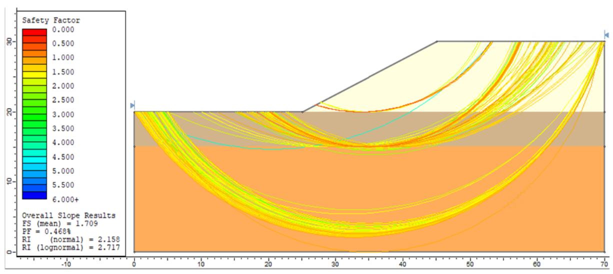

Figure 8. Example of three-layer failure mechanisms: LE results for mechanisms for failures in 1, 2 and 3 layers.

For the analysis of the undrained conditions, the undrained shear strengths of each layer, $c_{u,i}$, were treated as independent random variables. A specific combination was investigated, characterized by the following mean values: [μ_{cu1}, μ_{cu2}, μ_{cu3}] = [65, 55, 80] kPa. A uniform coefficient of variation ($V_{cu,i}$ = 30%) was applied to all three layers to represent their inherent variability.

In the "classical" approach, the estimated probability of failure is $4.68 \cdot 10^{-3}$. The variability of this P_f estimate is found to be 1.46%. This method required a significant computation time of 2.03 days to achieve these results.

For the MCS with DB approach, a probability of failure of $P_f = 2.53 \cdot 10^{-3}$ ($\beta = 2.80$) was determined after 250,000 realizations with a V_{Pf} value of 3.97%. The total calculation time for 250,000 realizations was 53.8 minutes.

Figure 9 shows the results from the DB Monte-Carlo model. As for the two-layer case, the failure boundary appears to be formed by three planes, corresponding to failure in 1, 2 and 3 layers defined by the following conditions:

$$c_{u1} < 24.1382 \qquad (9)$$

$$c_{u2} < 44.4583 - 0.4535 \cdot c_{u1} \qquad (20)$$

$$c_{u3} < 51.1164 - 0.2164 \cdot c_{u1} - 0.2634 \cdot c_{u2} \qquad (31)$$

The same arguments as for the 2-layer case can be extended to this case to explain why the results appear to give closely linear combinations of parameters.

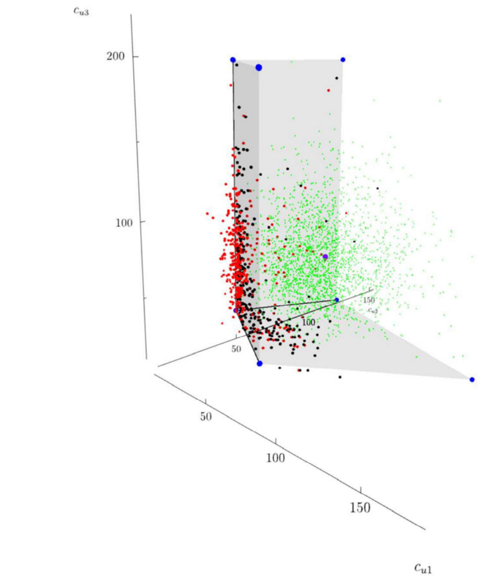

Figure 9. 3D plot showing Monte-Carlo cu pairs as green dots (slope is stable), or red dots (slope is unstable). The gray plane lines indicate the boundary dividing stable states from unstable states. The purple dot indicates the mean values of cu in all three layers. For clarity not all pass solutions (green dots) are plotted due to the large numbers involved.

Utilizing the failure conditions precisely determined by the DB analysis, a straightforward Monte Carlo Simulation (MCS) is performed to estimate the probability of failure (P_f) using 10^9 samples. In this MCS phase, where the underlying failure conditions are explicitly known, the calculated probability of failure is $P_f = 2.8 \cdot 10^{-3}$, corresponding to a reliability index of $\beta = 2.77$. The variability of this P_f estimate (V_{Pf}) is found to be 0.06%. This entire simulation, leveraging the known failure conditions, completes in less than 2 minutes.

6 MULTI-LAYERED PROBLEMS AND APLLICATION OF OPTIMIZATION

The preceding sections have illustrated the principles of three approaches to Monte-Carlo modelling of a layered slope problem. It can be shown that all three approaches can be extended to multiple layers or zones which in turn define multi-dimensional failure boundaries, and correspondingly more computationally demanding calculations to determine a probability of collapse to an acceptable level of accuracy.

For the latter two (DB and direct) methods, the challenge is to efficiently define the boundary set values or failure boundary surfaces. One promising methodology is to modify the optimization goals within the Discontinuity Layout Optimization procedure. Instead of seeking a mechanism of collapse that dissipates the least energy, the objective function can be amended to seek selected optimized layer strength combination sets to significantly and robustly reduce the search space of a Monte-Carlo assessment.

For example, a set of strengths across all layers that produce the lowest sum of strengths multiplied by cross-sectional area of layer could be sought. By amending the weightings in the optimization, key points on the failure boundary surface could be efficiently identified. An illustrative example of the lowest sum approach is shown in Figure 10 where the slope is divided into 308 individual zones. Here a minimum allowable strength of 10 kPa was defined for the optimization and the additional strength required is plotted. In this example each zone has a similar size, so the optimization minimizes the overall sum of additional strengths across all zones.

Application of this approach is the subject of ongoing further work.

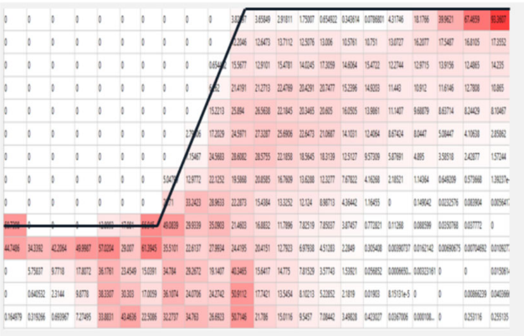

Figure 10. Example plot of multi-zone optimization Areas of darker red indicate areas where higher levels of additional strength are needed.

7 CONCLUSIONS

This study examined three approaches to probabilistic slope stability analysis using Monte Carlo Simulation (MCS): the classical MCS with Limit Equilibrium (LE) method, MCS combined with the Dynamic Bounds (DB) method, and a direct MCS approach with pre-defined failure boundaries. The findings reveal the significant computational burden of the classical approach, which often requires millions of simulations and can take several days. In contrast, the DB and direct methods achieve comparable reliability estimates with considerably less computational effort for problems with monotonic limit state functions.

The DB approach takes advantage of the monotonicity of the limit state function to drastically reduce the number of simulations needed, bringing it down to fewer than a hundred in some cases. Following this, a direct MCS, based on the DB results, enables rapid reliability estimation without further simulations.

These methods were successfully applied to two- and three-layer cohesive slope problems. In each of these cases, the failure surface could be approximated using linear conditions. However, for other problem types, the surfaces are expected to be non-linear. This suggests that the methodology is scalable and could, in principle, be extended to more complex n-layer systems. However, this extension would require sampling in an n-dimensional space, presenting new challenges in efficiently identifying key simulation points that define the dividing surface between stable and failure states.

A promising approach to managing this complexity involves the use of optimization strategies in Discontinuity Layout Optimization (DLO). For instance, modifying the objective function to focus on strength combinations that minimize the total additional strength across different zones can more effectively identify critical points on the failure boundary. This could significantly reduce the search space in high-dimensional problems and further improve computational efficiency.

It is important to note that the DB method is not restricted to DLO analyses. If the LEM software supports scripting and automation (for example, through Python), the MCS with the DB method can also be applied to LE-based modelling. This application provides similar efficiency gains in probabilistic analysis.

While this study assumed independence between soil parameters for simplicity, a key advantage of Monte Carlo simulation is its robust capability to incorporate and analyses complex correlations between variables. This makes it applicable to more detailed probabilistic assessments.

Finally, small differences in the estimated probability of failure between the approaches may not only result from approximations of the failure surface but also from inherent model uncertainties and fundamental differences between LE and DLO formulations. Further investigation into these sources of variability is warranted to enhance the robustness and reliability of advanced probabilistic slope stability assessments across all modelling platforms.

8 REFERENCES

Duncan, J. M., Wright, S. G. and Brandon, T. L. (2014): Soil Strength and Slope Stability; Second Edition; John Wiley & Sons.

prEN 1990-1:2023. Eurocode - Basis of structural and geotechnical design - Part 1: Design of new structures. CEN/TC250/SC10.

FprEN 1997-1:2023. Eurocode 7: Geotechnical design - Part 1: General rules. Draft for Formal Vote. CEN/TC 250/SC 7/WG 1 "Evolution of 1997 series".

European Commission, Joint Research Centre (JRC), van Den Eijnden, B., Knuuti, M., Lesny, K., Löfman, M., Mavritsakis, A., Roubos, A., Schweckendiek, T., Sciarretta, F., Ebener, A., Escher, K., Spross, J., Commend, S., Hehenkamp, M., Arnold, P., Wilhelm, S., Ene, A., Rimoldi, P. and Pereira, R., (2024): Reliability-based verification of limit states for geotechnical structures, Schweckendiek, T. editor, Publications Office of the European Union, Luxembourg, 2024, https://data.europa.eu/doi/10.2760/1342542, JRC139408.

Jiang, S., Huang, J., Huang, F., Yang, J., Yao, C. and Zhou, C. (2018): Modelling of spatial variability of soil undrained shear strength by conditional random fields for slope reliability analysis. Applied Mathematical Modelling, 63, 374–389. https://doi.org/10.1016/j.apm.2018.06.030

Rajabalinejad, M., Meester, L.E., van Gelder, P.H.A.J.M., and Vrijling, J.K (2011): Dynamic bounds coupled with Monte Carlo simulations. Reliability Engineering & System Safety 96 (2) 278-285.

Taylor, D. W. (1948): Fundamentals of soil mechanics, Wiley, New York