Proceedings of the 21st ICSMGE, Vienna, Austria, 14 – 19 June 2026. Pistrol, Adam & Schweiger (eds.)
Published by: ÖGG, Austrian Society for Geomechanics, Salzburg, Austria, ISBN 978-3-9503898-4-5
https://doi.org/10.53243/ICSMGE2026-1470

Effects of spatial variability of soft soil properties on the estimation of negative skin friction in groups of piles

Víctor Manuel Pineda-Núñez, Gabriel Auvinet
Institute of Engineering, National Autonomous University of Mexico, Mexico City, Mexico, vpinedan@iingen.unam.mx

Juan Félix Rodríguez Rebolledo
Department of Civil and Environmental Engineering, University of Brasilia, Brasilia, DF, Brazil

ABSTRACT: A stochastic analysis of the estimation of negative skin friction in groups of friction piles submitted to regional subsidence is presented. The spatial variability of the undrained resistance of the soil was represented as a one-dimensional vertical random field. The analytical method recommended in the current regulations of Mexico City to assess the behavior of friction piles foundations was used to estimate the negative skin friction. This method establishes a balance between the acting and resistant loads within the pile-soil foundation system, leading to a certain position of the so-called neutral level. The results of the probabilistic method show that the magnitude of negative skin friction, position of the neutral level and foundation settlements can vary significantly from those obtained by the deterministic method. It is concluded that introducing spatial variability of the properties in soils with a complex formation process in analyses of piled foundation leads to more realistic results.

KEYWORDS: spatial variability, soft soils, friction piles, random fields.

1 INTRODUCTION

Friction pile foundations are commonly used in soft soils submitted to regional subsidence, such as those of the lacustrine zone of Mexico City. In the usual design methods, soil properties are assigned based on average values for large soil volumes. This type of design exhibits a high degree of uncertainty due to the spatial variability of the soil properties (Reséndiz and Herrera, 1969).

The estimation of positive and negative skin friction developed on the piles and differential settlements is a complex process, and various analytical (Verruijt, 1969; Fellenius, 1998; Briaud, Jeong and Bush, 1991; Alberro and Hernández, 2000; Auvinet and Rodríguez, 2017; Zemanová, Anderle and Masopust, 2025) and numerical solutions (Chow, Lim and Karunaratne, 1996; Lee, Bolton and Al-Tabbaa, 2002; Pineda-Núñez, Auvinet and Rodríguez, 2016; Rodríguez, Cunha and Caicedo, 2018; Alarcón Posse et al., 2021) among others, have been proposed.

In this paper, the analytical method recommended in the current regulations of Mexico City is used to assess the behavior of friction piles foundations. This method establishes a balance between the acting and resistant loads within the pile-soil foundation system, leading to an assessment of the positive and negative skin friction developed on the piles, resulting to a certain position of the so-called neutral level. A stochastic analysis of pile foundations considering the spatial variability of undrained soil strength is performed. The variability in strength is represented by a vertical one-dimensional random field. The results of the stochastic analysis are compared with those obtained by the deterministic method and those of a Finite element model.

2 SPATIAL VARIABILITY AND RANDOM FIELDS

2.1 *Spatial variability in soils*

All natural soils exhibit variations in properties from point to point in the ground, attributable to inherent variations in composition and consistency during formation (Lumb, 1969). Even within nominally homogeneous soil layers, the soil properties may exhibit considerable variations from point to point (Vanmarcke, 1977). The spatial variability of soils is one of the main sources of structural malfunctioning of built systems (Breysse *et al*., 2004) and must be assessed by performing enough soil explorations, processing a large amount of data, and developing deterministic or probabilistic models of these variations that can be introduced in geotechnical analyses (Auvinet, 2019).

2.2 *Random fields*

The theory of random fields is a common approach for modeling spatial variability of soil properties (El-Ramly, Morgenstern and Cruden, 2002) and the most exact way to consider the stochastic nature of soil properties (Huber *et al*., 2010).

Let $V(X)$ be a geotechnical variable of either physical (e.g. water content), mechanical (e.g. undrained shear strength), or geometric type (e.g. thickness of a certain stratum), defined at points X of the domain $R^P (p = 1,2, or\ 3)$ considered. If at each point of the domain this variable is regarded as random, the set of these random variables constitutes a random field (Auvinet, 2002).

The following parameters and functions can be used to describe random fields:

- Expected value:

$$\mu_V = E\{V(X)\} \tag{1}$$

- Variance:

$$\sigma_V^2(X) = Var[V(X)] \tag{2}$$

The square root $\sigma_V(X)$ of the variance is known as the standard deviation.

- Coefficient of variation:

$$CV_V(X) = \sigma_V(X)/E\{V(X)\} \tag{3}$$

- Autocovariance function:

$$C_V(X_1, X_2) = Cov[V(X_1) - \mu_V(X_1)][V(X_2) - \mu_V(X_2)] \tag{4}$$

- Autocorrelation coefficient:

$$\rho_V(X_1, X_2) = \frac{C_V(X_1, X_2)}{\sigma_V(X_1)\sigma_V(X_2)} \tag{5}$$

2.2.1 *Karhunen-Loève Expansion*

The Karhunen-Loève expansion (Pineda and Auvinet, 2013) of a random field $V(X)$ is:

$$V(X) = E\{V(X)\} + \sum_{i=1}^{\infty} \sqrt{\lambda_i}\, \xi_i(\theta) \phi_i(X) \quad (6)$$

where X_1 and X_2 are the spatial coordinates, the set of functions ϕ_i forms a complete orthogonal basis of a Hilbert space and $\xi_i(\theta)$ is a set of random variables.

To obtain an approximation of the process the number of terms can be truncated. In general, a good approximation is obtained with few terms:

$$\hat{V}(X) = E\{V(X)\} + \sum_{i=1}^{M} \sqrt{\lambda_i}\, \xi_i(\theta) \phi_i(X) \quad (7)$$

where, $\hat{V}$ is the truncated expansion.

3 THE SUBSOIL OF MEXICO CITY AND ITS SPATIAL VARIATIONS

3.1 *The subsoil of Mexico City*

The Basin of Mexico is bordered by volcanic ranges. The late appearance of the Chichinautzin range led to the formation of several lakes named Zumpango, Xaltocan, Texcoco, Xochimilco, and Chalco (Figure 1). The Basin was rapidly silted up by river sediments stratified with volcanic ash and pumice from volcanic eruptions (Aguayo, Marín and Sánchez, 1989; Vázquez Sánchez and Jaimes Palomera, 1989).

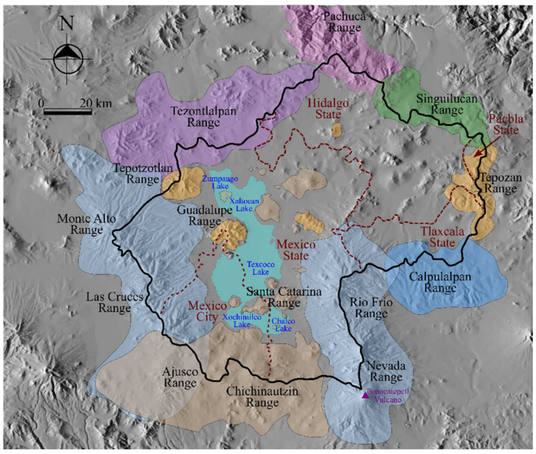

Figure 1. The Mexico Basin and some components of its geomorphology.

The distribution of sediments in the Mexico Basin is an important factor in the mechanical behavior of the subsoil, as it involves properties such as variations in texture, mineralogical and chemical composition, geometric shape of the strata, and their thickness. The characteristics of the subsoil in the typical lake zone of Mexico City are high compressibility (Mesri, Rokhsart and Bohort, 1975), low shear strength, high water content, and low permeability.

Water extraction through wells, to supply fresh water to the city, causes a loss of pressure in the aquifers and, therefore, an increase in the effective stresses in the soil mass, causing regional subsidence. This regional subsidence reaches rates of up to 40 cm/year in some areas. Regional subsidence is a factor that affects the static and dynamic properties of the soil (Ovando, Ossa and Romo, 2007). The most significant effect of regional subsidence on pile foundations is the development of negative skin friction on the walls of the foundation box (if any) and on the shafts of the piles or piers.

3.2 *Spatial variations in the subsoil of Mexico City*

The urban area of Mexico Valley is usually divided in three main geotechnical zones (Marsal, 1975): Foothills (Zone I), Transition (Zone II) and Lake (Zone III) (Figure 2). In the foothills, very compact but heterogeneous volcanic soils and lava are found. These materials contrast with the highly compressible soft soils of the Lake Zone. Generally, in between, a Transition Zone is found where clayey layers of lacustrine origin alternate with sandy alluvial deposits (Auvinet, 2019).

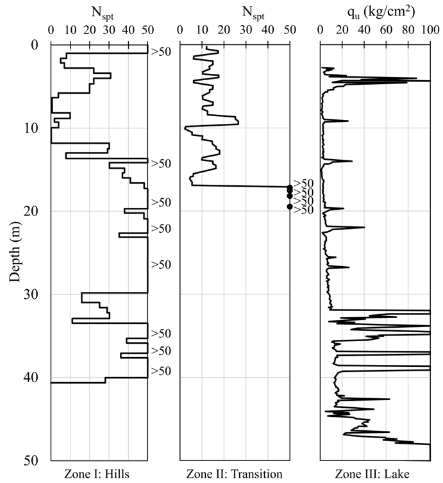

Figure 2. Typical soil profiles of the three geotechnical zones.

4 PILE GROUPS AND NEGATIVE SKIN FRICTION

4.1 *Pile groups*

The piles are often grouped into clusters or rows, each containing enough piles to support the load delivered by a single column or wall (Peck, Hanson and Thornburn, 1974). The interaction between piles depends essentially on their spacing, the ratio of their length and diameter, the stiffness of the pile compared to that of the surrounding soil, and the soil stiffness variability with depth (Niandou and Breysse, 2007). Moreover, the location within the group is important. Many researchers have shown that in small groups there is a clear distinction in the load magnitudes of piles located in the interior, at the edge, or in the corner (Zeevaert, 1973; Combarieu, 1988; Briaud, Jeong and Bush, 1991; Jeong, Kim and Briaud, 1997; Alberro and Hernández, 2000; Rodríguez, 2010; Pineda-Núñez, Auvinet and Rodríguez, 2016; Auvinet and Rodríguez, 2017), to name a few.

The behaviour of an interior pile within a very large group can be analyzed considering an "elementary cell", consisting of a pile and the surrounding soil (Schlosser, Jacobsen and Juran, 1984). This cell is laterally constrained in the horizontal direction, with allowable deformations in the vertical direction. (Rodríguez, 2010) corroborated this model for axisymmetric inclusions and piles using the finite element method (FEM), when boundary conditions become less important (large groups)

The behavior of edge and corner piles differs significantly from that of inner piles because they are more exposed to soil forces. Explicit modelling is recommended because their behavior depends on pile spacing, arrangement type, and especially the stiffness of the structure connecting their pile heads (Rodríguez, 2010; Pineda-Núñez, Auvinet and Rodríguez, 2016). Interacting factors obtained through parametric or field studies representative of the site can also be used to premultiply the lateral shaft strength; for example, this enables the magnitude of negative skin friction to be estimated (Jeong, Kim and Briaud, 1997; Pineda-Núñez, Auvinet and Rodríguez, 2016).

4.2 *Negative skin friction*

Negative skin friction (NF) can be defined as the downward stress that develops along the pile shaft when the surrounding soil consolidates due to piezometric drawdown or to recent surface loading (Auvinet and Rodríguez, 2017). The forces that resist the penetration of the pile in the soil are called positive skin friction (PF). The neutral plane or neutral level (z_0) is the depth where there are no relative movement between the soil and the pile or where the NF changes to PF. The acting loads on the pile are those applied at the pile head (R) plus the NF and the resisting loads are the PF plus the tip resistance (C_P) (Figure 3). There is an equilibrium between the acting and resisting loads on the pile, this equilibrium is maintained as the neutral level moves and there is an increase or decrease in the magnitude of the resisting or acting loads.

Negative skin friction induces shear stresses on the walls of the structural elements (box, slab), which can compromise the structural integrity of the element and may induce additional settlements throughout the structure. Regional subsidence favors a separation between the pile and the slab. This exposes the piles to shear forces due to earthquakes; this situation is critical when the foundation design considers that the load will be transmitted to the ground through the foundation slab (Auvinet, Rodríguez and Ibarra, 2009).

5 ESTIMATION OF NEGATIVE SKIN FRICTION IN PILES GROUPS

5.1 *Analytical model to estimate negative skin friction according to current regulations in Mexico City*

The analytical method for estimating NF recommended in the Complementary Technical Standards for the Design and Construction of Foundations (GCDMX, 2023), which are part of the Construction Regulations of the City, was considered. According to these standards, NF can be estimated through trial and error by adjusting the depth z_0 of neutral level until equality is achieved:

$$\frac{\Sigma Q}{N_p} + NF = PF + C_p \tag{8}$$

where, ΣQ are the permanent design loads (e.g., the self-weight of the structure), plus variable design loads with average intensity; Np is the number of piles or piers; NF is the dragload on inner piles equal to the minimum value of the shaft resistance from the pile head to the neutral level ($C_f|_{D_f}^{z_0}$), or the increase in effective stress at the neutral level ($\Delta\sigma'_{z_0}$) due to a probable reduction in pore pressure or an overload in the surrounding ground multiplied by the tributary area (A_T) at the neutral depth and; for edge and corner piles, the NF load equal to shaft resistance from the pile head to the neutral level ($C_f|_{D_f}^{z_0}$); PF load equal to the shaft resistance from the neutral level to pile tip or pier ($C_f|_{z_0}^{D_f+L_P}$) with a resistance factor equal to 1 and C_P is the pile toe resistance considering a resistance factor equal to 1, L_P is the length of the piles or piers, D_f is the depth of the footing, slab or foundation box and PF is the positive skin friction load considering a factor equal to 1.

This method assumes that the structure is sufficiently stiff for the load to be distributed approximately equally across all piles.

6 NEGATIVE SKIN FRICTION IN PILED GROUPS IN SPATIALLY VARIABLE SOILS

For this study, the NF developed in a pile group was evaluated in accordance with the regulations in force in Mexico City. The NF was considered due to a piezometric lowering resulting from the extraction of water from the aquifers. The uncertainty on the soil undrained resistance profile was considered. To demonstrate the importance of a probabilistic approach, the same analysis was carried out using a numerical deterministic model.

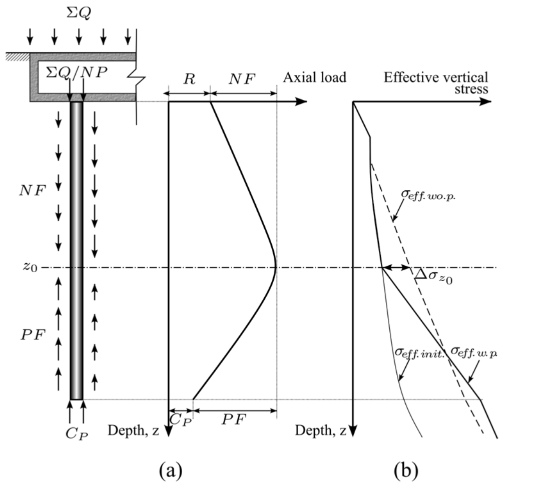

Figure 3. (a) Loads developed on a friction pile subjected to NF due to pore pressures drawdown; (b) Considerations for NF developed on a pile. (Auvinet and Rodríguez, 2017).

6.1.1 *Geotechnical model and piezometric conditions*

The study site is located at a point on the former Texcoco Lake. The stratigraphic profile is made up of five strata, the last one divided into four substrata (Rodríguez, 2010). From 0 to 2 m a Dry Crust (DC) is identified, formed by clays with an average water content (w) of 55 % and a very high pre-consolidation load due to drying and wetting periods. The water table level (WTL) is at a depth of 2 m. From 2 to 5 m is a clayey stratum called Crust (C) with a lower pre-consolidation load than the DC, but with an average w of 160 %. From 5 to 29 m depth is the Upper Clay Formation (UCF) which is subdivided into four substrata due to differences in water content and degree of pre-consolidation and thickness. UCF3.1 has a mean w of 340 % and an over-consolidation ratio, OCR, (pre-consolidation stress/vertical effective stress) of 1.6; UCF3.2 has a mean w of 350 % and an OCR of 1.1 and UCF3.3 and UCF3.4 have a mean w of 270 % and an OCR of 1.1. The latter substratum was divided into two to increase the accuracy of settlement estimation with an analytic-numerical method. The Hard Layer (HL) which lies between 29 and 31 m depth, is composed of clayey sands with a dense to very dense degree of compactness and a mean w of 30 %. Although the stratigraphic profile continues below, for the purposes of this paper it will only be considered up to a depth of 31 m.

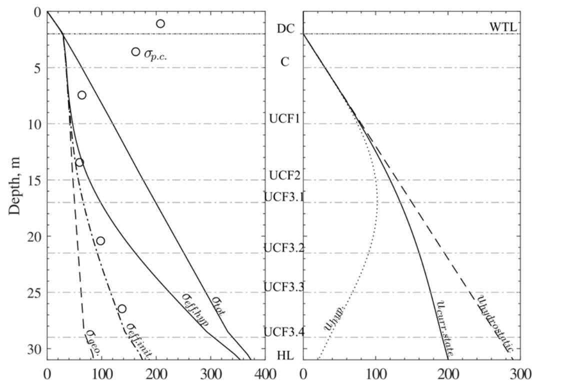

Figure 4. Stress and pore pressure conditions at the study site.

The WTL depth and initial pore pressure profile were obtained from measurements made in observation wells and open piezometers and reported in several soil mechanics studies. The initial piezometry is depressed with respect to hydrostatic. To account for regional subsidence, the hypotheses of future piezometric drawdown shown in Figure 4 were used. The hypothesis represents an extreme but possible case of drawdown.

6.1.2 *Piled foundation*

In Mexico City, friction pile foundations have been used either to reduce settlements (i.e. design in terms of deformations) or to support the structure's static and dynamic loads (i.e. design in terms of load capacity). Structures employing friction piles in Mexico City exhibit a range of heights and plan areas, extending from "light and short" structures with up to five levels to "heavy and tall" structures with up to 10 levels. These structures are characterized by plan areas ranging from 15 to 30 m.

For this study, a friction pile foundation slab for a 10-story building (load on slab of approximately 100 kPa) designed using bearing capacity criteria will be considered. The pile foundation has a plan area of 15 x 15 m and 25 piles with a square section of 0.4 m per side and 25 m in length (L_P) will be used to provide a 4 m space between the pile tip and the HL. The piles will be arranged in a square pattern with a spacing (S) of 3 m. The volumetric weight of the concrete is 24 kN/m^3.

6.2 *Probabilistic Model*

A *Matlab* code (MathWorks, 2022) was developed to simulate the random field representing the existing field properties, in this case the undrained soil resistance c_u. First, the trend of the original field is eliminated to obtain a residual field. Subsequently, an experimental correlogram (graphical representation of the autocovariance function that shows the linear dependence between the values of the property at different points of interest) is obtained. The correlation distance is determined from the area under the curve of the experimental correlogram. Once the correlation distance of the property is obtained, in this case in the vertical direction, the random field simulation is performed in one dimension using the development described in Section 2.2.1; this field has no trend (homogeneous field). Once the 1D random field simulation is obtained, the trend of the original field is reintroduced to represent the site conditions. For each realization of the random field, the NF value, the z_0 position, PF and C_P, are obtained in compliance with Equation (8).

6.3 *Deterministic Model*

For comparison purposes, a deterministic model (without considering spatial variability) was developed using the analytical model and the numerical modeling. The finite element software, *Plaxis 3D* (Plaxis BV, 2022), was used. The piles were modeled using volume elements, and the foundation slab was modeled using plate elements (Figure 5). It was assumed that the stiffness of the foundation slab is sufficiently rigid. This means that the load distribution is similar among all piles. The number of mesh elements in the model was approximately 300,000. The analysis was performed in terms of effective stresses, considering drained parameters. It was considered that the use of interface elements was not necessary because the behavior is more dependent on the compressibility of the material than on the shear strength of the soil (Rodríguez, 2010). The Mohr-Coulomb model was used to simulate the behavior of the DC, C, and HL strata, and the Soft-Soil model was used for the UCF strata.

The analysis stages are: Stage 1 is the initial state of stress, Stage 2 is the construction of the foundation with piles, Stage 3 is the application of a uniformly distributed load on the slab 100 kPa, in Stage 4, the load is maintained from stage 1 and the pore pressure is subjected to a drawdown with the hypothesis showed in Figure 3.

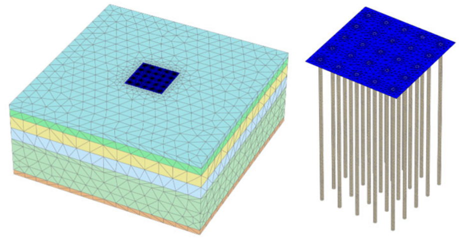

Figure 5. Numerical model of piled foundation.

7 RESULTS AND DISCUSSIONS

10,000 simulations of one-dimensional random fields of c_u were performed. Each field was used to estimate the NF with the corresponding position of z_0; likewise, the C_P and PF were obtained.

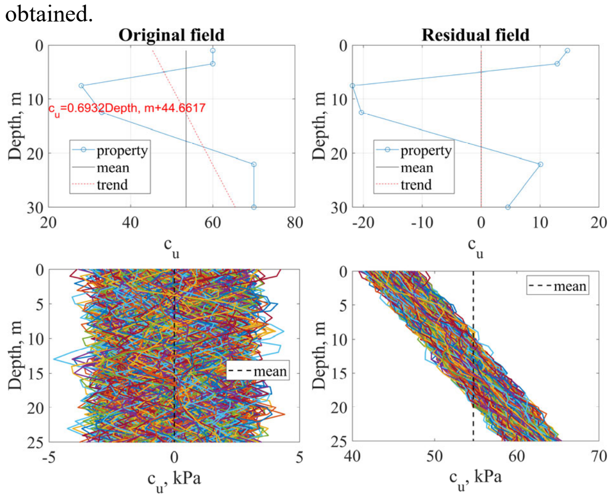

Figure 6. Original field (top left), residual field (top right), 10,000 simulations of the detrended (bottom left) and with trend (bottom right) fields.

Figure 7 shows the effect of the simulated 1D random fields on the position of the neutral level ($z_{0_{1DRF}}$) for the inner piles. This level varies from a maximum depth of $0.58L_P$ (14.51 m) to a minimum of $0.56L_P$ (13.94 m), with an average of $0.57L_P$ (14.23 m). If a deterministic model is considered, the neutral level ($z_{0_{det}}$) is at $0.58L_P$ (14.4 m). The above variation can be mainly attributed to the fact that the values of the simulated random field with trend have a range of 40 to 65 kPa. The deterministic model generally assumes average properties throughout the thickness, and therefore, the weighted average can be affected if the largest or smallest magnitude of a property corresponds to a thick stratum. To improve the accuracy of the C_f, the number of samples could be increased, especially at depths where greater uncertainty is observed.

A significant difference was observed in the position of the neutral level when using the analytical method for perimeter piles (edge and corner piles). This is because, when the neutral level is iterated without restricting the minimum value between $\Delta\sigma'_{z_0}$ and $C_f|_{D_f}^{z_0}$, the length at which FP develops must increase to maintain equilibrium.

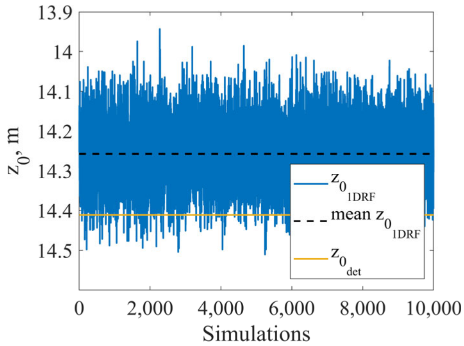

Figure 7. Neutral level position for the 10,000 simulations ($z_{0_{IDRF}}$) and for the deterministic model ($z_{0_{det}}$) for inner piles.

The probabilistic model accounts for variability through an autocovariance function that represents the degree of linear dependence between the values of the property of interest at two different points in the medium.

The results of the deterministic model, using numerical modeling, show that the problem of pile foundations is very complex since the group effect involves many variables. Some of them are the flexibility of the slab, the spacing between piles, and the position of the pile within the group. Figure 9 shows the results of the normalized axial load (Q_P/R, where Q_P is the axial load in the pile) for the loading stage (Stage 3) and the lowering stage (Stage 4). It is clearly observed that the load received by the pile at the head is transferred to the soil along the pile shaft. In general, the interior pile receives a lower load compared with the edge and corner piles. However, the flexibility of the slab or foundation box greatly influences the load distribution on the piles. This effect has been demonstrated in previous studies (Rodríguez, 2010 and Pineda-Núñez, 2016). The negative skin friction load can reach values as high as 1.8 times the load received by the pile at the head (stage 4). Additionally, it can be observed that the position of the neutral level is at the inflection point of the described curve of the normalized axial load. The neutral level average position of the numerical model with FEM3D ($z_{0_{det-FEM3D}}$) is 18.2 m, being 20.6 m for the interior pile, 17 m for the edge pile, and 16.98 m for the corner pile. It is important to note that the method proposed by current regulations does not consider foundation flexibility, whereas a model using FEM3D is affected by foundation stiffness. Consequently, the load on the pile head remains constant, and the load balance between NF and PF differs from the results of a numerical analysis.

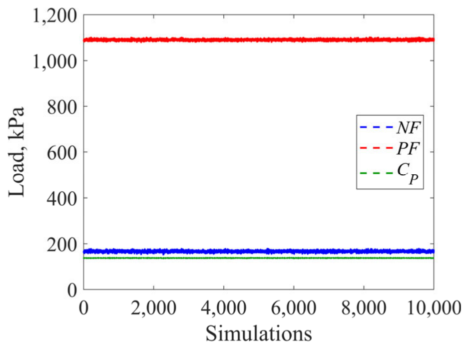

Figure 8. Loads due to NF, PF and C_P.

Regarding the settlement at the center of the foundation (x = y= 7.5 m), the deterministic methods estimate a settlement of 2.88 m for the analytical model and 2.23 m for the numerical method. The probabilistic model estimates an average settlement of 2.59 m. Similarly, random fields of soil compressibility parameters can be simulated to better understand the effects of uncertainty in the analysis of this type of foundation.

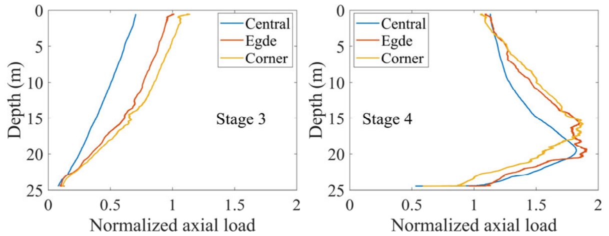

Figure 9. Normalized axial load in central, edge and corner piles obtained by FEM3D.

8 CONCLUSIONS

The present study demonstrates the usefulness of considering the inherent spatial variability in soils with a high degree of uncertainty. The inclusion of a one-dimensional random field to represent shear strength variability can be relatively straightforward without the need for cumbersome studies. With the inclusion of random fields in the analysis of geotechnical structures, the authors do not recommend decreasing the number of borings or sampling in the geotechnical exploration campaign. On the contrary, this type of analysis emphasizes that a geotechnical exploration campaign should be as complete as possible, especially in soils where there is a high degree of uncertainty due to the formation process.

9 REFERENCES

Aguayo, J., Marín, S. and Sánchez, F., 1989. Evolución geológica de la Cuenca de México. In: *Tópicos Geológicos de la Cuenca del Valle de México*. Sociedad Mexicana de Mecánica de Suelos. pp.25–42. (In Spanish)

Alarcón Posse, A.J., Rodríguez Rebolledo, J.F., Buriticá García, J.A., Caicedo Hormaza, B. and Rodríguez-Rincón, E., 2021. Validation of a 3D numerical model for piled raft systems founded in soft soils undergoing regional subsidence. *Soils and Rocks*, 44(1), pp.1–15. https://doi.org/10.28927/SR.2021.053620.

Alberro, J. and Hernández, R., 2000. Proceso de consolidación y fricción negativa inducida, en pilas, pilotes y muros. *Series del Instituto de Ingeniería*, (SID-619), p.63. (In Spanish)

Auvinet, G., 2002. *Decimosexta conferencia Nabor Carrillo: Incertidumbre en Geotecnia*. *Sixteenth Nabor Carrillo Lecture*. Querétaro: Sociedad Mexicana de Mecánica de Suelos.

Auvinet, G., 2019. Geotechnical Engineering in Spatially Variable Soft Soils. The Case of Mexico City. The 9th Arthur Casagrande Lecture. In: N.P. López-Acosta, ed. *XVI Pan-American Conference on Soil Mechanics and Geotechnical Engineering, Cancún, México*. Cancún: IOS Press. pp.5–107. https://doi.org/10.3233/ASMGE190004.

Auvinet, G. and Rodríguez, J.F., 2017. Criteria for the design of friction piles subjected to negative skin friction and transient loads. *Ingeniería, investigación y tecnología*, 18(3), pp.279–292. https://doi.org/10.22201/fi.25940732e.2017.18n3.025.

Auvinet, G., Rodríguez, J.F. and Ibarra, E., 2009. Effects of regional subsidence on foundations in Mexico City. In: G. Auvinet and M. Juárez Camarena, eds. *Geotechnical Engineering in urban areas affected by land subcidence*. Sociedad Mexicana de Mecánica de Suelos. pp.59–63.

Breysse, D., Niandou, H., Elachachi, S. and Houy, L., 2004. A generic approach to soil–structure interaction considering the effects of soil heterogeneity. *Géotechnique*, 54(2), pp.143–150.

Briaud, J.L., Jeong, S. and Bush, R., 1991. Group effect in the case of downdrag. In: *Geotechnical Special Publication*. Colorado: ASCE. pp.505–518.

Chow, Y.K., Lim, C.H. and Karunaratne, G.P., 1996. Numerical Modelling of Negative Skin Friction on Pile Groups. *Computers and Geotechnics*, 18(3), pp.201–224. https://doi.org/10.1016/0266-352X(95)00029-A.

Combarieu, O., 1988. Amélioration des sols par inclusions rigides verticales - Application à l'édification de remblais sur sols médiocres. *Revue Française de Géotechnique*, (44), pp.57–79.

El-Ramly, H., Morgenstern, N.R. and Cruden, D.M., 2002. Probabilistic slope stability analysis for practice. *Canadian Geotechnical Journal*, 39(3), pp.665–683. https://doi.org/10.1139/t02-034.

Fellenius, B., 1998. Recent advances in the design of piles for axial loads, dragloads, and settlement. In: *Proceedings of a Seminar by American Society of Civil Engineers, ASCE, and Port of New York and New Jersey*. p.18.

GCDMX, 2023. Normas Técnicas Complementarias para el Diseño y Construcción de Cimentaciones. p.117. (In Spanish)

Huber, M., Hicks, M.A., Vermeer, P.A. and Moormann, C., 2010. Probabilistic calculation of differential settlement due to tunnelling. In: L. Gucma, P. van Gelder and D. Proske, eds. *Proceedings of the 8th International Probabilistic Workshop, Szczecin 2010*. pp.1–13.

Jeong, S., Kim, S. and Briaud, J., 1997. Analysis of downdrag on pile groups by the finite element method. *Computers and Geotechnics*, 21(2), pp.143–161.

Lee, C.J., Bolton, M.D. and Al-Tabbaa, A., 2002. Numerical modelling of group effects on the distribution of dragloads in pile foundations. *Geotechnique*, 52(5), pp.325–335. https://doi.org/10.1680/geot.52.5.325.38704.

Marsal, R.J., 1975. The lacustrine clays of the valley of Mexico. *Series del Instituto de Ingeniería*, (E-16), p.30.

MathWorks, 2022. *Matlab R2022a (Versión 9.12.0.1884302) [Windows x64 bits]*. Available at: <https://la.mathworks.com/>.

Mesri, G., Rokhsart, A. and Bohort, B.F., 1975. Composition and compressibility of typical samples of Mexico City clay. *Géotechnique*, 25(3), pp.527–554.

Niandou, H. and Breysse, D., 2007. Reliability analysis of a piled raft accounting for soil horizontal variability. *Computers and Geotechnics*, 34(2), pp.71–80. https://doi.org/10.1016/j.compgeo.2006.09.006.

Ovando, E., Ossa, A. and Romo, M., 2007. The sinking of Mexico City: Its effects on soil properties and seismic response. *Soil Dynamics and Earthquake Engineering*, 27, pp.333–343. https://doi.org/10.1016/j.soildyn.2006.08.005.

Peck, R., Hanson, W.E. and Thornburn, T., 1974. *Foundation Engineering*. John Wiley and Sons.

Pineda, A. and Auvinet, G., 2013. Método del elemento finito estocástico en geotecnia. Enfoque espectral Stochastic Finite Element Method in Geotechnical Engineering. Spectral Approach. *Ingeniería Investigación y Tecnología*, XIV (númer, p.22. (In Spanish)

Pineda-Núñez, V.M., Auvinet, G. and Rodríguez, J.F., 2016. Fricción negativa en grupos de pilotes en la zona lacustre de la Ciudad de México. In: *XXVIII Reunión Nacional de Ingeniería Geotécnica - Mérida, Yucatán*. p.10. (In Spanish)

Plaxis BV, 2022. *Plaxis 3D Connect Edition (Version 2022.00.00.1733) [Windows x64 bits]*. Available at: <https://es-la.bentley.com/software/plaxis-3d/>.

Reséndiz, D. and Herrera, I., 1969. A probabilistic formulation of settlement-controlled design. 7th International Conference on Soil Mechanics and Foundation Engineering (México). Mexico City: International Society for Soil Mechanics and Geotechnical Engineering. pp.217–225.

Rodríguez, E., Cunha, R.P. and Caicedo, B., 2018. Behaviour of piled raft foundations systems in soft soil with consolidation process. In: A. McNamara, S. Divall, R. Goodey, S. Stallebrass and J. Panchal, eds. *Proceedings of the 9th International Conference on Physical Modelling in Geotechnics (ICPMG 2018)*. London: CRC Press. pp.1407–1411. https://doi.org/10.1201/9780429438646.

Rodríguez, J.F., 2010. *Modelado del comportamiento de pilotes e inclusiones sometidos a consolidación regional en la zona lacustre de la Ciudad de México*. PhD Thesis. Universidad Nacional Autónoma de México. (In Spanish)

Schlosser, F., Jacobsen, H.M. and Juran, I., 1984. Le renforcement des sols. *Revue Française de Géotechnique*, 29, pp.7–31.

Vanmarcke, E.H., 1977. Probabilistic Modeling of Soil Profiles. *ASCE J Geotech Eng Div*, 103(11), pp.1227–1246. https://doi.org/10.1061/ajgeb6.0000517.

Vázquez Sánchez, E. and Jaimes Palomera, L.R., 1989. Geología de la Cuenca de México. In: *Tópicos Geológicos de la Cuenca del Valle de México*. Sociedad Mexicana de Mecánica de Suelos. pp.1–24. (In Spanish)

Verruijt, A., 1969. A simplified elastic method for the calculation of negative skin friction on piles. In: *Specialty Sesssions Proceedings of 7th International Conference on Soil Mechanics and Foundation Engineering, México*. p.10.

Zeevaert, L., 1973. *Foundation Engineering for difficult subsoil conditions*. New York: Van Nostrand Reinhold Company.

Zemanová, A., Anderle, V. and Masopust, J., 2025. Settlement of piles with negative skin friction: design approaches according to part 3 of Eurocode 7. *Acta Polytechnica*, 64(6), pp.588–604. https://doi.org/10.14311/AP.2024.64.0588.