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Reloading Oedometer Modulus for Danish clay tills

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ABSTRACT: A new and general set of equations for assessment of the secant (reloading) oedometer modulus for preconsolidated Danish clay tills provides an improved prediction of the reloading modulus derived from incremental oedometer tests. The model requires information on Plasticity Index, undrained shear strength at unloading stress level (assessed using SHANSEP), preconsolidation stress and stress increment.

KEYWORDS: Clay Till, secant reloading oedometer modulus, undrained shear strength, SHANSEP

1 INTRODUCTION

In Danish practice, the oedometer modulus of Danish clay tills is usually determined from incremental-load oedometer tests. Tests are usually performed with loading to approximetely 80-90% of the estimated preconsolidation stress (taken as σ'_{pc}=3-4 times the field vane shear strength or σ'_{pc}~0.3q_{net} in CPT), followed by several cycles of unloading-reloading at decreasing unloading stress levels.

Based on a formula for recompression strain proposed by Brinch Hansen, Moust Jacobsen (1970) suggested a linear relationship between the Tangent Oedometer Modulus ($E_{oed,t}$) and the unloading stress, σ'_{unl}:

$$E_{oed,t} = \frac{\Delta\sigma'}{\Delta\varepsilon} = B + A\sigma'_{unl}$$
$$A \approx 211 \cdot e^{-1.256} \quad (1)$$
$$B \approx 0.6MPa \cdot e^{-1.256}$$

A common malpractice is related to uncritical use of Hansen (1978) and TEKNISK STÅBI (2011), where the coefficients for typical clay till are given as B=20MPa and A=1500. However, numerous examples show that no set of coefficients A & B is generally valid for clay tills e.g. Steenfelt & Christensen (1983) found B=5MPa and A=380-600 for one clay till (w=20%) and B=15MPa and A=1500 for a second clay till, whereas Sandgård et al. (1995) found B=30MPa and A=3000 for a third clay till location.

A commonly used "*rule of thumb model*" is to take the ratio (E_u/c_u) between the undrained Young's Modulus (E_u) and the undrained shear strength (c_u) as a constant, θ.

$$\frac{E_u}{c_u} = \theta$$
$$E_{oed} = \frac{2(1-\mu)}{3(1-2\mu)} \cdot E_u \quad , \mu=\text{Poisson's ratio} \quad (2)$$

According to Atkinson (2008) θ is often taken as θ=300 for a strain level of 0.1%. However, as both modulus E_u and undrained shear strength c_u depend on OCR (=$\sigma'_{pc}/\sigma'_{unl}$), and the ratio E_u/c_u varies with Plasticity Index (I_p) and strain level (ε) and test type, θ is not a "constant".

In Denmark the dominant test to determine the Oedometer Modulus is the incremental loading consolidation test. From such test the Reloading Oedometer Modulus is presented in terms of the Secant Oedometer Modulus $E_{oed,sec}$ calculated from the stress increments $\Delta\sigma'_{v,i} = \sigma'_{vi} - \sigma'_{unl}$ at each load increment at time of 100% primary consolidation:

$$E_{oed,sec,i} = \frac{\Delta\sigma'}{\Delta\varepsilon} = \frac{\sigma'_{v,i} - \sigma'_{unl}}{\varepsilon_{eop,i} - \varepsilon_{total}} \quad (3)$$

σ'_{unl} is the unloading stress (reference stress level) from which stress increments *i* is calculated. ε_{total} is the strain after unloading.

In the Danish Code of Practice (DS415:1984) the following equation is given for assessment of the Oedometer Modulus for inorganic clays using field vane shear strength c_v. It is not unusual in Danish Practice to see this equation applied for clay till as well (c_v~c_u):

$$E_{oed,sec} = \frac{4000}{w} \cdot c_v \quad , w=\text{water content} \quad (4)$$

To improve the assessment of the Oedometer Modulus for clay till, (when no or insufficient number of oedometer tests have been conducted for a site) HOFOR Earth Manual JOR 103 (2001) suggests deriving the Secant Oedometer Modulus $E_{oed,sec}$ as a function of the undrained shear strength of the clay till:

$$E_{oed,sec} = \left[0.0013 \cdot \left(\frac{c_u}{1kPa}\right)^2 + 0.0636 \cdot \left(\frac{c_u}{1kPa}\right) + 7.9478 + 1.5 \cdot \frac{\sigma'_{unl}}{1kPa}\right] \cdot 1MPa \quad (5)$$

However, none of the usually adopted models (eqs.1,2,4,5) present predictions of Oedometer Modulus that span satisfactorily the full test range of undrained shear strengths and unloading stress levels. Therefore, new models are proposed in this study to improve the assessment of the reloading Secant Oedometer Modulus for Danish clay tills, $E_{oed,sec}$. The new models consider unloading stress σ'_{unl}, preconsolidation stress σ'_{pc}, Plasticity Index I_p, the undrained shear strength in unloading $c_{u,unl}$ and the reloading stress increment $\Delta\sigma'$.

2 DANISH CLAY TILLS

Danish clay tills comprise a silty, sandy, gravelly clay of glacial origin and are mostly encountered in the Eastern part of Denmark. The fractions of clay, silt and sand are highly variable (e.g. Figure 1); clay 15-25%, silt 25-50%, sand 40-60%. Plasticity Index typically ranges 5-15%, void ratio typically ranges 0.25-0.4. Specific gravity G_s~2.7. Carbonate content varies 0-20%. Preconsolidation stress for Danish clay tills is typically in the range σ'_{pc}=500kPa to 2000kPa.

It is obvious that the variability in "clay till" properties will constitute a challenge for any modelling of the Oedometer Modulus, and any "general model" will show a significant scatter. However, if such a general model is calibrated for a particular clay till, the prediction of $E_{oed,sec}$ can be improved and hence the scatter reduced (e.g. *B1-model* in this study).

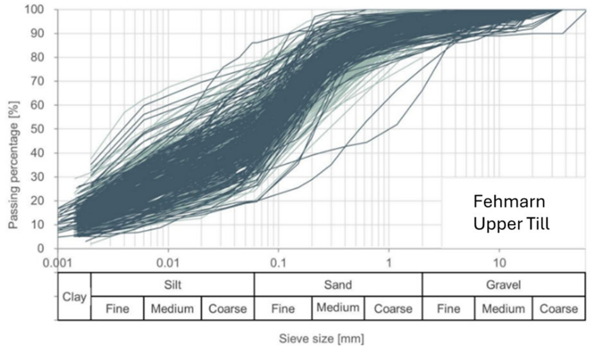

Figure 1. Examples of grain distribution curves for clay till. Fehmarn Upper Till (from Fehmarn General GIR).

3 UNDRAINED SHEAR STRENGTH

The SHANSEP variation of the undrained shear strength (c_u) with effective vertical stress (σ'_v) by (Ladd & Foot, 1974) has been adopted in Danish Practice for Danish clay tills e.g. Steenfelt & Foged (1992):

$$\frac{c_{u,SHANSEP}}{\sigma'_v} = \alpha \cdot (OCR)^{\lambda} \qquad 6)$$

The undrained shear strength $c_{u,SHANSEP}$ corresponds to the undrained shear strength at the unloading stress $\sigma'_v=\sigma'_{unl}$ ($OCR=\sigma'_{pc}/\sigma'_{unl}\geq 1$). In the new models for $E_{oed,sec}$, presented in this study, the undrained shear strength $c_{u,SHANSEP}$ is used to address the undrained shear strength of the sample at the unloading stress σ'_{unl} in the test:

$$c_{u,unl} = c_{u,SHANSEP} = \alpha\sigma'_{unl}(OCR_{unl})^{\lambda} \qquad (7)$$

The coefficients α and λ was assessed for clay tills in relation to the bridge and tunnel crossing the Great Belt (PI=3-9%) as α=0.42 and λ=0.85 for CAU_c triaxial tests (Steenfelt & Foged (1992), see Figure 2. The Great Belt coefficients appear to describe a lower bound envelope to the 246 CAU_c triaxial tests on Danish clay tills. There appears to be no recognizable effect of the Plasticity Index (I_p=PI) as shown in Figure 3.

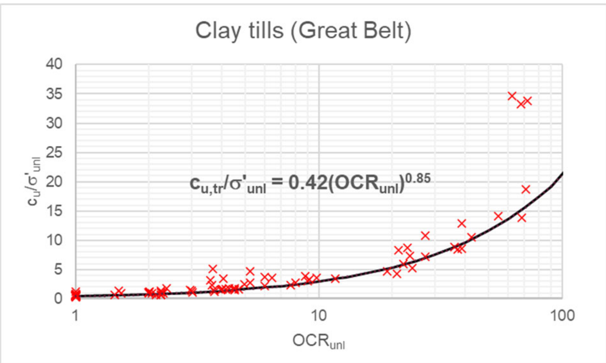

Figure 2. "Standard" SHANSEP equation evolved from undrained, anistorpically consolidated triaxial compression tests (CAU_c) on Great Belt clay till (Storebælt Bridge & Tunnel project)

Table 1 shows the SHANSEP coefficients adopted for the clay till sites providing data to this study. α=0.42 and λ=0.85 have been adopted for most of the sites. Sites with a significant number of test data is named (Fehmarn, Great Belt) and within the Copenhagen area (Nordhavnen, Carlsberg). Minor sites within the Copenhagen area are compiled in "Copenhagen" and remaining data compiled in "other locations". Figure 4 shows the site specific SHANSEP equation for the clay till "Fehmarn Upper Till" used for the *B1-model* in this study.

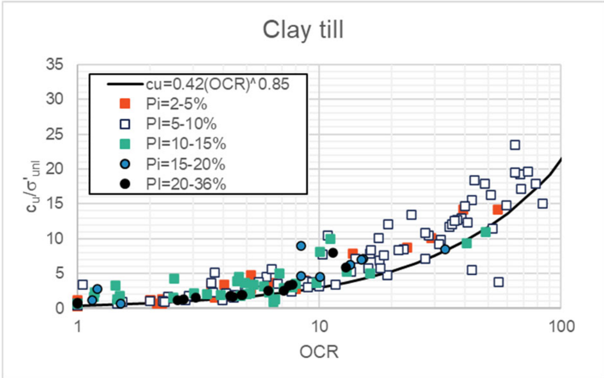

Figure 3. SHANSEP equation with α=0.42 and λ=0.85 compared to undrained shear strength assessed in 246 CAU_c triaxial tests on Danish clay tills from locations listed in Table 1.

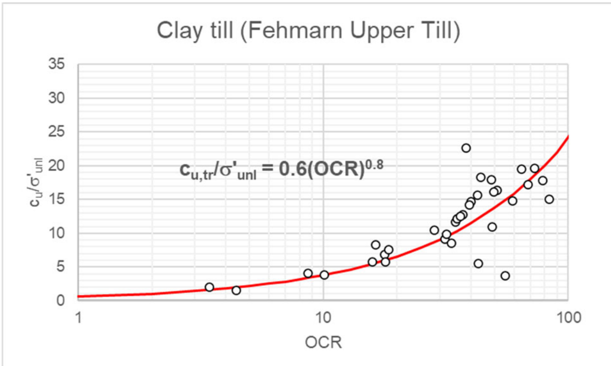

Figure 4. SHANSEP equation with α=0.6 and λ=0.8 for Fehmarn Upper Till. This model is used in the *B1-model* in this study

Table 1. Adopted SHANSEP coefficients.

	PI(%)	α	λ
Fehmarn Upper Till	4-13	0.60	0.80
Great Belt	3-9	0.42	0.85
Nordhavnen	10-15	(0.42)	(0.85)
Carlsberg	10-16		
Copenhagen	6-23		
Other locations	8-14		
Fehmarn Lower Till	8-23		
Fehmarn Chalk Till	6-8		
Fehmarn Lowermost Till	23-60		

4 SECANT OEDOMETER MODULUS

In relation to the immersed tunnel project at Fehmarn Belt the following model (χ*-model* eqs.8-10) was developed by COWI (2022) and adopted to calculate the Secant Oedometer Modulus, $E_{oed,sec}$ for clay till:

$$E_{oed,sec,\chi} \geq \left[10 + \chi \cdot \left(\frac{\sigma'_{pc}}{1kPa}\right)^{0.4} \cdot \left(\frac{\sigma'_{unl}}{\sigma'_{pc}}\right)^{\psi}\right] \sigma'_{pc} \qquad (8)$$

$$\psi = 0.31 \cdot \left(\frac{I_p}{1\%}\right)^{0.44}, \; I_P = 5.5\text{-}60\% \qquad (9)$$

$$\chi = 45.157 + 0.0256 \cdot \left(\frac{\Delta\sigma'}{1kPa}\right) \leq 55 \qquad (10)$$

The preconsolidation stress ($\sigma'_{pc,C}$) derived from Casagrande interpretation of the oedometer test is used, or, if specimen unloading is performed from a stress level $\sigma'_{max} > \sigma'_{pc,C}$, the preconsolidation stress felt by the sample is taken as σ'_{max}.

$$\sigma'_{pc} = max(\sigma'_{pc,C}, \sigma'_{max}) \qquad (11)$$

A comparison of Secant Oedometer Modulus $E_{oed,sec}$ calculated in incremental oedometer tests and values predicted using the χ-model is shown in Figure 5 (log-log plot) and Figure 6 (linear plot). The same data have been used in both plots. Given the highly variable classification parameters for clay till, it is unavoidable that modelling of stiffness and strength using simple models will show some scatter.

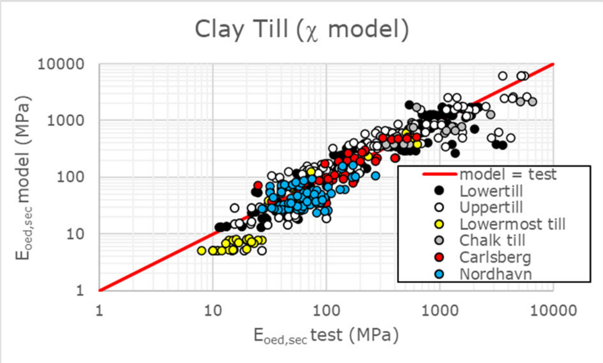

Figure 5. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in tests and as modelled using the χ-model

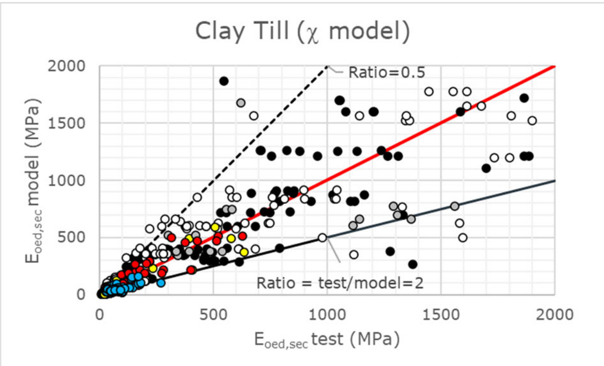

Figure 6. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in tests and as modelled using the χ-model. Legend as in Figure 5

Figure 7 and Figure 8 show that at low undrained shear strength (c_u<150kPa) the *χ-model* (eqs.8-10) tends to underestimate the Secant Oedometer Modulus, $E_{oed,sec}$. The undrained shear strength at time of unloading, $c_{u,unl}$ is calculated for each specimen using the relevant SHANSEP equation.

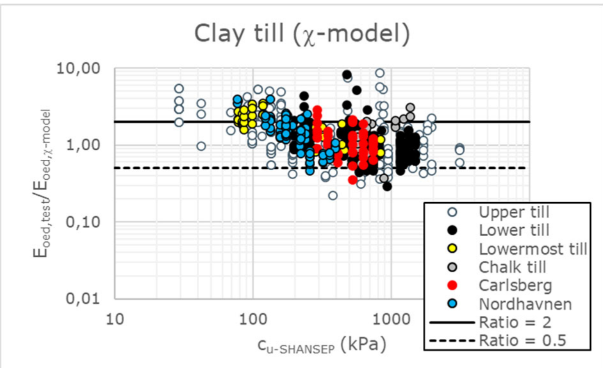

Figure 7. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in tests and as modelled using the χ-model. $c_{u,SHANSEP} = \alpha\sigma'_{unl}(OCR_{unl})^{\lambda}$

To improve the prediction of the Secant Oedometer Modulus, the χ-model is supplemented by a model (*B2-model*) using the undrained shear strength c_u as input:

$$E_{oed,sec,B2} = 900 \cdot \sigma'_{unl} \cdot \left(\frac{c_{u,SHANSEP}}{\sigma'_{unl}}\right)^{0.375} \qquad (12)$$

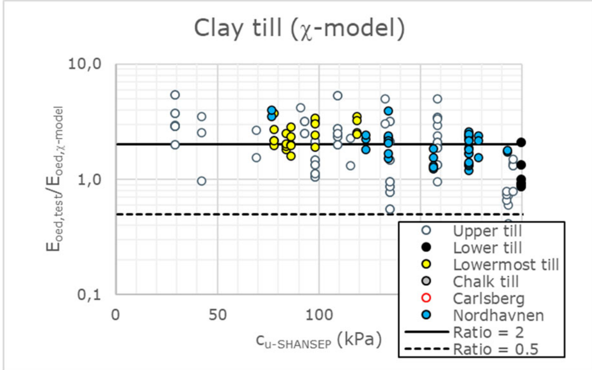

Figure 8. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in tests and as modelled using the χ-model. $c_{u,SHANSEP} = \alpha\sigma'_{unl}(OCR_{unl})^{\lambda}$

Figure 9 shows a comparison of $E_{oed,sec}$ from tests ($E_{oed,test}$) and values modelled by (a) JOR103 (eq.5), (b) K-equation from DS415 (eq.4), and (c) $E_{oed,sec,B2}$ (eq.12). The prediction of the Secant Oedometer Modulus has improved for c_u<150-200kPa using *B2-model* (Figure 9c and Figure 10a) compared to the *χ-model* (Figures 7-8) but not for c_u>150-200kPa, thus suggesting that a combo of the two models (*χ-B2 model*) may be better. Figure 9 shows that both JOR103 (eq.5) and K-equation (eq.4) have a higher tendency to overestimate the Secant Oedometer Modulus than the B2-model (eq.12).

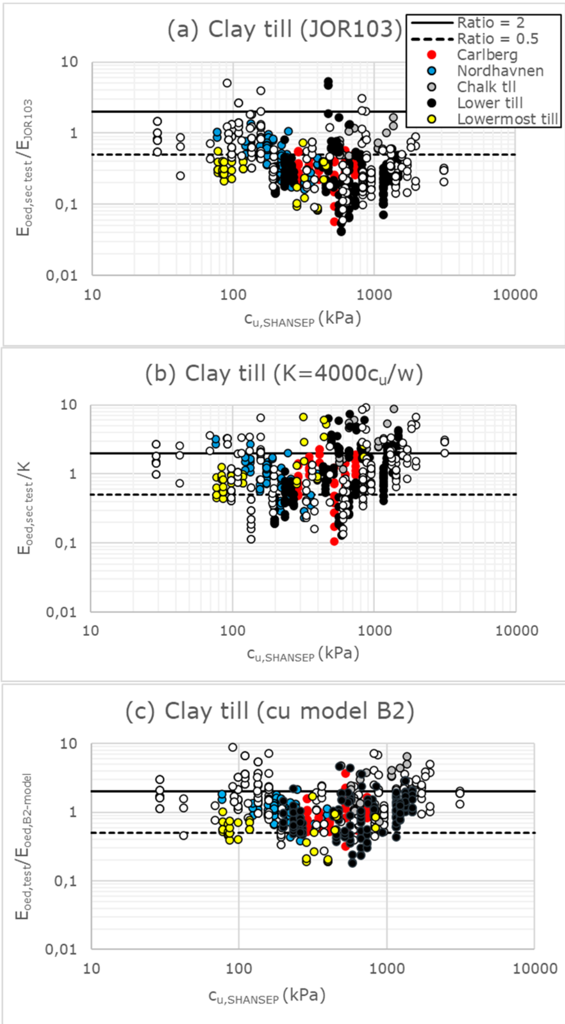

Figure 9. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in tests and as modelled using (a) JOR103 (eq.5), (b) K-equation (eq.4) and (c) B2-model (eq.12). $c_{u,SHANSEP} = \alpha\sigma'_{unl}(OCR_{unl})^{\lambda}$

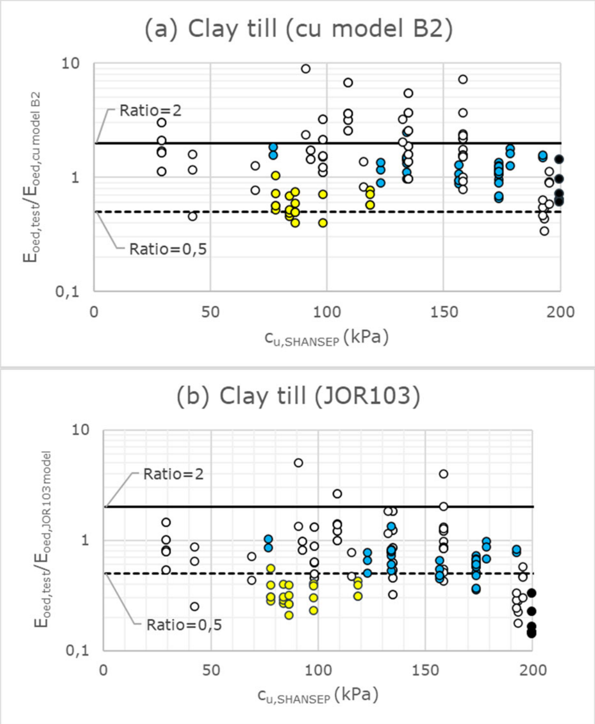

Figure 10. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in tests and as modelled using B2-model (a) and using JOR103 (b) using undrained shear strength $c_{u,SHANSEP} = \alpha\sigma'_{unl}(OCR_{unl})^{\lambda}$<200kPa. Legend as in Figure 8.

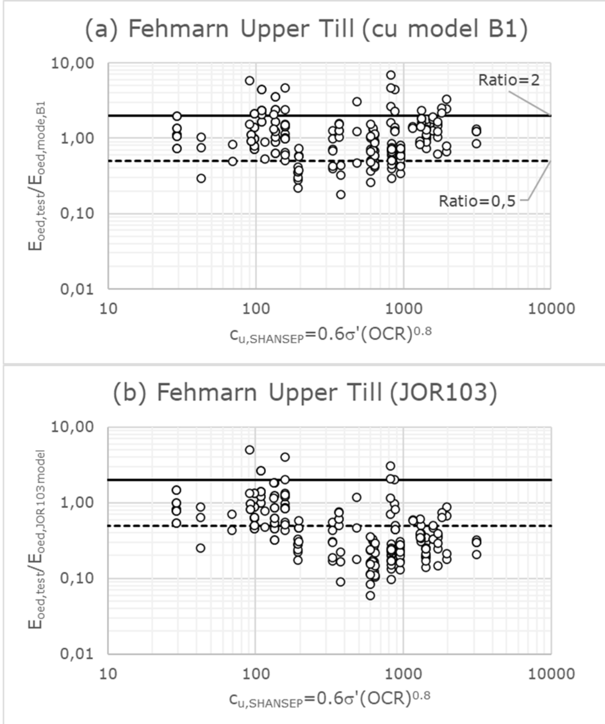

Figure 11. Comparison of Secant Oedometer Modulus in tests on Fehmarn Upper Till, with values obtaining using (a) B1-model and (b) JOR103, using relevant SHANSEP undrained shear strength given by $c_{u,SHANSEP} = 0.6\sigma'_{unl}(OCR_{unl})^{0.8}$

Figure 10 shows a comparison of $E_{oed,sec}$ from tests ($E_{oed,test}$) and values modelled by *B2-model* and JOR103 for c_u<200kPa (same test data as in Figure 8). The *B2-model* is more conservative than JOR103. If sufficient data are available, the *B2-model* can be calibrated for a specific clay till e.g. *B1-model* for Fehmarn Upper Till:

$$E_{oed,sec,B1} = 1392 \cdot \sigma'_{unl} \cdot \left(\frac{c_{u,SHANSEP}}{\sigma'_{unl}}\right)^{0.375} \quad (13)$$

Figure 11a shows a comparison of $E_{oed,sec}$ from tests ($E_{oed,test}$) and values modelled by the *B1-model,* $E_{oed,sec,B1}$, for Fehmarn Upper Till. Using the same data, Figure 11b) shows a comparison of $E_{oed,sec}$ from the same tests and values modelled by JOR103. Figure 11a shows that using a clay till specific model (here *B1-model* for Fehmarn Upper Till) improves the accuracy of the model compared to *B2-model* (Figure 9).

Comparing Figure 11a and Figure 11b it appears that for Fehmarn Upper Till *B1-model* and JOR103 are fairly consistent for c_u<200kPa, but for larger undrained shear strength JOR103 significantly overestimates $E_{oed,sec}$.

Figure 12 shows the ratio between $E_{oed,sec}$ from tests and predicted $E_{oed,sec}$ using the clay till specific *B1-model* $E_{oed,sec,B1}$ for Fehmarn Upper Till and values from *B2-model* for other clay tills, $E_{oed,sec,B2}$. Although neither of the B-models account for the reloading stress increments Δσ', Figure 12 shows that predictions appear unaffected by the magnitude Δσ' for clay till with c_u<200kPa.

To obtain better coverage of the full c_u-range, it is suggested to predict $E_{oed,sec}$ using a combination of the *χ-model* with the general *B2-model* (or a clay till specific model like the *B1-model*), i.e. combo *χ-B model*:

$$E_{oed,sec,\chi-B} = \max\left(E_{oed,sec,B}, E_{oed,sec,\chi}\right), \; c_{u,unl}<150\text{kPa} \quad (14)$$

$$E_{oed,sec,\chi-B} = E_{oed,sec,\chi}, \; c_{u,unl}\geq 150\text{kPa} \quad (15)$$

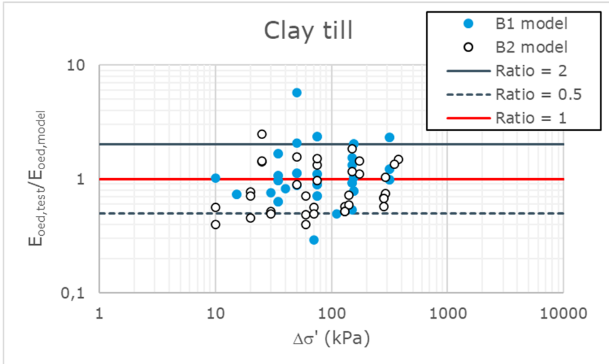

Figure 12. Comparison of Oedometer Modulus, $E_{oed,sec}$ in tests with c_u<150kPa using B1-model for Fehmarn Upper Till and B2-model for other clay tills.

Figure 13 shows the distribution of the ratio $E_{oed,test}/E_{oed,sec,model}$ using the χ-B model (eqs.12-15), JOR103 (eq.5) and K-model (eq.4) to assess $E_{oed,sec,model}$ in 531 test cases. For the χ-B model Figure 13 shows that the ratio $E_{oed,test}/E_{oed,sec,\chi-B}$ is larger than 2 in 64 cases (~12%) and less than 0.5 in 34 cases (~6%).

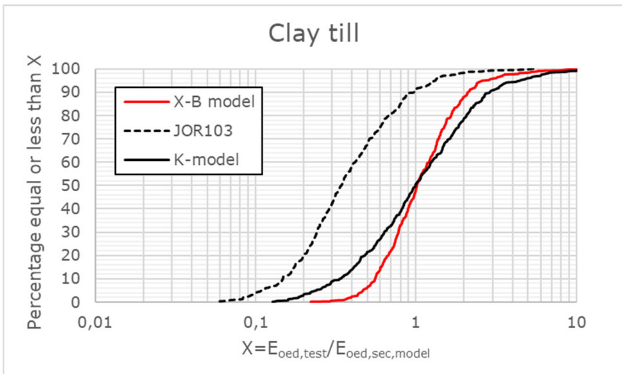

Figure 13. Distribution of $E_{oed,test}/E_{oed,sec,model}$ using 531 values of $E_{oed,test}$

Using the combo-model (χ-*B1 model* and/or χ-*B2 model*), a comparison with test results is shown in Figures 14a-14d with the ratio $E_{oed,test}/E_{oed,sec,\chi\text{-}B}$ shown as function of c_u in unloading, I_P, reloading stress increment $\Delta\sigma'$ and σ'_{pc}. Figure 14 shows that there appears to be no evident influence on the accuracy of the prediction of $E_{oed,test}$ within the test ranges of c_u=29-3125kPa, I_P=4-60%, $\Delta\sigma'$=10-3800kPa or σ'_{pc}=84-5976kPa.

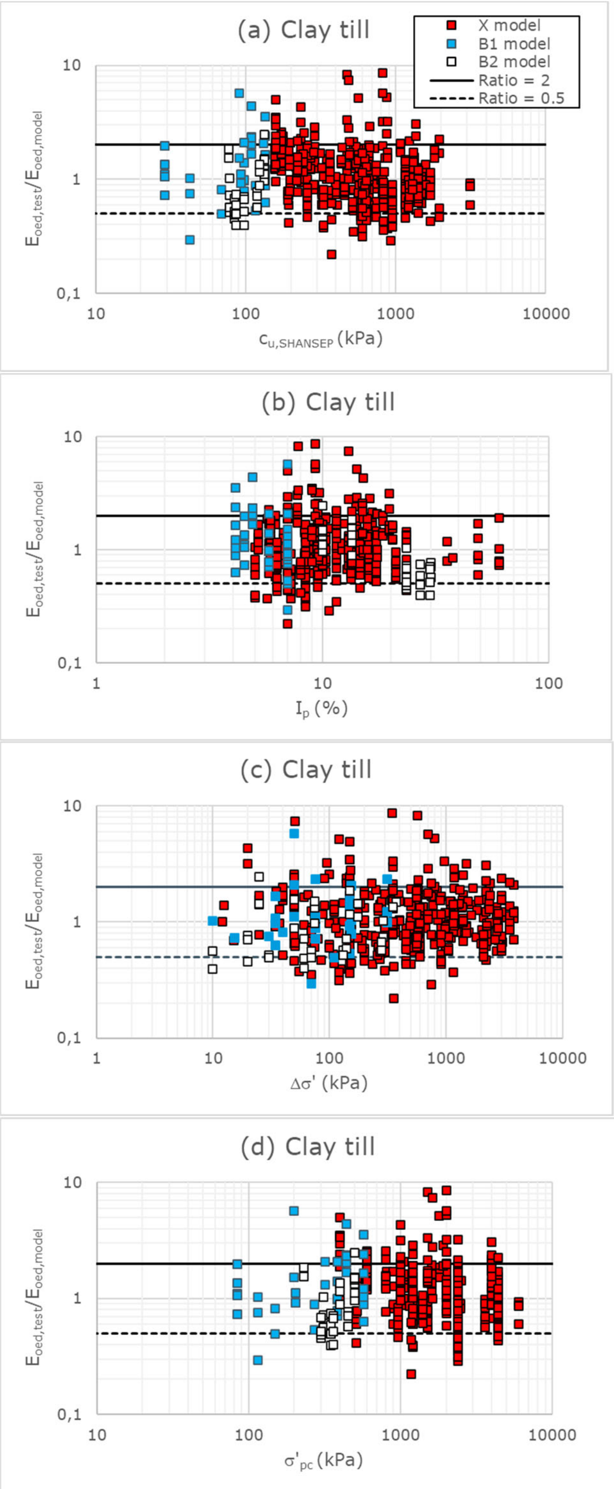

Figure 14. Comparison of Secant Oedometer Modulus in tests and $E_{oed,sec}$ using combo models: χ-B1 model for Fehmarn UT and χ-B2 model for other clay tills. 531 values of $E_{oed,test}$. B1-model and B2-model applied for c_u<150kPa and. χ model applied for c_u> 150kPa

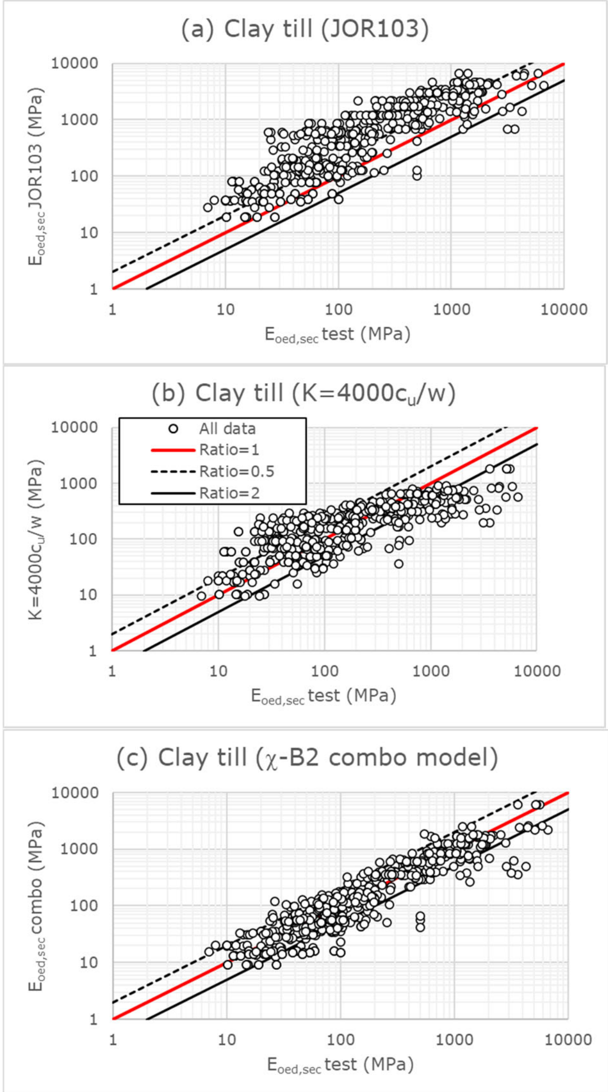

Figure 15. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in all tests and modelled using (a) JOR103, (b) K-equation (DS415) and (c) χ-B2 model. Ratio=$E_{oed,test}/E_{oed,model}$

Figures 15-17 shows a comparison of all test data ($E_{oed,test}$) to the predicted secant reloading oedometer modulus using the combo model χ-*B2 model* ($E_{oed,sec,\ \chi\text{-}B2}$) to K-model (DS415) and JOR103 ($E_{oed,sec,JOR103}$). Figures 15-17 also allows for a direct visual comparison of the accuracy of the various models.

Figure 15a shows that JOR103 generally overestimates the test values, but that there appears to be no apparent influence of the magnitude of the Secant Oedometer Modulus. For the K-model (Figure 15b) the K-model appears to tend to be more conservative for $E_{oed,test}$>100MPa more unconservative for $E_{oed,test}$<100MPa. The χ-*B2 model* (Figure 15c) predictions appear to be evenly distributed within the range 0.5 $E_{oed,test}$ to 2$E_{oed,test}$ for all levels of $E_{oed,test}$.

Figure 16 shows that χ-*B2 model* (Figure 16c) predictions are evenly distributed within the 0.5-2$E_{oed,test}$ for all levels of undrained shear strength. Same goes for the K-model (Figure 16b) but with significantly larger scatter and larger number of values >2$E_{oed,test}$. The JOR103 model (Figure 16a) shows increasing overestimation with increasing undrained shear strength.

Figure 17c shows the same trend as in Figure 16, with χ-*B2 model* predictions evenly distributed within the 0.5-2$E_{oed,test}$ for all levels of I_P, larger scatter for K-model (Figure 17b) and marked effects on prediction quality for JOR103 (Figure 17a).

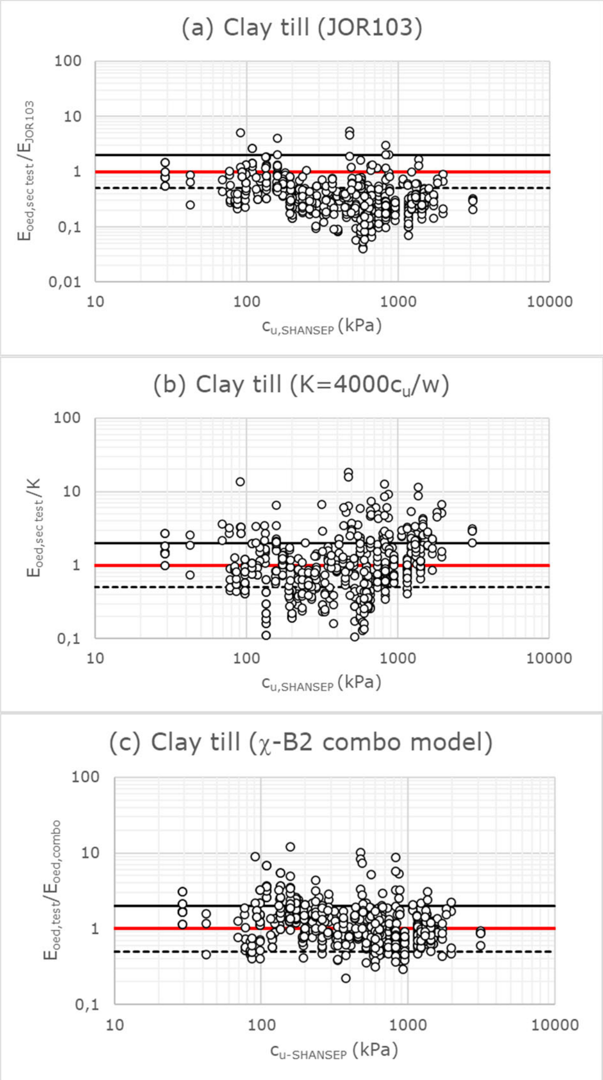

Figure 16. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in all tests and modelled using (a) JOR103, (b) K-equation (DS415) and (c) χ-B2 model, as function of the c_u in unloading. Legend as for Figure 17

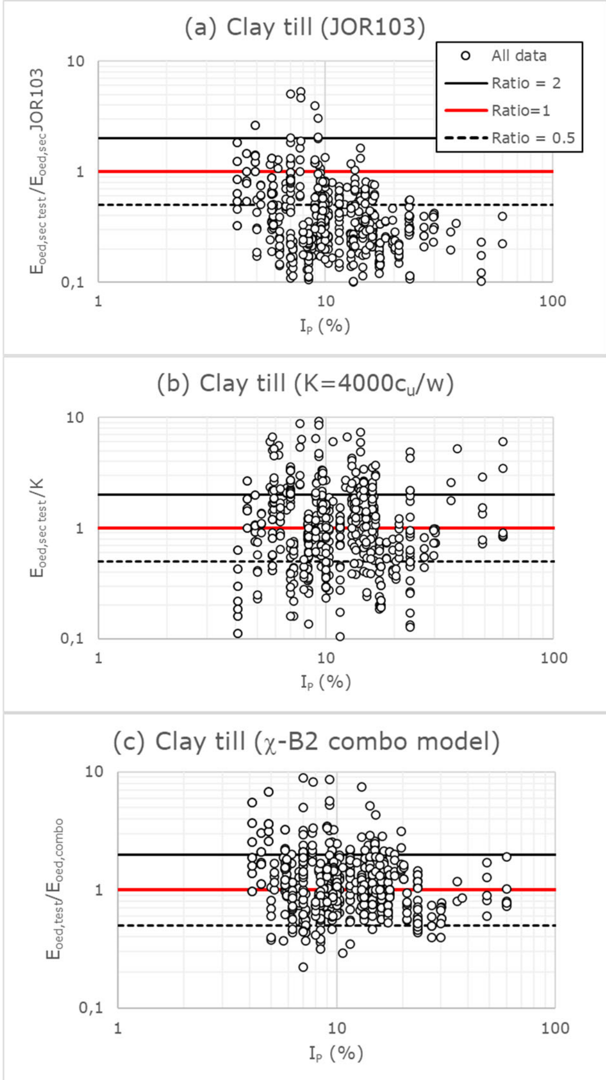

Figure 17. Comparison of Secant Oedometer Modulus, $E_{oed,sec}$ in all tests and modelled using (a) JOR103, (b) K-equation (DS415) and (c) χ-B2 model, as function of the Plasticity Index, I_p

5 CONCLUSIONS

The combination of Secant Oedometer Modulus $E_{oed,sec,\chi}$ from *χ-model* (eqs.8-10) and $E_{oed,sec,B}$ from *B1/B2-models* (eqs.12-13) i.e. the combo *χ-B model* (eqs.14-15), enables an improved assessment of the reloading Secant Oedometer Modulus compared to two of the equations often used in Danish Practice.

$$E_{oed,sec,\chi-B} = \max\left(E_{oed,sec,B}, E_{oed,sec,\chi}\right) \text{ , } c_{u,unl}<150\text{kPa} \quad (14)$$

$$E_{oed,sec,\chi-B} = E_{oed,sec,\chi} \text{ , } c_{u,unl}\geq 150\text{kPa} \quad (15)$$

$$E_{oed,sec,B2} = 900 \cdot \sigma'_{unl} \cdot \left(\frac{c_{u,SHANSEP}}{\sigma'_{unl}}\right)^{0.375} \text{ , B=B2} \quad (12)$$

$$E_{oed,sec,B1} = 1392 \cdot \sigma'_{unl} \cdot \left(\frac{c_{u,SHANSEP}}{\sigma'_{unl}}\right)^{0.375} \text{ , B=B1} \quad {}^*(13)$$

*eq.13 applies only for Fehmarn Upper Till

In more than 80% of the investigated cases the predicted Secant Oedometer Modulus using *χ-B model* $E_{oed,sec,\chi\text{-}B}$ fall within the range of 0.5 to 2.0 times the test value $E_{oed,test}$. In ~6% of the 531 cases analyzed the *χ-B model* provides predictions larger than 2 times $E_{oed,test}$.

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