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Enhanced assessment and mitigation design of slopes and embankments using globally optimized strengthening profiles

Colin Smith
School of Mechanical, Aerospace and Civil Engineering, University of Sheffield, United Kingdom,
c.c.smith@sheffield.ac.uk

Jozef Jurko
Colas Central Europe, Budapest, Hungary

ABSTRACT: Uncertainty is a major feature in geotechnical design, particularly in situations involving complex geometry, stratigraphy and loadings. This can impact the planning of the site investigations, and design of mitigation/strengthening measures to avoid collapse. In this paper the use of a robust global optimization technique is applied to the collapse analysis of a slope in order to identify the zones within the soil body where strength parameter variation has the greatest influence on overall stability. This approach generates a visual map that can be used to indicate areas where site investigation could be optimally focused. It can also be used to determine where it would be most efficient to add strengthening measures in situations where the slope requires stabilization, or in the design of new slopes or embankments using discrete zones of e.g. lime/cement stabilized soil, it can be used to determine the optimal target strength for these zones. An overview of the underlying theory is presented, and the potential benefits of the approach are illustrated through consideration of a practical case study of a slope failure on a clayey mine spoil heap.

KEYWORDS: Slope stability, optimization, stabilization, design.

1 INTRODUCTION

Uncertainty is a major feature in geotechnical design, particularly in situations involving variability in ground properties, complex geometry, and varied loadings. For slope and embankment stability problems, this can impact the planning of the site investigations, and design of mitigation/strengthening measures to avoid collapse.

While application of statistical approaches can assist in addressing issues of uncertainty, other numerical techniques can also play a useful role. In this paper the use of a robust global optimization technique is applied to the collapse analysis of a slope in order to identify the zones within the soil body where strength parameter variation has the greatest influence on overall stability.

Such information can then be used to design an efficient site investigation campaign and/or to target specific zones for strengthening. In the case where a slope or embankment is to be constructed, it enables efficient targeting of materials and associated techniques such as lime/cement stabilization to specific layers/zones of soil.

The approach is well suited to computational Limit Analysis approaches that already employ optimization to identify the critical collapse mechanism. Such methods include Finite Element Limit Analysis (FELA) and Discontinuity Layout Optimization (DLO, Smith & Gilbert 2007). The latter approach will be utilized in this paper. The results are generated using a development version of the LimitState:GEO software (LimitState 2021). In this paper this method will be termed Optimized Strength Distribution (OSD).

Previous related work has focused more on topology optimization-based approaches and optimal design of foundations, for example (Kammoun et al. 2019, and Li et al. 2024).

An overview of the underlying theory is first presented and the potential benefits of the approach are illustrated through description of how it could be applied to a practical case study of slope failure on a clayey mine spoil heap.

2 CONVENTIONAL SLOPE ANALYSIS

A common approach to stability assessment of slopes is to undertake a c-ϕ reduction analysis which gives the factor of safety against whole slope collapse on soil strength. Coupled with appropriate design code partial factors, such an analysis can determine whether the slope requires strengthening to meet the required safety level.

However, such an approach treats the whole slope as one single entity. A more informed approach is to determine where in the system strength is actually required, and where it is less critical.

Within the context of optimization based plastic analysis/design, such a problem can be formulated. In this paper the DLO analysis approach applied to the undrained stability of a slope/embankment is presented.

3 DISCONTINUITY LAYOUT OPTIMIZATION

3.1 *Overview of standard theory*

Stages in the standard DLO procedure are briefly outlined schematically in Figure 1 for the plane strain translational case. In the procedure, the plastic limit analysis problem is couched in terms of the potential slip-line discontinuities which connect nodes used to discretize the region under consideration. The critical layout of discontinuities is identified using optimization to minimize the energy dissipated in the mechanism.

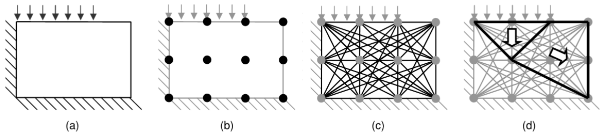

Figure 1. Stages in DLO procedure (after Smith & Gilbert 2007): (a) starting problem (surcharge applied to block of soil close to a vertical cut); (b) discretization of soil using nodes; (c) interconnection of nodes with potential discontinuities; (d) identification of critical subset of potential discontinuities using optimization (giving the layout of slip-lines in the critical failure mechanism).

The DLO formulation has a kinematic form and an exact mathematical dual equilibrium static form. In the development of the theory in this paper it is clearer to work with the equilibrium static form. The general equilibrium static formulation for 2D problems can be stated as follows (Smith and Gilbert 2007, 2022):

$$\max \lambda \tag{1}$$

subject to:

$$\mathbf{B}^{\mathbf{T}}\mathbf{t}+\lambda\mathbf{f}_{\mathrm{L}}-\mathbf{q}=-\mathbf{f}_{\mathrm{D}} \tag{2}$$

$$\boldsymbol{N}^{\boldsymbol{T}}\mathbf{q}\leq\mathbf{g} \tag{3}$$

where λ is the unknown load factor at collapse, $\mathbf{f}_{\mathrm{D}}$ and $\mathbf{f}_{\mathrm{L}}$ are vectors containing respectively specified dead and live loads at discontinuities, $\mathrm{t}^{\mathrm{T}}=\{\mathrm{t}_1^{\mathrm{x}},\mathrm{t}_1^{\mathrm{y}},\mathrm{t}_2^{\mathrm{x}},\mathrm{t}_2^{\mathrm{y}}\ldots\mathrm{t}_{\mathrm{n}}^{\mathrm{y}}\}$ where $\mathrm{t}_{\mathrm{j}}^{\mathrm{x}}$ and $\mathrm{t}_{\mathrm{j}}^{\mathrm{y}}$ can be interpreted as *x* and *y* direction stress function gradients acting at node *j* (*j*=1...*n*), and where ***q*** is here a vector of forces: $\mathrm{q}^{\mathrm{T}}=\{\mathrm{S}_1,\mathrm{N}_1,\mathrm{S}_2,\mathrm{N}_2,\ldots\mathrm{N}_{\mathrm{m}}\}$, where S_i and N_i represent respectively the shear and normal force acting on discontinuity *i*. $\mathbf{B}^{\mathbf{T}}$ is a suitable equilibrium matrix containing direction cosines and $\mathbf{N}^{\mathbf{T}}$ is a suitable yield matrix. For an undrained problem with uniform soil strength c_{u}, **g** is a vector ($c_{\mathrm{u}}l_1$, $c_{\mathrm{u}}l_1$, $c_{\mathrm{u}}l_2$, … $c_{\mathrm{u}}l_{\mathrm{m}}$), where l_{i} is the length of discontinuity *i* and there are *m* discontinuities. Equation 3 essentially implements the equation - $c_{\mathrm{u}}\leq S\leq c_{\mathrm{u}}$.

In qualitative terms, the method seeks to maximize the live loads (equation (1)) while not violating yield on any slip line (equation (3)) where shear and normal forces are computed via the stress functions which ensure equilibrium in equation (2). While at first sight this may appear to be a lower bound formulation, it is an upper bound since yield is not checked everywhere (Smith and Gilbert 2022).

The method is fully flexible in that it can use any distribution of nodes. However, in this paper a rectangular distribution of nodes will be adopted.

3.2 *Modified OSD theory*

It is assumed that the overall domain is rectangular and that a grid of rectangular sub-zones is laid out in a uniform pattern across the domain, each one centred on a node *j*. All sub-zones will thus sit adjacent to each other. Each sub-zone has an area *A* and an additional shear strength $r_j\geq 0$ above a specified baseline value. The aim is to minimize the sum of additional strengths, under a fixed load ***f***, while ensuring that yield is not violated along any discontinuity.

The revised kinematic formulation for 2D translational problems can be stated as follows:

$$\min A\sum_{j=1}^{n} r=\min \boldsymbol{r}^{T}\boldsymbol{a} \tag{4}$$

subject to:

$$\mathbf{B}^{\mathbf{Tt}}+\mathbf{f}-\mathbf{q}=0 \tag{5}$$

$$\boldsymbol{N}^{\boldsymbol{T}}\mathbf{q}\leq\boldsymbol{g}_{\boldsymbol{c}}+\boldsymbol{P}^{\boldsymbol{T}}\boldsymbol{r} \tag{6}$$

where **r** is a vector of LP variables representing the additional shear strength r_{j} of a sub-zone centered at node *j*, **a** is a column vector of values between 0 and 1 specifying the proportion of soil occupying that sub-zone.

Equation (6) requires a constraint for every node (corresponding to each row in ***P***). The relevant element in ***P*** essentially lists the length of line that crosses the relevant sub-zone for each discontinuity.

In this case the objective function gives the extra cost defined as area × strength. It is assumed that greater strength results in proportionally greater cost.

4 EXAMPLE

4.1 *Problem specification*

An example slope stability problem is depicted in Figure 2. The domain has a width of 120 m and elevation from the base to the highest point at the left hand side running from 520 m to 539 m and on the right hand side running from 520 m to 562 m. The soil has a unit weight of 18 kN/m^3 and an undrained strength of 25 kPa at an elevation of 561m which increases linearly with depth at a rate of 1.5 kPa/m.

4.2 *Conventional stability analysis*

A standard DLO stability analysis returns a factor of safety on strength of 1.16, which is insufficient to comply with Eurocode 7 VC3 + M2 partial factors.

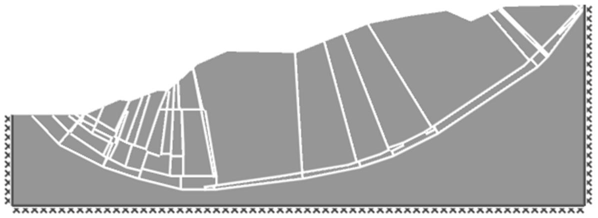

Figure 2. Example slope stability problem, and critical slip-line mechanism result derived using conventional DLO analysis.

4.3 *OSD analysis*

In Figure 3 is an analysis where the slope domain has been divided into 9 vertical x 16 horizontal patches and the optimization goal has been to find the optimal additional strength for each patch such that the sum of the additional strengths for all patches is minimized as discussed in Section 3.2. The baseline strength was selected as the original strength profile divided by the Eurocode M2 factor of 1.4. It is noted that the additional strength in any sub-zone was not allowed to fall below 0 kPa.

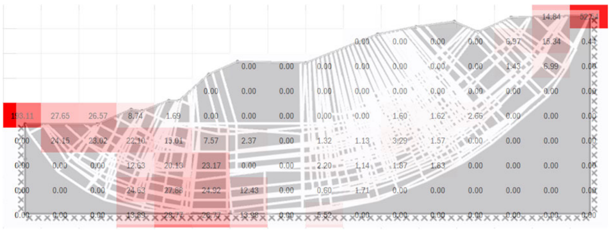

Figure 3. Example slope stability problem, and failure mechanism corresponding to result of optimized local analysis. Rectangular blocks indicate reinforcement zones. Number in each zone indicates additional strength in kPa required for overall stability. Colour shade is proportional (in range 0 to 60kPa) to the additional strength required

The number in each sub-zone in the image indicates the extra strength in kPa above the baseline that is required, and the colour intensity, proportional to this number, is added to assist visualisation. It can be seen, for example, that, as might be expected, there is little to gain by strengthening the soil in the right hand lower corner of the domain and that in this case focus is primarily required below and to the right of the toe region.

5 POTENTIAL APPLICATIONS

The OSD approach presented in this study offers significant potential for improving geotechnical design and risk management, particularly in complex and uncertain environments. Its ability to identify localized zones of influence on slope stability enables more targeted and effective interventions. Key application areas include the following.

5.1 *Enhanced Site Investigation*

The method allows for optimized planning of site investigations by highlighting zones with the greatest impact on slope stability. In practice, this means boreholes and CPT probes can be concentrated in critical areas—such as slope toes—while reducing unnecessary exploration in less sensitive zones.

It would be possible to take the results of an initial site investigation and further constrain the optimization with these values to determine where the remaining optimal locations would be. With rapid and simple computation coupled with immediate initial analysis from, for example, a CPT, it should be possible to do this in real time during a site investigation campaign.

5.2 *New slope construction*

In circumstances where a new slope is to be constructed, there is significant scope to vary the strength of specific components, for example, through use of lime/cement stabilization, selective choice of fill, or similar techniques. Thus, the minimum amount of additional strengthening would be needed. Zone sizes can be selected to correspond to onsite practical requirements.

5.3 *Existing slope stabilization*

Where an existing slope is to be stabilized, the map provides guidance as to where, for example, soil nails might be most profitably placed. In principle the optimization could be rerun with a soil nail in place, guiding a sequential placement.

5.4 *Slope Stability Remediation*

In remediation projects, especially following slope failures, the technique can be used to prioritize stabilization efforts. By identifying zones requiring higher strength improvement, engineers can focus on localized reinforcement (for example, soil nailing, grouting, soil replacement or drainage improvements), rather than applying uniform measures across the entire slope.

5.5 *Design Optimization*

For new infrastructure projects, the method can assist in optimizing slope geometry and material selection. By simulating various strength distributions, designers can evaluate the material cost and safety margin ratio, leading to more resilient and economical designs.

5.6 *Risk monitoring*

The approach can be embedded into the monitoring of slopes. As new data becomes available (e.g., from inclinometers or piezometers), the optimization can be rerun to assess risk development.

6 PRACTICAL EXAMPLE

A relevant example illustrating the importance of targeted slope stability analysis is documented by Jurko & Droniuc, 2024 (from which the example problem in Figure 2 was drawn). The paper discusses geotechnical challenges, risk acceptance, and the implications for infrastructure development on mine spoil heaps, such as shown in Figure 4.

Figure 4. Slope stability problems arising on a clayey mine spoil heap.

In the case of a slope failure during excavation of a 24 m deep cutting and bridge construction on a clayey mine spoil heap, the geotechnical challenges were driven by:

- Highly variable soil consistency due to anthropogenic deposition.
- Unpredictable water-soil interaction and porosity.
- Stratigraphic uncertainty and inconsistent strength parameters.

It is envisaged that the OSD method presented in the current paper could have assisted the engineering decisions in the following ways:

1. Pre-Failure Risk Identification:
   Before excavation of the 24 m deep cutting, the method could have been applied to simulate various strength distributions across the slope. By dividing the slope into patches and optimizing for minimum required strength, engineers could have identified zones with higher risk of failure.

2. Targeted Site Investigation:
   Instead of uniform borehole placement, the optimization results could have guided a targeted site investigation strategy. CPTs and boreholes could be concentrated in zones flagged as critical, improving data relevance and reducing post-failure investigation costs.

3. Post-Failure Remediation Planning:
   After the failure, the method could have been used to reassess the slope and simulate various remediation scenarios. This would allow engineers to compare interventions and select the most cost-effective and technically sound solution, minimizing further disruption to the bridge construction.

7 DISCUSSION

While the core method outlined in this paper involves a simple global optimization of the required additional strength across the whole domain, it is straightforward to vary the objective function to target specific parts of the domain or to vary the weighting applied to each sub-zone. This would allow the engineer to assess different scenarios according to available data or differing estimates of uncertainty.

Furthermore, while the illustrative example presented in Section 4 modelled a uniform soil with a linear variation of strength with depth. It would be perfectly possible to apply the approach to more complex layered stratigraphies and potentially to examine multiple scenarios in cases where there was uncertainty about the in-situ soil parameters.

Finally, the approach could have beneficial applications in the back analysis of failed geotechnical constructions more generally such as foundations or retaining walls. Ultimately, this methodology has the potential to transform reactive remediation into proactive risk management.

8 CONCLUSIONS

The theory behind a robust global optimization technique to identify the zones within the soil body where strength parameter variation has the greatest influence on overall undrained stability has been outlined.

This method provides a robust framework for complementary assessments of slope stability in complex and uncertain geotechnical environments. By discretizing the slope domain and minimizing the required strength across localized zones, the method generates a targeted slope strengthening map that enhances both design and remediation strategies. Whether applied during early design stages or in response to observed instability, the slope strengthening map facilitates more efficient resource allocation, improved safety margins, and reduced remediation costs. Ultimately, this methodology has the potential to transform reactive remediation into proactive risk management.

Further work is ongoing to extend and validate the approach for drained conditions and to document direct experience with its application to ongoing practical field problems.

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