

# Numerical modelling of action effect factoring in geotechnical design to Eurocode 7

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**ABSTRACT:** The second edition of Eurocode 7, EN1997-1 (2024), provides guidelines for the application of partial factors when undertaking a numerical analysis. It requires both an input factoring and an output factoring approach, where the latter involves application of factors on the effects of actions, e.g. cross-sectional quantities acting on structural elements, using partial factor sets VC4 and M1. This paper compares and contrasts the possible variants of this approach and the consequent results through a series of practical examples. These include the conditions under which the action effect is actually computed, which can include the ULS, SLS or working state and the variant permitted under Clause 8.2 (9) Note 2, where the action factors are initially applied to the resistance side of the equation, i.e. to the strength properties of the structure. This enables beneficial utilization of load redistribution capacities, but can also result in potential stiffness reduction, and may require caution where brittle components are present in the system.

**KEYWORDS:** ULS, SLS, Action Effect, Partial Factors, Eurocode 7.

## 1 INTRODUCTION

The second edition of Eurocode 7, EN1997-1 (BSI 2024), provides guidelines for the application of partial factors when undertaking a numerical analysis. In general, it requires both an input factoring (IFA) and an output factoring (OFA) approach.

The former involves application of factors on ground strength and variable actions, using partial factor of verification case VC3 (of EN1990-1, BSI 2023) and set M2 (of EN1997-1). The latter involves application of partial factors on the effects of actions, e.g. cross-sectional quantities acting on structural elements, using partial factors of verification case VC4 and set M1, where all factors on ground strength are 1.0.

This paper compares and contrasts the possible variants of the VC4 + M1 factoring approach and the consequent results through a study of a propped sheet pile wall example in a frictional soil. The design of the prop and wall strength will be considered.

For numerical methods Eurocode 7 EN1997-1, Table 8.1, specifies the procedure for verification of ultimate limit states with numerical models, and the partial factor sets to be used (which may be altered by National Standard Bodies). The key factors relevant to the example problem used in this paper are summarized in Table 1.

Table 1. Selected factoring approaches and partial factors to be used with numerical models

Factoring Approach	Partial factor on $\tan\phi'$	Partial factor on action effects
Input VC3 + M2	1.25	1.0
Output VC4 + M1	1.0	1.35

## 2 EXAMPLE PROBLEM

### 2.1 Problem geometry and material properties

This paper will examine the issue in the context of the example geometry depicted in Figure 1 and for a pile section GU15N with properties given in Table 2 (ArcelorMittal 2022). The specified pile system is chosen to be oversized to Eurocode 7.

For simplicity and to allow some checking against simple hand calculations, a frictionless wall is assumed. While this is less relevant to real world scenarios, it is not considered to affect the illustrative points made in this paper. Further, in the design of the wall to Eurocode 7, both ULS (Ultimate Limit

State) and SLS (Serviceability Limit State) criteria have to be considered. However, this paper will focus upon the ULS.

The prop is considered to provide a normal force to support the wall, but for simplicity it is assumed to provide no moment or shear support. In most of the analyses presented here the prop is simulated as a rigid support to the top of the wall in the horizontal direction, with the exception of subsection 3.3.2 where the prop is simulated as a horizontal force.

Table 2. Pile properties used in the example problem

Property	Value used in the analysis
Pile section	GU15N
Elastic section modulus	1530 cm <sup>3</sup> /m
Plastic section modulus	1840 cm <sup>3</sup> /m
Steel grade	S320
Plastic moment strength $M_p$	589 kNm/m

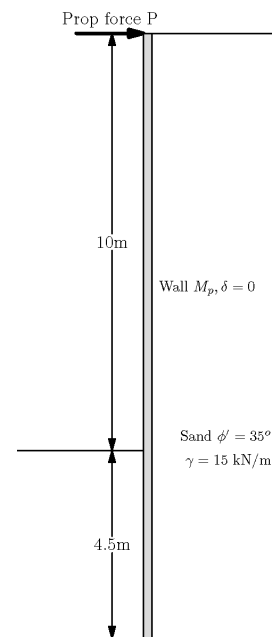


Figure 1. Problem Geometry

## 2.2 Modelling tools

For the modeling, two 2D numerical analysis tools are utilized and compared: Limit Analysis using Discontinuity Layout Optimization (DLO, Smith and Gilbert, 2007), as implemented in the software LimitState:GEO (LimitState 2021) and the finite-element (FE) software ZSoil (ZACE, 2024).

The LimitState:GEO DLO model provides a plastic analysis capability and is highly suited to directly and straightforwardly modelling the ultimate limit state, while ZSoil provides working state elasto-plastic analyses and can also be used to iterate towards the ultimate limit state. In the DLO Limit Analysis approach, it is necessary to specify one or more properties that can be increased or decreased in order to achieve an ULS. Most commonly this is an applied load but can also be a material or structural strength.

The LimitState:GEO model employed a nodal spacing of 0.5m on a square grid within the soil bodies and at 0.25m spacing on boundaries. The sheet pile was modelled using the Engineered Element feature in the software, with the relevant specified plastic moment.

For the 2-D-ZSoil-calculations isoparametric 4-node-quadrilateral elements with 1st order interpolation and an enhanced assumed strain procedure were used for modelling the soil. 2-node-layered beam elements were chosen for the sheet pile and 2-node truss elements for the prop. Interface elements allow considering effects like contact separation and friction. To facilitate comparison with the DLO limit analysis model, a Mohr-Coulomb-model with associated flow rule was applied for the soil. The Young's modulus was set to 200000 kN/m<sup>2</sup> for the soil, and to 200 GPa for the prop and sheet piles. The Poisson's ratio was set to 0.3 for both soil and sheet pile. The steel is modelled as elastic – perfectly plastic material. Several iterative solvers are provided for dealing with the material nonlinearities of the model. For most of the analyses a relatively coarse mesh was used. Checks with a fine mesh showed slightly more conservative results.

After the calculation of the primary stress state (fictive time steps are used to handle construction stages), the wall and prop are wished in place. Then the excavation is performed in one step. For some of the analyses the excavation was divided into several substages in order to enhance numerical stability. With the help of a built-in strength reduction procedure (or by using reduced soil strength parameters from the beginning) IFA can be checked.

Bending moments in the wall will be considered as negative if leading to bending away from the retained soil body and positive for the reverse, which may occur in the embedded section of the wall.

## 3 ULS DESIGN TO VC4 + M1

### 3.1 General considerations

In principle the Eurocode calculations aim to minimize the risk of an ULS on the pessimistic assumption that for any given assessment case the partial factored parameters represent a real but low probability scenario.

In the following analyses, two interpretations of the results can be made. For an existing design, the Eurocode only requires compliance to be demonstrated, i.e. to show that with the partial factors applied the system does not reach an ULS. In an FE analysis this can be demonstrated by showing the working state is stable and means it is unnecessary to continue further with e.g. a strength reduction which might result in loss of numerical convergence.

The alternative is to determine the most efficient design that complies with Eurocode 7. This can involve an FE-analysis

that iterates as close as possible to a 'real ULS' or a Limit Analysis that directly determines the 'real ULS'. The 'real ULS' is then interpreted as a way of determining a pessimistic low probability set of parameters to be avoided. Partial factors are then applied in reverse to generate the actual design values used to specify the soil and/or structure.

### 3.2 VC4 + M1 (conventional OFA)

In this standard application of the output factoring approach, the VC4 + M1 combination requires that the soil strength properties are factored by 1.0 and that calculated action effects are factored by 1.35.

This presents a challenge for ULS analysis, in that given the wall is overdesigned, it is not directly possible to conduct a ULS analysis by a simple hand type calculation assuming a rigid wall and prop to determine moments. This would generally only be available for a rigid structure of a length that would place it on the point of failure. Two numerical approaches could be taken as follows.

#### 3.2.1 Working state analysis

The FE-analysis for this profile and assuming a rigid prop results in a predicted prop force of 113 kN/m and extreme bending moments of -266 and +159 kNm/m, as shown in Figure 2. Figure 3 shows a map of the accompanying absolute displacements. With an OFA partial factor of 1.35 on the structural forces and moments, a conservative design would specify a prop with a strength of 153 kN/m and the wall with a bending strength of 359 kNm/m to comply with VC4 + M1.

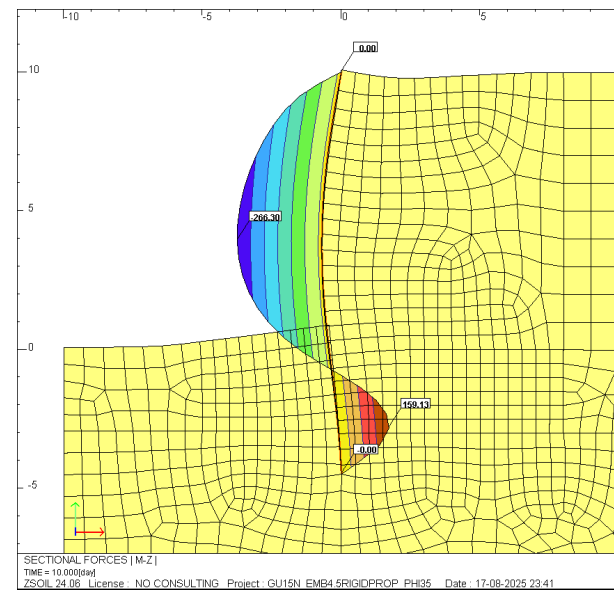


Figure 2. Bending moments in the wall for M1 partial factors from the FE analysis.

The FE-results depend strongly on the stiffness relations between soil and structural elements. Modelling of a very stiff, practically rigid wall results in significantly larger forces and moments and a change in the bending moment distribution as shown in Figure 4 for a very fine mesh. In this model, the extreme moment is  $M = -1081$  kNm/m, and the corresponding prop force is  $P = 299$  kN/m. A similar result can be obtained using the DLO model, assuming a rigid wall and finding the mobilization of soil strength (applied equally to each side of the wall) that leads to overall equilibrium. Such an analysis predicts an extreme bending moment of -1223 kNm/m, and prop force of 351 kN/m but relies on the assumption of uniformly mobilized soil strength.

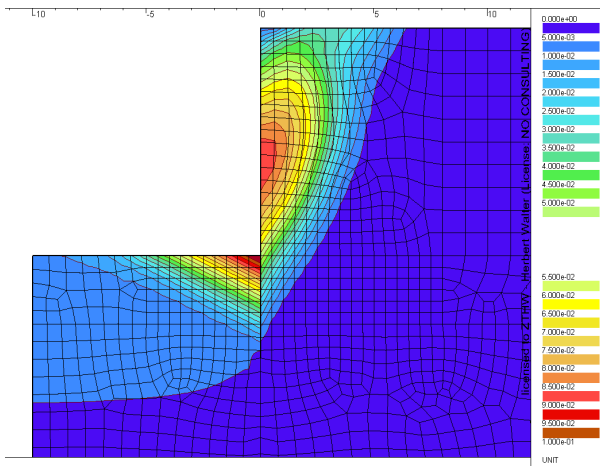


Figure 3. Absolute displacements after excavation from the FE-analysis using M1 partial factors.

### 3.2.2 Alternative ULS analysis

An alternative is to adopt the ‘stress probe’ approach proposed by Smith & Gilbert (2011a, 2011b), and considered in more detail for retaining wall analyses in Smith (2012) and Smith (2013). This will be carried out using the DLO analysis.

Conceptually, the idea here is to separately ‘test’ or ‘probe’ each part of the structure by locally overloading it according to the parameter of interest, through the application of an artificial perturbing force or moment at the selected point and increasing the force or moment until failure occurs. This then induces an ULS which then determines the actions and resistances mobilized from the rest of the soil structural system for that particular failure mode. These can be extracted from the analysis and the required structural member strength determined as the difference between the two. For the VC4 + M1 factoring approach, this value is then additionally factored by 1.35.

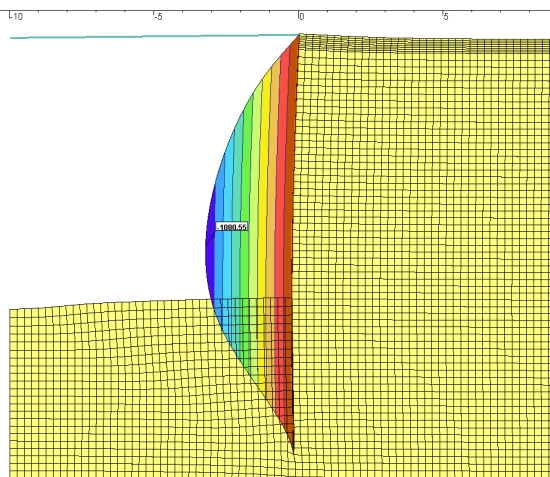


Figure 4. Moment distribution in the wall for an almost rigid wall.

The starting structure is modelled with initially specified design values for prop and wall strength, and the stress probe approach will check this design for compliance with the code at the relevant induced ULS state or provide an alternative more efficient design value of the parameter. In the below example, the previously determined factored working state values of 153 kN/m prop strength and the 359 kNm/m wall strength were adopted. For the latter value the original GU15N pile section strength could have been selected, but as will be seen below, this would not have affected the results.

For the prop, an ULS is induced by applying an additional hypothetical horizontal force  $H$  from right to left at prop level.

Assuming the prop strength from the initial design calculation of 153 kN/m,  $H$  is found to be 75 kN/m, which gives a required prop strength of  $78 \text{ kN/m}$ , and a required minimum design strength of  $1.35 \times 78 = 105 \text{ kN/m}$  to comply with VC4 + M1, lower than the initial working state value which thus also complies with the code.

For this stress probe analysis focused on the prop, the maximum bending moments were found to be  $-329 \text{ kNm/m}$  and  $+166 \text{ kNm/m}$ , both less than the specified strength of 359 kNm/m. As these values were not the subject of the ‘stress probe’ they do not require to be factored.

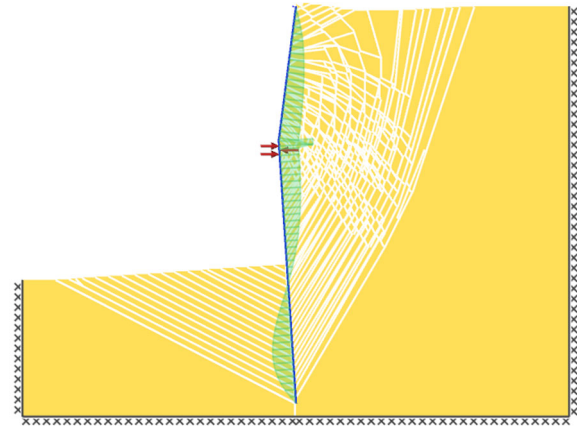


Figure 5. Moment probe using VC4 + M1.

Now, applying the ‘stress probe’ procedure for the bending moment, a ULS is induced by applying a paired couple to the wall at a range of levels (Smith, 2012 and Smith, 2013). The critical point was found to be at a height of 5 m above excavation level and a stress probe applied at this location induces a bending failure as shown in Figure 5. Using an  $M_p$  value of 359 kNm/m from the initial analysis, an imposed moment from the paired couple of 154 kNm/m was required to generate an ULS. This gives a required  $M_p$  of 205 kNm/m and a design  $M_p$  of  $1.35 \times 205 = 277 \text{ kNm/m}$  which is less than the initially assumed strength of 359 kNm/m.

The predicted bending moment in the embedded portion of the wall was a slightly higher value of 211 kNm/m and the prop force was 115 kNm/m. Again, as these values were not the subject of the stress probe, they do not require to be factored.

The specified design with 153 kN/m prop strength and the 359 kNm/m wall strength therefore complies with VC4 + M1 according to the stress probe approach but is oversized to this partial factor set.

Based on the above results, an acceptable more efficient design to VC4 + M1 could have a prop strength of 115 kN/m and a wall bending strength of 329 kNm/m to comply with both factoring calculations. However, an iterative approach, varying the prop and wall strength values further could be used to find a more efficient set of strengths that complies with the VC4 + M1 stress probe approach.

### 3.3 VC4 + M1 (variant of OFA)

The variant permitted under EN1997-1 Clause 8.2 (9) Note 2, where the action factors are initially applied as an input factor to the resistance side of the equation, i.e. to the strength properties of the structure, is considered here. It is noted that if the structural elements have been designed according to the original working state OFA calculations of Section 3.2.1, then the requirements of the OFA-variant have been automatically fulfilled as well.

However, similar to the stress probe approach, the OFA variant approach can be used to enable beneficial utilization of load redistribution capacities and achieve a more efficient ULS design. Two numerical approaches could be taken as follows.

### 3.3.1 'Strength reduction' approach using DLO-analysis

In a DLO analysis, determination of the minimum strength at which an ULS occurs is essentially equivalent to the strength reduction approach in an FE analysis.

For the prop, the strength can be reduced to 78 kN/m before an ULS is induced. When factored by 1.35, this gives a design strength of 105 kN/m. Note that because this is applied at a single location it gives the same answer as determined by the stress probe approach in Section 3.2.2. In this case, the corresponding maximum bending moment is found to be 329 kNm/m.

For the wall, the bending strength can be reduced to 206 kNm/m before an ULS is induced. The corresponding failure mechanism, which involved two hinges, is given in Figure 6. When factored by 1.35, this gives a design strength of 278 kNm/m, again, less than the 359 kNm/m working state value. This is a slightly different answer as determined by the stress probe approach in Section 3.2.2 since the mechanism involved more than one plastic hinge and for the 'stress probe' approach, essentially only one hinge is factored whereas in the OFA variant approach all potential hinges locations are factored within an individual structural element. For this analysis, the corresponding prop force was 112 kN/m.

As before, when considering the two results together an acceptable design to VC4 + M1 could have a prop strength of 112 kN/m and a wall bending strength of 329 kNm/m to comply with both calculations. However, it should be possible to iterate further to lower values.

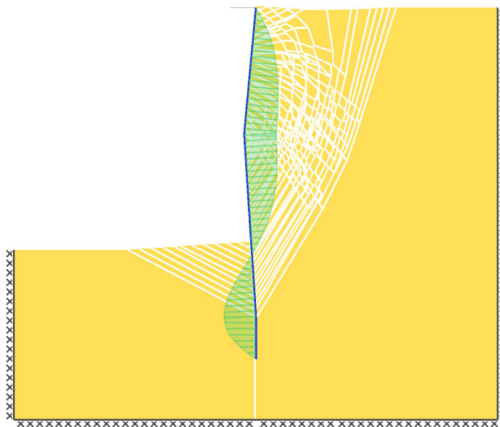


Figure 6. ULS failure for VC4 + M1, using the OFA variant strength reduction approach with DLO.

### 3.3.2 Strength reduction approach using FE-analysis

For the prop, a similar procedure can be performed with FE-analysis: the prop is replaced by a sufficiently high horizontal force. This force is then reduced in steps until failure is detected. Using the coarse mesh, the minimum prop force was found to be  $P = 82$  kN/m and the corresponding moment  $M = -346$  and  $+159$  kNm/m; with the fine mesh  $P = 83$  kN/m,  $M = -356$  and  $+147$  kNm/m. These results show that it is more difficult to get close to ULS than with limit analysis. Factors which contribute to the differences are tolerances for iterative procedures on a local and global level, interface stiffness properties, ambiguities due to softening etc. For this example, the results with FE-analysis were always more conservative in

terms of required structural strength, and usually more conservative with the fine mesh than with the coarse mesh.

For the bending strength, the procedure is as follows: if one puts the partial factor on the yield stress, it is obvious that the calculation would still result in a moment of  $-266$  kNm/m, since the plastic moment strength of the wall is considerably higher, even with reduced yield stress:  $W_{pl} \cdot f_t / 1.35 = 1840 \text{ cm}^3 \times 32 \text{ kN/cm}^2 / 1.35 = 436$  kNm/m.

As discussed in Section 3.1, one could even go further than Eurocode 7 requires and try to reduce the yield stress until a 'real' ULS is reached: a moment of  $266$  kNm/m corresponds to a steel yield stress of  $f_t = 146$  MPa. However, it turned out that reducing the yield strength below this value was not very successful: stable results were achieved using  $f_t = 140$  MPa, with  $f_t = 130$  MPa the results were already unstable (no convergence, but also no clear divergence) with extreme moments of  $-237$  kNm/m and around  $+200$  kNm/m. The corresponding prop force was approximately  $P = 115$  kN/m. Ideally, a ULS should not be reached earlier than when the positive and negative moments reach the same value. (This was the case for the value  $M = \pm 206$  kNm/m in the DLO-analysis.) Since there is still a considerable difference between them, the calculated moment is still higher than the one corresponding to 'real' ULS. With the given steel grade of S320 no factor on  $266$  kNm/m is needed. The required steel strength would be not higher than  $140 \text{ MPa} \times 1.35 = 189$  MPa, resulting in the required plastic moment strength.

Finally, whereas the stiffness of the steel is not affected by the yield stress, other constitutive relations show an influence of strength parameters on stiffness. In such a case special consideration has to be given to side effects of stiffness changes.

## 4 DISCUSSION

### 4.1 Overview

It can be seen that the highest bending moment and prop force values will generally occur at the working state in comparison to an ultimate limit state, particularly in the case of a very stiff wall.

To recap, the FE-analysis for the selected pile profile and assuming a rigid prop yields a prop force of  $113$  kN/m and extreme bending moments of  $-266$  and  $+159$  kNm/m for the M1 material factors (essentially no factoring of soil strength). Factoring of these values by  $1.35$  gave a baseline possible design compliant to VC4 + M1 with  $153$  kN/m prop strength and  $359$  kNm/m wall strength which was then further examined by various approaches to enhance efficiency. The required prop and wall strengths from these various approaches are summarized in Table 3.

Table 3. Summary of selected results from Section 3

Analysis approach	Required design prop strength <sup>1</sup> (kN/m)	Required design bending strength <sup>2</sup> (kNm/m)
Conventional OFA: working state (FE-analysis)	153	359
Conventional OFA: stress probe (DLO)	105	275
OFA variant: working state (FE-analysis)	153	359
OFA variant: strength reduction (DLO)	105	278

<sup>1</sup>assuming wall bending strength of  $359$  kNm/m.

<sup>2</sup>assuming prop strength of  $153$  kN/m.

While in all OFA-cases the bending moment values determined were lower than the initially specified GU15N wall section, one could have chosen a smaller section if IFA were not dominant.

The following sections address the considerations that would determine this choice. Practical aspects such as the availability of certain sheet pile types to the construction firm are ignored in these considerations. For other geotechnical tasks the considerations may be of practical value, e.g. for the design of tunnel support like shotcrete linings at young age.

#### 4.2 Use of working state and ULS assessment values

Because both the stress probe and OFA variant strength reduction approaches induce an ULS, plastic failure of either the wall or prop leads to plastic redistribution of stresses and typically a reduction in bending moment and/or prop force compared to the working state. However, when the resulting values are factored up by 1.35 the value may or may not exceed the working state value depending on the geometry, stiffness and strength of the structure and soil.

One question that arises is whether the working state values should be factored by 1.35. Since the values do not arise from an ULS analysis then it could be argued that it is inappropriate to apply ULS partial factors. However, in general, applying this factor would not be incorrect, but simply overconservative since the system is not at an ULS in the working state and could be a long way from the ULS. Using the OFA-variant is an elegant way of avoiding factoring of already conservative values.

The choice of bending strength could also fall under SLS considerations which are beyond the scope of this paper. However, it is unlikely that one would wish to select a plastic moment of resistance that corresponded to the maximum predicted working state bending moment and instead, some additional margin of safety may be desired to avoid reaching plastic deformation in order, for example, to be able to reuse the sheet piles.

#### 4.3 VC4 + M1 stress probe vs OFA variant strength reduction approach

Two approaches have been presented here, one corresponding to a direct VC4 + M1 application of factors via a 'stress probe' and one corresponding to the OFA variant approach, employing strength reduction. In many cases these will give the same results. For cases where, for example, two plastic hinges form in the wall, results will differ.

To reiterate, conceptually what is happening in the 'stress probe' approach is that one is examining the structure section by section and giving it a 'push' at each location that effectively augments the existing action effect that is already in place. If a small 'push' takes it into an ULS state, then this is unacceptable.

The actual action effect at the working state may be difficult to compute as the scenario will generally be statically indeterminate and may require assumptions of e.g. elastic behaviour so there will be inherent uncertainties in the predicted values.

However, if a 'push' takes the system to an ultimate limit state, then the underlying action effect (now beneficially changed due to mobilizing additional strength in the system at ULS), including the 'push' is easier to compute and will have a reduced uncertainty associated with it. This is still an unacceptable state and in the Eurocode, a factor of 1.35 on the underlying (now ULS based) action effect is required to provide an adequate margin of safety.

In contrast the OFA variant approach is essentially a material factoring approach with the 1.35 factor applied as a structural material factor to the prop strength and/or bending strength of the entire wall. This is more conservative than the

'stress probe' approach as it assumes for example that all parts of the wall have a reduced strength rather than an equivalent 'loss of strength' on an independent point by point basis which is how the 'stress probe' approach could be interpreted. Indeed, it is noted that while the two approaches are conceptually different, mathematically they are very similar. The result of the stress probe approach applied to an individual point on a sheet pile wall will be identical to applying strength reduction only to that point (rather than to the whole wall simultaneously).

Overall, it can be seen that the OFA variant approach is simpler to apply since it requires only a single numerical model to be run for each structural strength parameter to be investigated.

For a single parameter representing a 'point' value such as the prop strength in the current example, both approaches are essentially equivalent and will generate the same results.

Finally, both approaches maintain the philosophy of limit state design by applying ULS factors to a ULS based analysis.

#### 4.4 Scope of factoring at ULS

In both the conventional OFA 'stress probe' approach and the OFA variant approach, the action effect factor of 1.35 was not applied simultaneously across the whole system.

In the 'stress probe' approach the factor is applied on a point by point basis throughout the structural elements, while in the OFA variant approach the factor is applied simultaneously across each individual structural element property in turn and values of strength elsewhere remain as specified in the original design.

OFA factoring ensures that there is some safety for weak structural elements in the case of sound soil, and it could therefore be argued that it is inappropriate to factor all structural quantities simultaneously.

#### 4.5 Optimal design to ULS

Both the stress probe and the OFA variant approaches require an initial wall design which is then to be checked to determine whether it complies with the Eurocode. This is generally straightforward.

However, if a wall is to be designed, then an iterative approach may be needed since the selected initial properties of the prop will affect the moment assessment and vice versa. This will apply to any case where there is more than one structural strength in the system.

#### 4.6 VC3 + M2 (IFA)

While the focus of this paper has been VC4 + M1, it is of value to briefly consider the VC3 + M2 combination. This will often be the critical partial factor set that determines the final design.

For example, using the FE analysis working state approach, and applying a material factor of 1.25 to the soil strength, the calculated prop force is 167 kN/m and the maximum wall bending moment is 497 kNm/m.

Thus, in this case the VC3 + M2 combination would be governing from the FE-analysis perspective (and could require the originally specified GU15N section). However the ULS analysis approaches (either stress probe or structural strength reduction) described in Sections 3.2 and 3.3 could also be used for VC3 + M2 with a factor of 1.0 applied to the action effects and would typically result in lower required strengths compared to the working state.

#### 4.7 Other issues

It should be noted that the discussions in this paper primarily assume plastic behaviour in the system. For a system containing brittle components additional care is required in the design.

It is acknowledged that a prop may be considered a ‘brittle’ component of the system as it can be subject to buckling. For the purposes of this paper the prop has been assumed to be a non-brittle element.

Where possible numerical calculations were checked against hand calculation assuming simple Rankine earth pressures states. These were broadly comparable, especially where the failure mechanism involved rotation about the wall base which closely generates a classical Rankine earth pressure distribution.

## 5 CONCLUSIONS

This paper has explored in detail the issues around the application of the Eurocode OFA VC4 + M1 partial factor set for numerical ULS design. Two approaches have been discussed, a ‘stress probe’ approach and the more straightforward but conservative OFA variant approach based on a strength reduction analysis. Both approaches maintain the principle of limit state design by considering ULS states when determining parameters.

The alternative approach of applying partial factors to working state calculations (original OFA) has been discussed and in general would be considered a valid approach, but one that may return over-conservative values.

Compliance with the IFA VC3 + M2 partial factor set and SLS considerations would still also be necessary but were not discussed in detail in this paper.

## 6 DISCLAIMER

While both authors of this paper have participated in several Eurocode working groups during the development of the new Eurocode, any views expressed in this paper are the opinions of the authors and do not necessarily represent official Eurocode guidance.

## 7 REFERENCES

- ArcelorMittal 2022. Piling Handbook, 9th edition (2016), revision 2022. ArcelorMittal Commercial RPS. Available at: <https://sheetpiling.arcelormittal.com/download-center/piling-handbook> [Accessed 6th August 2025].
- British Standards Institute 2023. Eurocode - Basis of structural and geotechnical design. Part 1, BS EN 1990-1:2023.
- British Standards Institute 2024. Eurocode 7 - Geotechnical design. Part 1: General rules, BS EN 1997-1:2024.
- LimitState 2021. *LimitState:GEO Manual, Version 3.6.1* [Online]. Available at: <https://www.limitstate.com/geo> [Accessed 6th August 2025].
- Smith, C.C. 2013. Application of Computational Limit Analysis in Ultimate Limit State Design. *Modern Geotechnical Design Codes of Practice*, 195-213.
- Smith, CC. 2012. Theoretical basis of ultimate limit state design. *Geotechnique Letters* 2, 205–208.
- Smith, CC. and Gilbert, M. 2007. Application of discontinuity layout optimization to plane plasticity problems. *Proc. R. Soc. A* 463, 2461–2484.
- Smith, CC. and Gilbert, M. 2011a. Ultimate limit state design to Eurocode 7 using numerical methods Part I: methodology and theory. *Ground Engineering*. 25–30.
- Smith, CC. and Gilbert, M. 2011b. Ultimate limit state design to Eurocode 7 using numerical methods Part II: proposed design procedure and application. *Ground Engineering*. 24–29.
- ZACE, 2024. ZSoil.PC. Version 24.06, Zace Services Ltd, Lausanne, Switzerland.