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Failure mechanisms of pile groups under generalized loading conditions

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ABSTRACT: This paper examines the failure mechanisms of pile group foundations subjected to very large axial loads combined with lateral forces and moments. It investigates the interaction between horizontal load and moment capacities across different axial load levels, identifying a critical axial load beyond which failure is dominated solely by vertical mechanisms. The study also discusses how increasing lateral loads reduce moment capacity and introduces a new failure locus that defines the minimum moment capacity under high axial compression. These findings provide deeper insight into the coupled failure behavior of pile groups under combined loading conditions.

KEYWORDS: pile groups, failure domain, foundation design.

1 INTRODUCTION

One of the most challenging tasks in piling engineering is the assessment of the bearing capacity under multi–axial loading, i.e., the combined action of vertical (Q), lateral (H), and moment (M) loads leading to a geotechnical failure mechanism. In recent years, several closed–form and ready–to–use solutions have been developed for the failure envelopes of pile groups, based on limit analysis theorems or limit equilibrium methods. These envelopes, also referred to as interaction diagrams, describe how the components of the resultant action at failure interact with each other.

Prior to the development of such models, conventional design approaches typically assessed vertical and lateral capacities independently. The vertical capacity of a pile group was traditionally evaluated as the combination of axial load and moment that mobilises the axial capacity in compression (N_u) or uplift ($-S_u$) on the outermost pile. This defines the so–called yielding domain, which corresponds to the axial failure of at least one pile rather then the collapse of the group. An example is shown in Figure 1a for a row of equally spaced, identical piles, assuming $S_u = -0.75 \cdot N_u$.

Di Laora et al. (2019) provided an exact solution for pile groups under combined axial load and moment, with M acting perpendicular to an axis of symmetry of the pile layout.

Their model assumes rigid–plastic, uniaxial pile behavior with hinged connections to a rigid cap clear from the soil. Under these assumptions, the failure domain takes the form of a convex polygon with $2 \cdot p$ sides, where p is the number of piles. Each vertex corresponds to a distinct collapse mechanism involving rotation about a point between adjacent piles, while each edge represents a kinematic mechanism around a single pile.

Furthermore, Di Laora et al. (2019) proposed a lower–bound approximation that includes the plastic moment capacity at the pile head (M_s), by modelling the pile–cap connections as rigid–plastic internal fixities.

When the moment vector is not perpendicular to an axis of symmetry of the pile layout, the solution provided by Di Laora et al. (2019) represents an upper bound. For this condition the exact solution based on limit analysis theorems has been later provided by Cesaro et al. (2024).

Compared to the conventional domain, these failure loci – also shown in Figure 1a for the previously described row of four identical piles – allow identification of the failure mechanism for any load path in the Q–M plane and typically define a larger domain, enabling more rational and cost–effective design. In this example, a reinforcement ratio of 3% is assumed to represent the domain that includes the pile head plastic moment capacity.

The theoretical framework by Di Laora et al. (2019) has been validated against centrifuge tests on model foundations in kaolin clay by de Sanctis et al. (2021).

Subsequently, the solution was extended to the three–dimensional force space Q–H–M via an incremental limit–equilibrium–based algorithm proposed by Iovino et al. (2021) and Di Laora et al. (2022). In this extension, axial force distribution among piles is computed under elastic–perfectly plastic assumptions, and the corresponding horizontal capacity is derived based on pile reinforcement. A key contribution of the work by Di Laora et al. (2022) is the introduction of a lower bound approximation of the failure domain, which can be constructed using only the lateral capacities of individual piles (evaluated at N_u, $-S_u$), with minimal computational effort (Figure 1b). The methodology for checking foundation safety is based on projecting the current load state onto the Q–M and Q–H planes. This approach is referred to as "weak coupling model", implying negligible interaction between H and M. In such cases, the external moment is resisted solely by axial loads on the piles ($M_s = 0$).

However, in many practical situations – especially under complex loading conditions – a more refined representation of the H–M interaction becomes crucial (Sakellariadis and Anastasopoulos 2024, Potini et al. 2025, Di Laora et al. 2025). The simplest way to include this interaction is through the analytical model proposed by Di Laora et al. (2025), which enables the construction of the H–M interaction diagrams at any fixed value of Q, using only six reference points (Figure 1c). Notably, through this solution, it can be observed that increasing H leads to a reduction in M, in the first quadrant. This model can be defined as "fully coupled", in contrast to the weak coupling assumption.

It is important to note that the H–M interaction limits the mobilization of the moment capacity derived from axial loads on piles – denoted as M_N – particularly when the lateral load exceeds the group's capacity under free–head conditions.

When M_s is negligible compared to M_N – as in case of tall wind turbine foundations – the H–M interaction becomes irrelevant, and M_N can be fully mobilised regardless of H.

It must also be emphasized that plastic moments may appear in the Q–M plane as a misleadingly indication of additional moment capacity. For design considerations, one should be aware that this condition applies exclusively in the absence of horizontal loading. Indeed, with an increase in H, plastic moments hinder the mobilization of the M_N contribution associated with axial loads on the piles.

From a theoretical standpoint, a new locus can be defined in *Q–M* plane, representing the minimum capacity that the foundation can provide at each value of *Q*. A critical axial load (Q_{crit}) can thus be defined as the load level at which $M_s = M_N$, below which the available moment capacity drops to zero for $H > 0$.

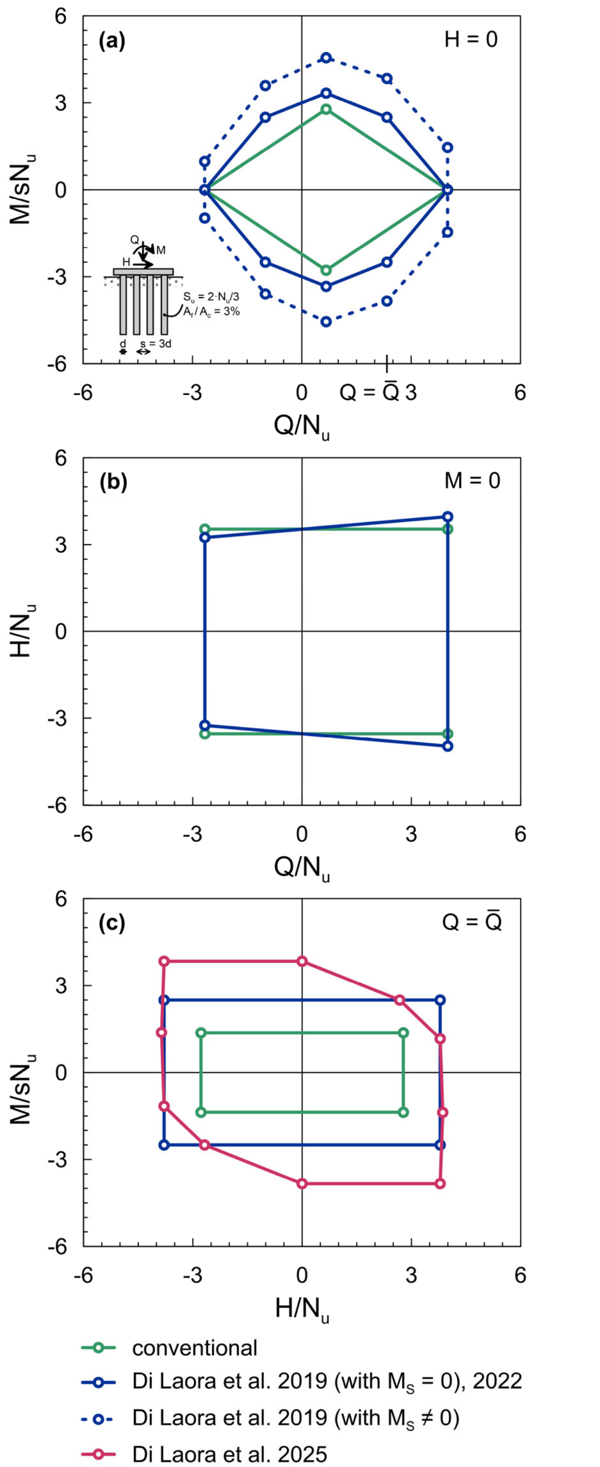

Figure 1. Conventional failure domains and recent analytical solutions in the (a) (*Q–M*), (b) (*Q–H*), and (c) (*H–M*) planes.

This work presents: (a) the analytical solution proposed by Di Laora et al. (2025) for the construction of the interaction domain in the *H-M* plane, and (b) a comparison with the numerical results by Sakellariadis & Anastasopoulos (2024).

2 CONSTRUCTION OF THE *H-M* DOMAIN

2.1 *Horizontal load – moment loci*

In absence of a lateral force, Di Laora et al. (2019) defined the maximum moment a pile group can withstand as the sum of the moment capacity due to the axial loads upon piles and the cross-sectional capacity of the pile heads. For simplicity, let us consider the same row of piles as in Fig. 2, focusing on the external vertical load value at which the ultimate moment is maximum. Considering the sectional moment capacity of the piles, the moment can increase up to the value $M_S + M_N$ corresponding to a load distribution where two piles fail in compression, two in tension and all the piles heads have achieved the sectional moment capacity, with M_y having the same direction of the external moment. Under these loads, some pile-soil horizontal contact stresses will develop along the pile, so that their integral must give zero net horizontal load and a moment equal to M_y at the head.

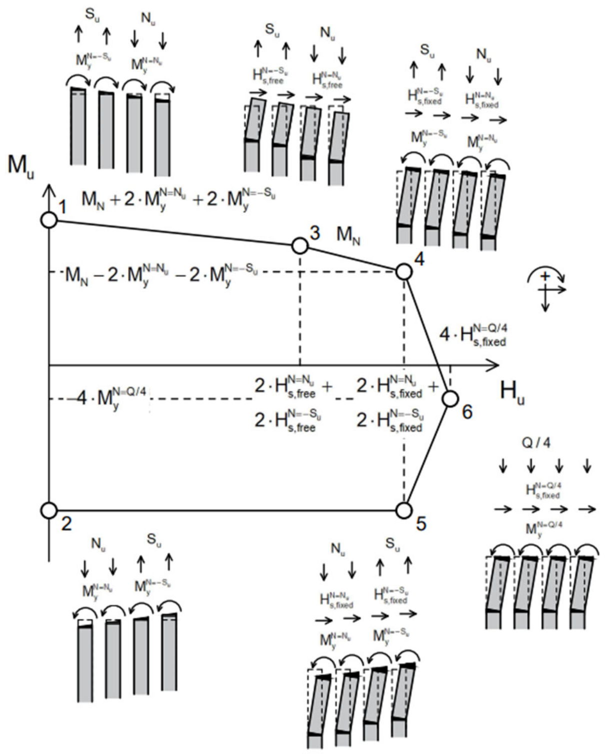

Figure 2. Construction of the proposed *H-M* interaction diagram at failure for a row of 4 identical piles.

We are therefore able to draw points 1 and 2 on the *H-M* interaction diagram at failure of the pile group under the assigned *Q*, representing the ultimate moment, in absence of horizontal load, in both directions (Fig. 2).

Note that from point 1 it is not possible to apply a positive horizontal force while maintaining the same external moment, because this would result in a moment increasing with depth, violating the failure criterion of pile section, as the moment has already attained its maximum value at the head. A compatible load scenario is represented by piles with no head moment – as if they were hinged to the cap – and subjected to their ultimate horizontal load corresponding to the free head condition with zero eccentricity. In this case, failure occurs through the formation of a plastic hinge at some depth, attaining a deeper location for piles failing axially in compression and a shallower one for piles in tension. Such load distribution is represented by point 3 in Fig. 2.

An increase in the horizontal force on piles, to let them fail through a double-hinged mechanism (fixed-head piles), is still possible; nevertheless, this implies counterclockwise moments at piles' top resulting in a decrease of the external moment (point 4). Note that the deepest plastic hinges will form at larger depth compared to the free-head case represented by point 3.

As concerns the negative moment, from point 2 it is possible to apply a positive horizontal force up to the value as that for point 4, i.e. the one corresponding to the double-hinged mechanism. In this case the yielding moment at pile heads will represent a favourable contribution for the external moment and the moment along the pile decreases with depth until the formation of the second plastic hinge. Thus, it is possible to

define the coordinates of point 5. However, such horizontal force is still not the maximum sustainable by the pile group. Suppose, under the same vertical load Q, to apply a positive horizontal translation while imposing zero rotation.

Piles are subjected to a vertical load, assumed for simplicity to be $N = Q/4$ upon each pile, and will fail through a double-hinged mechanism with the same counterclockwise yielding moment [$-M_y(Q/4)$]. In this scenario, the rotational equilibrium is satisfied by a counterclockwise (negative) external moment equal to the sum of pile head yielding moments; the sixth and last point can be therefore placed on the H-M interaction diagram.

It is possible to demonstrate that this last scenario corresponds to the maximum horizontal force the pile group is able to carry for continuously inhomogeneous soil. However, pile-to-pile interaction entails higher axial forces in edge piles and can lead to a slight decrease in such value.

It is worth noting that the constructed domain is an odd function (i.e. symmetrical with respect to the axes' origin) as $M\text{u}(-H) = -M\text{u}(H)$.

In addition, if M_S is negligible compared to M_N, the constructed domain reduces to the rectangular failure envelope proposed in Di Laora et al. (2022).

It is worth noting that all points are limit equilibrium solutions as they are kinematically admissible. Points 1 to 5 can also be derived from the upper bound theorem of limit analysis, while point 6 is an exception because, in this case, the vertical load distribution on the piles is defined a-priori. Moreover, these points do not violate the failure criterion of each pile. However, in this case, the normality condition does not hold due to the shape of the cross-sectional domain of the piles, so that it cannot be demonstrated that they represent an exact solution. As a further comment, the proposed locus is a polyline connecting some critical points. By contrast, the failure surface from 1 to 5 is curved, deriving from the shape of the failure envelope of the single pile. However, given the convexity of the failure surface, the points along the straight lines represent a lower bound close to the actual collapse surface. Thus, the proposed approach can be considered a reasonable option from an engineering perspective.

2.2 *Comparison with FEM results*

Di Laora et al. (2025) demonstrated the prediction capability of the proposed approach through the comparison with rigorous 3D finite element analyses, modelling explicitly the moment-axial force interaction for a reinforced concrete section. In the numerical study the piles are embedded in a homogenous clay deposit and the soil obeys to the Tresca failure criterion.

It is worth highlighting that the objective is the validation of the procedure to derive the ultimate domains of the pile group starting from the axial and lateral capacities of the single pile. In principle, such values should be consistent with the numerical model to make the analytical domain of the group comparable with the numerical predictions. Thus, the proposed approach has general validity, and the single pile axial and lateral capacity reflects the constitutive features on a case-by-case basis.

However, to further validate the proposed framework, here is shown a comparison with the FEM results presented by Sakellariadis & Anastasopoulos (2024).

The authors analyzed the response of different pile group layouts, ranging from single piles to 2×1, 3×1, 4×1 and 4×3 pile group configurations with variable pile spacing. The piles are considered wished in place, with diameter D = 1 m, and length L = 15 m. They are made of (C25/30) reinforced concrete (RC) (assuming 1% longitudinal reinforcement ratio and transverse reinforcement minimum requirements) and are embedded in saturated Perth sand. Regarding soil conditions, the sand considered is dense (D_R = 80%, γ' = 10.3 kN /m^3). The key attributes of the FE model are presented reported by the authors. The soil is modelled with hexahedral elements, employing the extended hypoplasticity model (Von Wolffersdorff, 1996), enhanced to incorporate inter-granular strains (Niemunis & Herle, 1997), and implemented in Abaqus. It is noteworthy that the authors' numerical analyses have been validated against extensive centrifuge testing results.

In Figure 3, the numerical results are shown in terms of single pile response under axial load. As expected for a pile in cohesionless soil, punching behavior can be observed under compression. In these cases, the ultimate load in compression can conventionally be defined as the load corresponding to a settlement of 10% of the pile diameter.

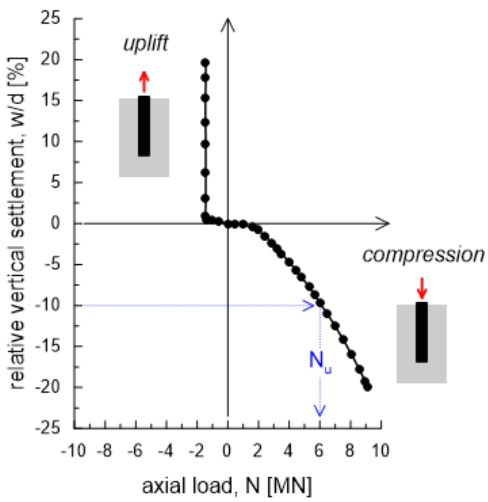

Figure 3. Single pile axial behaviour from Sakellariadis & Anastasopolulos (2024).

Knowing the failure loads in compression and in uplift of the single pile, it is possible to derive the failure domain, for H = 0 in the Q-M plane. In Figure 4 the domain is shown for a row of 3 piles spaced at 6 diameters.

The numerical results provided by the authors additionally permit the determination of the horizontal capacity of the piles for both free-head and fixed-head conditions.

Thus, the failure domain in the H-M plane, for a fixed value of Q it can be constructed.

Figure 5 demonstrates the excellent agreement between the analytical solution proposed by Di Laora et al. (2025) and the numerical results presented by the authors.

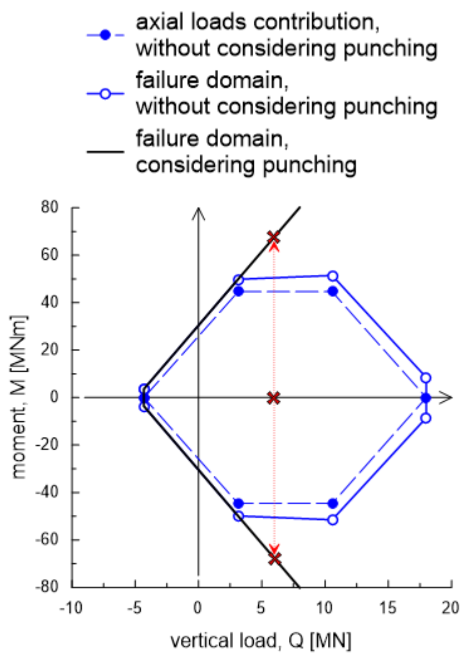

Figure 4. Failure domain for $H = 0$ in the *Q-M* plane, for a row of 3 piles spaced at 6 diameters.

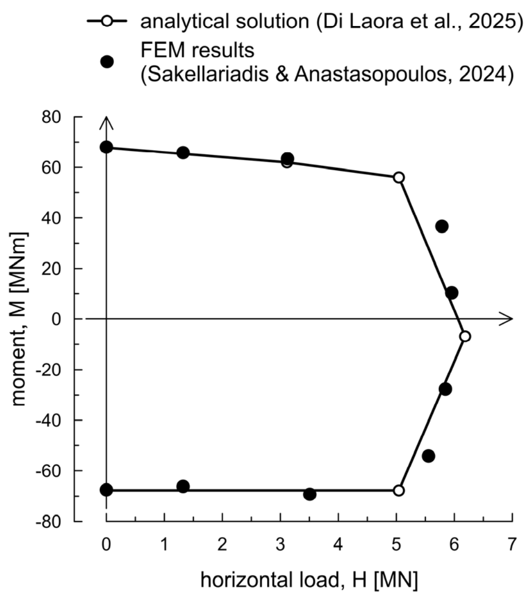

Figure 5. Comparison with FEM results from Sakellariadis & Anastasopolulos (2024).

3 CONCLUSIONS

This work investigates the failure behavior of pile group foundations under generalized loading conditions. A new physically motivated solution for the domain of a pile group in the *Q-H-M* force space is presented in which the contribution of sectional yielding moments of the piles to the moment capacity of the foundation is not negligible compared to the contribution of piles' ultimate axial loads. The approach is based on limit equilibrium and, for a given value of the external axial load, allows to determine the inter-action diagram by means of 6 points in the *H-M* plane. These are function of the axial capacity of the piles in compression and uplift and the piles' lateral capacity. The proposed failure loci are validated against results of numerical FE analyses presented by Sakellariadis & Anastasopoulos (2024).

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