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Ultimate pullout capacity of horizontal plate anchors in shallow rock masses

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ABSTRACT: This study introduces a semi-analytical method for rigorously evaluating the ultimate pullout capacity of horizontal plate anchors embedded in shallow rock masses, employing the kinematic approach of limit analysis. While traditional analytical methods based on variational approaches have been widely used to estimate the bearing capacity of anchor plates in the context of limit analysis, they require approximations of the shear strength envelope to maintain analytical feasibility. The proposed semi-analytical approach addresses this limitation by directly incorporating the generalized Hoek–Brown failure criterion, eliminating the need for closed-form shear strength envelopes or their approximations. Results indicate that the semi-analytical and variational approaches produce nearly identical outcomes when the strength envelope is accurately approximated. The study explores the influence of rock and anchor properties on pullout capacity through pullout capacity factors and failure mechanisms. The proposed method overcomes the limitations of conventional approaches, offering an effective and flexible framework for evaluating anchor performance in rock masses.

KEYWORDS: Ultimate pullout capacity, Shallow anchor plate, Hoek-Brown failure criterion, Variational analysis, Limit analysis.

1 INTRODUCTION

Anchors play a crucial role in resisting uplift forces in both offshore and onshore geotechnical applications. Considerable research has been devoted to the pullout behavior of anchors in soils, utilizing experimental investigations, numerical simulations, and theoretical frameworks such as limit equilibrium and limit analysis methods (Chen et al. 2025; Merifield and Sloan 2006; Meyerhof and Adams 1968; Murray and Geddes 1987; Rowe and Davis 1982; White et al. 2008). Despite this progress, studies focusing on anchors embedded in rock masses remain relatively scarce, even though they are of significant practical importance in underground excavation support, slope stabilization, and offshore foundation systems (Serrano and Olalla 1999).

The challenge arises from the nonlinear and pressure-dependent strength characteristics of rock masses, which require more advanced failure criteria than those commonly applied to soils. Earlier approaches often relied on the Mohr–Coulomb framework with partially curved strength envelopes reflecting limited tensile strength (Ganesh, 2024; Perazzelli and Anagnostou 2017). However, this simplification is unable to adequately represent the inherent nonlinear strength behavior of fractured rocks. By contrast, the Hoek–Brown failure criterion (Hoek and Brown 2019; Hoek and Brown 1980; Hoek et al. 2002) has become widely accepted as a more robust description of rock strength, as it directly incorporates field and laboratory characterizations through straightforward material constants.

A practical difficulty, however, is that the original Hoek–Brown criterion is formulated in the principal stress plane, where no closed-form shear strength envelope exists. This complicates its application within limit equilibrium or limit analysis frameworks. Curve-fitting approximations of the shear strength envelope have been proposed to overcome this limitation, but they inevitably introduce uncertainties and reduce the physical interpretability of the rock mass parameters. Previous attempts include Serrano and Olalla (1999), who applied early forms of the Hoek–Brown criterion via variational methods, and Rahaman and Kumar (2022), who analyzed the uplift capacity of circular anchors using a lower-bound finite element approach. Nevertheless, the direct application of the generalized Hoek–Brown criterion to strip plate anchors remains underexplored.

To address this gap, the present study develops a segment-based semi-analytical framework within the kinematic approach of limit analysis. Unlike conventional methods that approximate the shear strength envelope, the proposed approach preserves the fundamental form of the Hoek–Brown criterion in the principal stress space. This enables a more accurate estimation of the ultimate pullout capacity and a rigorous characterization of the associated failure mechanisms for shallow strip anchors in rock masses.

2 ANALYSIS BACKGROUND

2.1 *Hoek–Brown failure criterion*

The Hoek–Brown failure criterion is one of the most widely used empirical models for describing the nonlinear strength of fractured rock masses (Hoek and Brown 1980; Hoek et al. 2002). The Hoek–Brown criterion directly accounts for the influence of intact rock strength (σ_{ci}), rock type (m_i), rock disturbance (D), and rock mass quality expressed by the Geological Strength Index (GSI). The generalized Hoek–Brown failure criterion is expressed as:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a \tag{1}$$

where

$$m_b = m_i e^{\left(\frac{GSI-100}{28-14D}\right)} \tag{2}$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}}\right) \tag{3}$$

$$s = e^{\left(\frac{GSI-100}{9-3D}\right)} \tag{4}$$

Although the criterion provides a more realistic representation of rock mass strength compared with linear models, its formulation in the principal stress space ($\sigma_1 - \sigma_3$) complicates direct implementation in theoretical frameworks. To ensure analytical feasibility, many previous studies have employed an approximated shear strength envelope derived from the Hoek–Brown criterion, typically expressed in terms of parameters A and B over a selected stress range:

$$\tau = A\sigma_{ci}\left(\frac{\sigma_n + \sigma_t}{\sigma_{ci}}\right)^B \tag{5}$$

However, regression-based curve fitting may deviate from the original Hoek–Brown strength envelope, since its accuracy is highly sensitive to the assumed stress range. This drawback has motivated recent studies to pursue approaches that retain the Hoek–Brown criterion in its original form while ensuring computational tractability for stability analysis (Michalowski and Park 2020; Park and Michalowski 2019), often through the use of a parametric reformulation of the criterion (Balmer, 1952; Kumar, 1998):

$$\sigma_n(\delta)=\sigma_{ci}\left\{\left(\frac{1}{m_b}+\frac{\sin\delta}{m_b a}\right)\left[\frac{m_b a\left(1-\sin\delta\right)}{2\sin\delta}\right]^{\frac{1}{1-a}}-\frac{s}{m_b}\right\} \tag{6}$$

$$\tau(\delta)=\sigma_{ci}\left\{\frac{\cos\delta}{2}\left[\frac{m_b a\left(1-\sin\delta\right)}{2\sin\delta}\right]^{\frac{a}{1-a}}\right\} \tag{7}$$

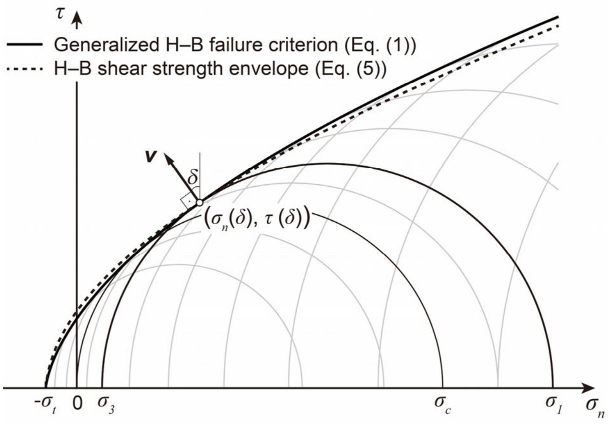

Figure 1. Generalized Hoek–Brown failure criterion (solid line) and its approximated shear strength envelope (dashed line).

2.2 *Problem statement*

A strip plate anchor of width b embedded at depth H in a shallow rock mass is considered under plane-strain conditions (Figure 2), given that its length-to-width ratio exceeds 10 (Murray and Geddes 1987). The surrounding rock mass is assumed isotropic and homogeneous, with strength governed by the Hoek–Brown criterion. The anchor is subjected to a vertical pullout load P_u, expressed as a uniform pressure p_u, while an additional surcharge q is applied at the ground surface. Effects of shaft resistance and weight are neglected to isolate the plate anchor response.

The anchor plate is assumed to be of negligible thickness and unbonded to the rock mass, excluding adhesion or grouting effects, thereby providing a conservative estimate of pullout resistance. The stability analysis adopts the kinematic approach of limit analysis (upper-bound theorem) (Chen, 1975; Drucker and Prager 1952), where ultimate capacity is determined by equating external work to internal dissipation within an admissible failure mechanism. The rock mass is modeled as rigid–perfectly plastic, and the method yields a rigorous upper bound for anchor pullout capacity, consistent with justifications in the literature (Michalowski and Park 2020).

The pullout capacity factor (F_p) is a dimensionless stability index that characterizes the resistance of a strip anchor against vertical uplift. It is defined as the critical ratio between the applied pullout pressure and the overburden stress due to embedment depth:

$$F_p=\left(\frac{p_u}{\gamma H}\right)_{crit} \tag{8}$$

where p_u is the uniform pullout pressure acting on the anchor, γ is the unit weight of the rock mass. In this study, the kinematic approach of limit analysis is used to determine an upper-bound estimate of F_p. This provides a rigorous assessment of the minimum stability margin, beyond which failure is expected to occur.

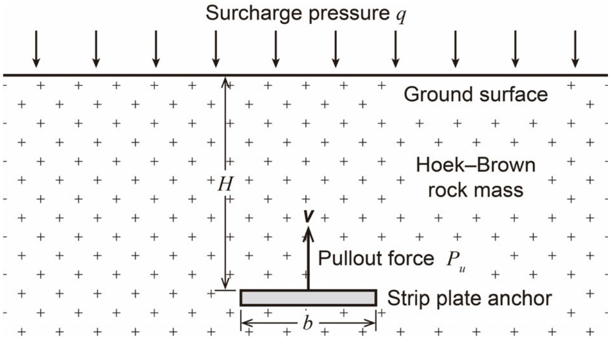

Figure 2. Strip plate anchor embedded in a shallow rock mass.

3 STABILITY ANALYSIS

3.1 *Piecewise linear failure mechanism (PLFM)*

Figure 3 illustrates the failure mechanisms of an embedded strip plate anchor in a rock mass governed by the generalized Hoek–Brown criterion. The rock block above the anchor is assumed to move upward as a rigid body without slip along the anchor interface. First, in the left-hand side, the piecewise linear failure mechanism (PLFM) approximates the failure surface of an uplifted anchor by a set of connected straight-line segments extending from the anchor edge (j = 1) to the ground surface (j = n, thereby a total of n segment). Each segment represents a kinematically admissible velocity discontinuity, and the geometry of the failure mechanism is defined by the inclination of these segments (δ_j). This piecewise construction allows a flexible approximation of a curved failure surface, and the accuracy of the solution increases as more segments are introduced.

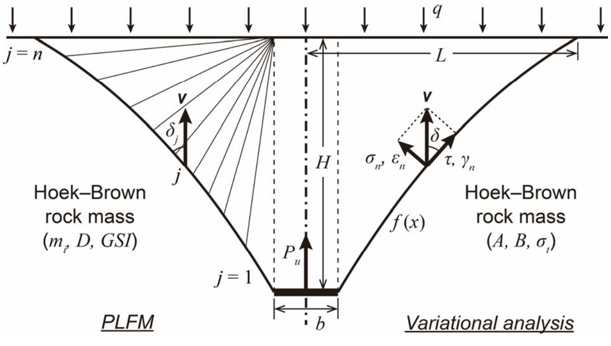

Figure 3. Anchor failure mechanisms: piecewise linear failure mechanism (left) and continuous analytical failure mechanism (right).

The stability of the plate anchor is evaluated using kinematic limit analysis, which balances the rate of external work against the rate of plastic dissipation. External work contributions arise from the anchor pullout force, the self-weight of the failed rock mass, and any surcharge applied at the ground surface. Internal dissipation is calculated along the piecewise failure surface, where stresses are obtained from the Hoek–Brown envelope through parametric expressions. The pullout capacity factor is determined by minimizing the dimensionless pullout load with respect to the failure geometry.

3.2 *Variational analysis*

Variational analysis, widely used in problems such as tunnel and slope stability, provides an optimal failure mechanism by minimizing an objective function. For plate anchors embedded in a Hoek–Brown rock mass, previous studies (Huang et al. 2011; Serrano and Olalla 1999; Zhao et al. 2018) have applied this approach to estimate ultimate pullout capacity.

In this method, the plastic potential is defined from the Hoek–Brown strength envelope, and the normality flow rule relates the stress components to the velocity field along the failure surface. The failure mechanism is represented by a continuous profile function $f(x)$, which characterizes the shape of the failed block. Plastic strain rates are expressed in terms of the slope of $f(x)$, allowing stress components to be linked with the Hoek–Brown parameters. The rate of internal plastic dissipation is obtained by integrating along the discontinuity surface, while the external work rates are derived from the pullout force, surcharge pressure, and the self-weight of the failed block. Applying the Euler–Lagrange equation leads to a differential equation governing the failure profile $f(x)$. Solving this equation with boundary conditions at the ground surface yields a closed-form solution for the failure mechanism and the associated pullout capacity.

The resulting expression provides the pullout capacity factor in terms of normalized embedment depth and block geometry, without the need for iterative optimization. Unlike the piecewise linear failure mechanism, the variational approach delivers a complete analytical solution. However, its accuracy remains sensitive to the assumed confining stress range used in approximating the Hoek–Brown criterion. To address this, the present study incorporates a previously reported stress range specification, thereby enhancing the reliability of the variational solution.

4 RESULTS AND DISCUSSION

The semi-analytical segment-based approach and the variational analysis with continuous failure surface were applied to evaluate the ultimate pullout capacity of strip plate anchors embedded in shallow rock masses governed by the Hoek–Brown failure criterion. The results are presented in terms of normalized capacity factors and failure mechanisms.

Unlike the PLFM approach, the variational analysis cannot directly incorporate the generalized Hoek–Brown failure criterion in the principal stress space (Equation (1)). Instead, it requires the derivation of a Hoek–Brown shear strength envelope expressed in the shear–normal stress ($\tau - \sigma_n$) plane (Equation (5)). To achieve this, the shear strength envelope was constructed following the procedure described by Park (2025), in which the range of confining stress is assumed up to $0.55\gamma H$. Within this stress range, the nonlinear Hoek–Brown relation can be adequately represented in terms of equivalent shear strength parameters, allowing for compatibility with the variational framework. This additional step highlights a key distinction between the two methods: while PLFM can directly integrate the generalized Hoek–Brown criterion into its kinematic formulation, the variational analysis requires an intermediate transformation of the failure criterion to a shear strength form.

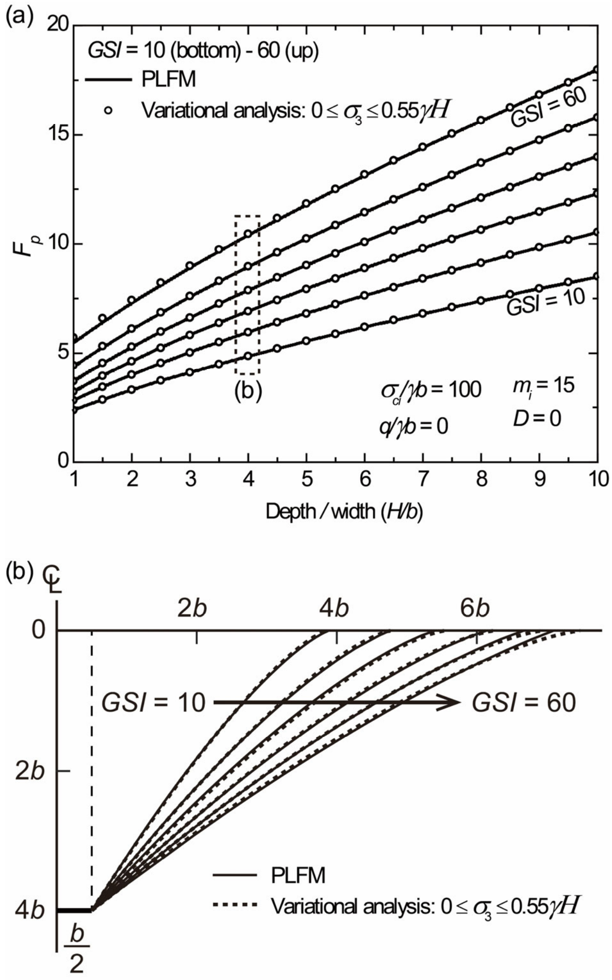

Figure 4. Analysis outcome: (a) pullout capacity factor as a function of *H/b* and (b) anchor failure mechanism.

Figure 4(a) presents the variation of the pullout capacity factor F_p with embedment ratio *H/b*, comparing results obtained from the piecewise linear failure mechanism (PLFM) with those from the variational analysis. Different ground conditions, expressed in terms of the Geological Strength Index (*GSI* = 10–60), are considered. The results demonstrate that the pullout capacity increases consistently with *GSI* and embedment depth. Importantly, the solid lines (PLFM) closely follow the circular markers (variational analysis), indicating excellent agreement between the semi-analytical approximation and the variational solutions. This validates the reliability of PLFM in capturing the influence of rock mass strength parameters on anchor performance.

Figure 4(b) illustrates the corresponding collapse mechanisms for the selected depth-to-width ratio (*H/b* = 4), as highlighted by the dashed box in Figure 4(a). The figure compares the predicted failure envelopes for varying *GSI* values. Both PLFM (solid lines) and variational solutions (dashed lines) exhibit similar curvatures of failure surfaces, with the mechanism extending outward from the plate edge toward the ground surface. As *GSI* increases, the failure surface becomes larger and more outward-spreading, reflecting enhanced rock mass strength and a larger mobilized resistance. The near coincidence of the two approaches across all *GSI* levels again emphasizes that PLFM provides an efficient yet accurate tool to replicate the theoretically derived variational profiles.

5 CONCLUSIONS

This study presented a kinematic limit analysis framework for evaluating the pullout capacity of horizontally embedded strip plate anchors in Hoek–Brown rock masses. A piecewise linear failure mechanism was employed to approximate the nonlinear failure surface, and the associated work rates were formulated explicitly. The method directly incorporates the generalized Hoek–Brown failure criterion, thereby eliminating the uncertainties commonly introduced when the strength envelope is simplified into equivalent Mohr–Coulomb parameters. The stability was assessed through minimization of the pullout capacity factor.

In parallel, a closed-form variational analysis was utilized to provide an analytical benchmark. While the variational approach yields elegant solutions without the need for segmental optimization, its accuracy is sensitive to the assumed stress range used in deriving the shear strength envelope. Comparative results demonstrated that the PLFM provides consistent and reliable predictions, while also offering flexibility for extension to different anchor geometries or loading conditions.

Analysis outcomes further revealed that both embedment ratio and rock mass quality have a pronounced influence on anchor performance, with deeper embedment and stronger rock masses significantly enhancing resistance. The PLFM approach thus offers a rigorous and flexible framework for design and assessment of plate anchors in fractured rock. Future work may extend this methodology to various circumstances (*e.g.*, inclined anchors with non-symmetric mechanisms) and field verifications.

6 ACKNOWLEDGEMENTS

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