

Shallow foundation design using FDM method applied to eccentric and inclined loading study

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ABSTRACT: A large number of real-scale foundation loading tests were performed in-situ in the 80s by LPC (acronym in French for *les Laboratoires des Ponts et Chaussées*), i.e. French Road and public Works laboratories, establishing a rich and integral experimental database. By comparing these results with numerical findings using FDM method, the aim of this study is to introduce a more realistic numerical model to improve understanding of the behavior of shallow foundations submitted to complex (inclined and / or eccentric) loadings. Two sites in France with different soil types covering sandy at Labenne and fine soils at Jossigny are principally studied. The validation of the numerical models involves comparing the computed and measured results based on the load–settlement relationship, as well as the stress distribution at the base of the loaded footing. By highlighting the difference between the results of existing methods, this paper also intends to optimize the bearing capacity factors used in footing design, as their current design bearing capacities under such combined loadings can be considered as overly conservatives. The conclusions obtained from this study contribute to the improvement of calculation method for the shallow foundations submitted to planar eccentric and inclined loading.

KEYWORDS: shallow foundation, loading test, FDM modeling, eccentric/inclined loading.

1 INTRODUCTION

Shallow foundation is a simple structure implemented commonly from individual houses to bridges or in retaining structures or offshore developments. However, in day to day practice the design methods for these structures are mainly based on an empirical traditional approach that is found somewhat to be inaccurate or inappropriate (Poulos et al., 2001). Furthermore, from a technical perspective, deep foundations have been the subject of extensive research into planar eccentric and inclined loads or combinations of V-H-M tensor, whereas these studies are not clearly estimated for footings (Murff, 2012), which often occurs due to the sliding or recentering. Some research studies have also been conducted over time, utilizing small-scale physical models in combination with analytical studies as in (Meyerhof, 1953), (Gottardi et al., 1999). Alternatively, the study based on full-scale foundation loading tests, conducted on experimental sites in France, with various soil types, loading modes, and footing configurations, is also summarized in (Canépa & Garnier, 2004). This study enriches the experimental database and contributes to better apprehending the behavior of shallow foundations.

Another approach to investigate the subject is the use of numerical methods. The Finite-Difference Method (FDM) implemented in FLAC Software developed by Itasca is a well-known approach to numerical geotechnical modelling. It uses an explicit method of calculation which takes over the system of differential equations with algebraic expressions at discrete points, then explicitly apply algorithms for solving them (Cundall & Board, 1988). This allows to perform calculations without requiring important storage, also provides solutions for stiffness matrix suitable for the non-linear processes modelling (Masouleh & Fakharian, 2008). Additionally, both small and large strain deformation modes can also be used to model nonlinear processes (Ghee & Guo, 2010). This thus constitutes the FDM as a relevant and efficient alternative for modeling foundation loading tests.

Hence, the paper presents a numerical modelling study using the FDM approach in FLAC with its latest 3D version, to replicate square foundation loading tests on sandy and silty soils. This is followed by a simulation incorporating variations in loading modes. The comparison between the measured data

and modeling results in terms of the load–displacement relationship, and the stress distribution under the footing base is then carried out to justify the quality of the numerical models. The findings from this study will provide a comprehensive understanding of the footing behavior under eccentric/inclined loading, as well as the design optimization of shallow foundations.

2 LOADING TEST MODELLING ESTABLISHMENT

2.1 Soil input and constitutive model

The French Road and public Works laboratories (LPC) from 1978 to 1990, had conducted a major experimental campaign involving over a hundred shallow foundation loading tests on sites across France with various soil types and loading modes as presented in (Canépa & Despresles, 1990a, 1990b). The test footing consists of steel plates designed on purpose in the shape of a 1x1 meter square with a modeled thickness of 8 centimeters, placed on the ground surface (no embedment) and equipped with several pressure sensors located at the base of the foundation.

Table 1. Soil information in Labenne and Jossigny.

Layer depth/ thickness (m)	Young's modulus, E (MPa)	Density, γ (kg/m ³)	Saturated density, γ_{sat} (kg/m ³)	Friction angle, φ (°)	Cohesion, c (kPa)
Labenne					
0 - 1	29.0				
1 - 3	41.4	1600	n/a	32.5	0
3 - 6	37.8				
Jossigny					
0 - 1	14.5	1950	1971	21.3	12.9
1 - 3	24.3				
3 - 5	115.0	2100	2053	23.0	20.6
5 - 8	64.0	1960	1984	29.4	5.8
1x1 m² steel plate					
0.08	210000	7850	n/a	n/a	n/a

Two experimental sites were then studied for this presented work, Labenne and Jossigny, selected based on their representativeness in terms of soil type as well as the availability of site information, particularly the results obtained from the foundation loading tests. Firstly, the site of Labenne located in the south-west of the country is characterized homogeneously by dune sandy soil. The groundwater level in this area is at a great depth (below 40 meters), and is therefore considered negligible. The second site situated in Jossigny features silty soil in the first 3 meters of depth, then the clay layer found in the next 2 meters and finally, sandy soil. The water table here varies from 1 to 2 meters of depth.

Moreover, many different characterization tests, including in-situ investigation and tests in laboratory, were also performed for soils from these sites. The soil information is summarized from diverse works as (Amar et al., 1994), (Bakir et al., 1994), and presented in the Table 1 above. One should note that the soil elastic modulus values are derived from Menard pressuremeter test data using the normalized indicative modulus correlation values from French standard (AFNOR, 2013). These soil parameters are used as input to establish the numerical model of the loading test.



Figure 1: Foundation loading tests on Labenne performed by LCP

In addition, the classical elasto-plastic model is applied to simulate soil behaviors. The failure envelope adheres to the Mohr-Coulomb criterion, incorporating both shear yield and tension cutoff functions (Itasca, 2024). The parameters introduced in Table 1 utilized as model inputs are mainly the elastic modulus, the Poisson ratio and the shear failure parameters such as the friction angle and the cohesion.

2.2 Interface parameters

In order to define the soil/footing interaction, it is assumed that the interface between them is extremely rigid, with no other deformation generated than their own. Indeed, only mechanical phenomena like sliding or tensile/shear bonding are taking place. The interface normal and shear stiffness are estimated by the Equation (1) proposed by (Itasca, 2024):

$$k_n = k_s \geq 10 \frac{K + \frac{4}{3}G}{\Delta Z_{min}} \quad (1)$$

Where k_n, k_s are respectively the normal and shear stiffness of the interface, ΔZ_{min} the smallest height of adjacent zone to the interface; K, G are respectively the soil bulk and shear modulus.

Table 2. Soil/foundation interface parameters.

Site	Normal stiffness, k_n (MPa/m)	Shear stiffness, k_s (MPa/m)	Interface friction angle, φ_i (°)	Interface cohesion, c_i (kPa)
Labenne	5000	5000	22	0
Jossigny	2700	2700	14	0

The friction angle at the interface is defined equal to two-thirds of the value for soil and interface cohesion is defined equal to zero, as recommended by (AFNOR, 2013). Table 2 shows the defined values of interface parameters.

2.3 Geometry of the numerical model

As an in-situ test, the geometry of the loading test model is contemplated very extensively since the ground is modeled as a continuous medium with an infinite horizontal dimension and a semi-infinite vertical dimension, meaning it has no fixed dimensional limits. From the same modeling case of a 1x1 meter square footing, (Mestat & Berthelon, 2001) proposed setting both dimensions of the model to 10 meters, i.e. ten times larger than the foundation size, to assure the boundary conditions with zero normal displacement. Furthermore, (Salden, 1980) introduced also the ratio of the distance between the loaded footing edge and the soil's lateral sliding surface, a to the foundation's width, B as a function of the soil's friction angle, φ' . According to the relations proposed by various authors (Figure 2), the distance a for the case of soils in Labenne and Jossigny is recommended to be no less than 3.5 times the footing width.

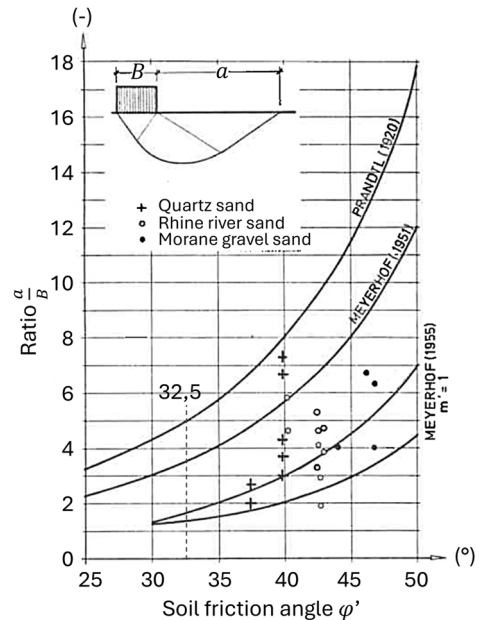


Figure 2: Relation between the soil's sliding surface distance normalized to the foundation's width and the soil friction angle (Salden, 1980)

However, the dimensions of the model should not be too small, as this can cause the scale effect, nor should they be too large, as this would consume computer memory and running time. Therefore, a parametric study was proposed to optimize the model size, as shown in Table 3.

Table 3. Parametric study of model geometry

1x1 m ² footing model size proposed	Horizontal dimension H (m)	Vertical dimension V (m)
Size 1	10.0	9.3
Size 2	5.0	7.3
Size 3	3.5	6.2

Additionally, the test shallow foundation is in shape of a symmetrical 1x1 meter square. For this symmetrical problem, three numerical models are also considered: full, half and quarter. In order to reduce calculation time, (Oliveira, 2022)

performed only a quarter of the geometry is modelled, based on the axisymmetric nature of the problem. These aspects will be analyzed in the next section.

2.4 Boundary conditions and meshing system

The normal velocity vectors at the lateral and bottom borders are set to zero to ensure no displacement in the normal direction at the model boundaries over time. The meshing system is created to be small and dense in the area around the footing, gradually enlarging towards the boundaries with a gradual size ratio from 0.9 and 1.1 (Zhou, 1997). Figure 3 below shows the schematic cross-section of a numerical quarter-model

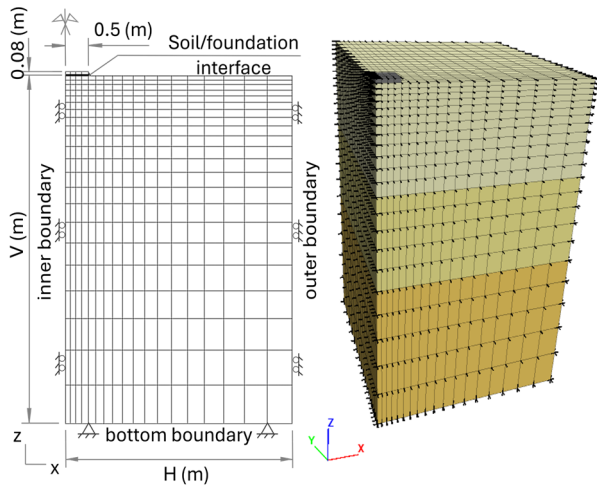


Figure 3: Geometry of the loading test quarter-model cross-section

3 LOADING SIMULATION ASSESSMENT

3.1 Centered and vertical loading

Experimental data from several foundation loading tests under simple loads, on different sites, performed by LCP in the 80s are compared with computed results, by stress-displacement relation, in order to understand the impact of geometric changes on the numerical model. Indeed, the FDM models with different geometrical sizes and symmetrical characteristics are then numerically set up under vertical and centered solicitations. The numerical results are calculated as the average values of stress and displacement obtained from the soil/footing interface of the loading test model.

Firstly, Figure 4a below presents the load-settlement curves obtained from the three numerical model sizes proposed (Table 3) in comparison with those measured from the loading tests in Labenne. A mean adjusted experimental curve is modelled, using the least-squares method. It illustrates that the shapes of the numerical curves for different model sizes are similar, and that they are also comparable to the experimental data, indicating good predictive accuracy. Secondly, these experimental load-settlement curves in Labenne are also confronted with the FDM calculation curves with the three geometry models: full, half, and quarter. Figure 4b shows that there is almost no difference between the numerical results, and that they are always found in the range of measured data.

From this parametrical study, it can be concluded that the geometric effect is not significantly evident in numerical results, since changes in model size may have little influence on the area around the foundation. Moreover, FDM modelling demonstrates at first a good reproducibility of the loading test under centered and vertical loading.

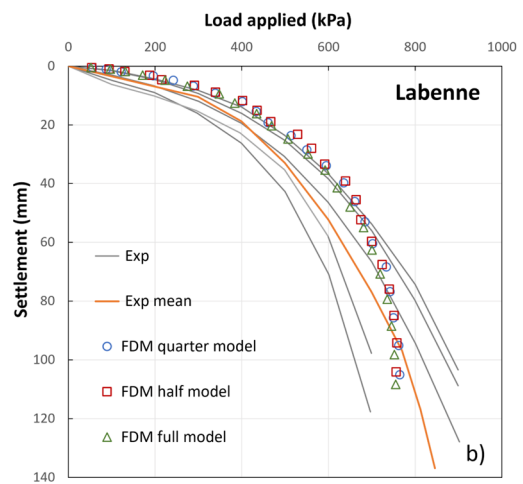
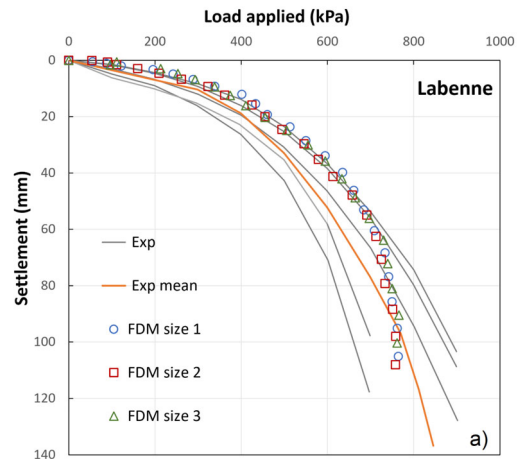
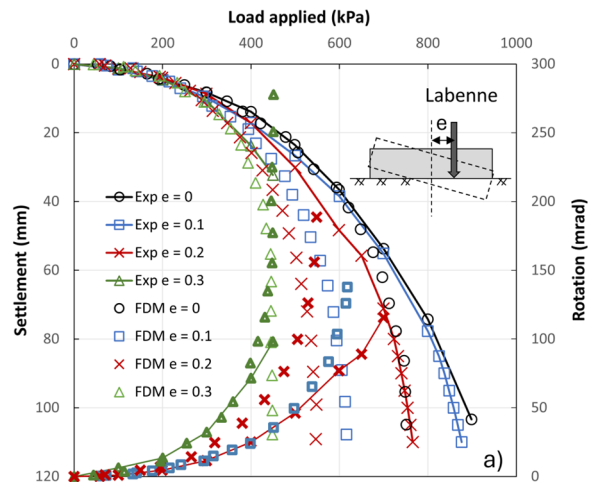


Figure 4: Effect of a) geometrical size and b) symmetrical characteristic on load-settlement relation

3.2 Eccentric and vertical loading

Vertical solicitations with three eccentricity levels ($e = 0.1, 0.2$ and 0.3) in one direction are simulated on the footing model and results are compared to those from simple case ($e = 0$). Half models in size 2 are applied, with the cutting-plane coinciding with that of eccentricity. The numerical results of the loading test under these eccentric loads are presented in this section. Figures 5a and 5b) show the load-footing settlement and load-footing rotation relationships, comparing with experimental data for the Labenne and Jossigny sites, respectively.



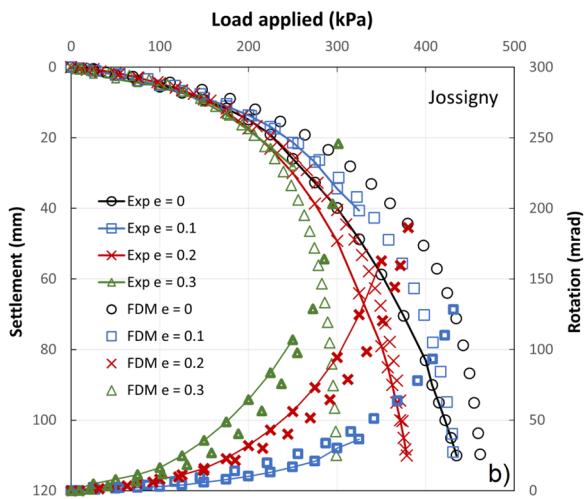


Figure 5: Load-settlement relationship of footings under eccentric effect on a) sandy soil at Labenne and b) silty soil at Jossigny

For the load-settlement relation, both experimental and numerical curves show consensus about the decrease in this relationship when eccentricity increases. Indeed, they fall and lose resistance quickly, except for a small rise in the $e = 0.1$ and $e = 0.3$ experimental data in Jossigny, which may be due to experimental inaccuracies. For the load-rotation relation, experimental and numerical curves from Jossigny site show a strong correlation, while only the $e = 0.3$ curves display comparable results on Labenne site. In conclusion, for both silty and sandy soil, increasing eccentricity can lead to a reduction in bearing capacity and an increase in the rotation of eccentrically loaded foundation.

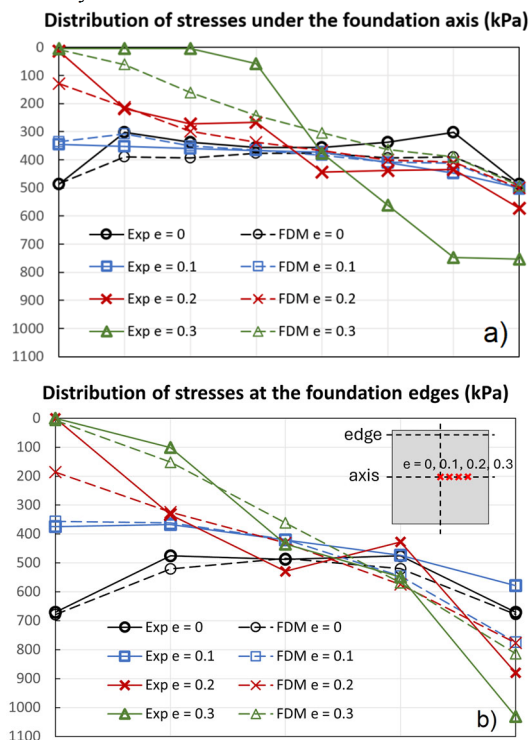


Figure 6: Distribution of stress along a) symmetrical axis and b) edges of footing in the plane of eccentricity

The distribution of stress along the footing symmetrical axis and along the footing edges in the plane of eccentricity, is respectively illustrated in Figures 6 a) and 6 b) for the loading tests in Jossigny. Again, experimental data measured by

pressure sensors equipped at the foundation base when the conventional failure occurred, i.e. settlement reaches one-tenth the foundation width ($s = 10\%B$) is used as reference to verify the numerical results. Generally, the experimental and numerical results show a relative agreement, except the only one, $e = 0.3$ along the symmetrical axis. Experimental data in this case changes quite drastically, possibly due to a overturning of foundation in two directions during the loading test.

In both cases, on the axis and on the edge of the footing, a stress distribution transition is principally observed from a symmetrical distribution to an asymmetrical one as the eccentricity increases, evolving from a rectangular shape to a trapezoidal one and then to a triangular one. Stress increases on the eccentric side and in reverse. Furthermore, it is also found, especially in the simple load case, that contact stress reaches its maximum at the corners (700 kPa), then decreases at the edges (500 kPa) and gets low value in the middle zone of footing (350 kPa). Indeed, there are certain loading test modeling studies arrive at the same conclusions of stress distribution as this one, such as from (Cajka, 2003) and (Nater et al., 2003).

3.3 Centered and inclined loading

The FDM shallow foundation models are also numerically analyzed under centered loading with three inclination angles ($\delta = 10^\circ, 20^\circ$ and 30°) and compared with the simple case ($\delta = 0^\circ$). Half models in size 2 are again applied. Figure 7a and 7b demonstrate the load-footing settlement as well as the load-footing horizontal displacement relationships for soil at Labenne and Jossigny, respectively. Similarly, numerical findings are contrasted with available experimental data.

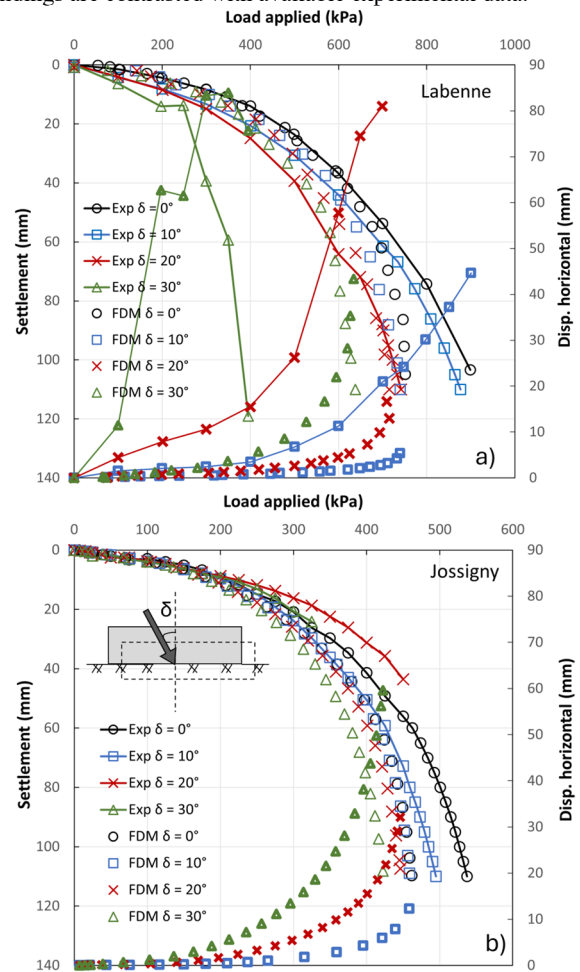


Figure 7: Load-settlement relationship of footings under inclined effect on a) sandy soil at Labenne and b) silty soil at Jossigny

For the load-settlement relationship, in almost all cases, the numerical study shows good agreement with the measurements. Generally, an increase in the load inclination angle causes a diminution in foundation bearing capacity. However, on Jossigny site again, the experimental curves for 10° and 20° inclinations rise slightly, even more than the simple case. In contrast, on Labenne site, the experimental curve for 30° inclination witnesses a drastic descent, likely due to an important sliding of foundation during the experiment. On the other hand, for the load-horizontal displacement relationship, the numerical results for sandy soil at Labenne significantly underestimate the experimental data, while the measured result is not available for comparison on Jossigny site.

In conclusion, based on the load-settlement relationship, and on the stress distribution at the base, the numerical simulation, under simple and complex (inclined and eccentric) loadings, provides generally the acceptable computed results when confronted with the experimental data on two sites covering sandy at Labenne and fine soils at Jossigny. Thus, this can confirm the relevance as well as the reliability of the foundation loading test modeling using FDM method. The assessment of footing bearing capacity under such combined loadings will thereafter be revisited in relation to other existing methods.

4 RESULT ANALYSIS

Based on the established FDM loading test models, parametrical study is conducted by varying the load eccentricity or inclination angle to better understand how shallow foundations behave under these complex loads. Available methods from different research and standards are also presented and then put in comparison with the obtained numerical results, by means of the evaluation of bearing factors.

The ultimate load or the bearing capacity of a footing is conventionally defined as the minimum value between the load measured when the settlement reaches one-tenth of the foundation width, $s = 10\% \cdot B$ and that when horizontal displacement reaches one-twenty foundation width, $d_h = 5\% \cdot B$, i.e. respectively 100 millimeters and 50 millimeters for 1x1 m² footing. Therefore, the eccentric or inclination bearing factor is calculated as the ratio of these ultimate loads:

$$i_{e/\delta} = \frac{q_{e/\delta}}{q_0} \quad (2)$$

Where $i_{e/\delta}$ is the eccentric or inclination bearing factor, $q_{e/\delta}$ is the bearing capacity of shallow foundation under eccentric or inclined effect, and q_0 is that under the simple load.

4.1 Eccentric effect

Figure 8 suggests the relationship between the eccentric factor, i_e to the eccentricity ratio, e/B for a given foundation width, B . The load eccentricity increases in the range from 0 to maximum 0.5 corresponding respectively to the variation of load application point from the center to the extreme edge of the footing. This relationship is presented by comparing the FDM results with experimental data for Jossigny (hereafter JO) and Labenne (hereafter LA).

In comparison with the study of (Meyerhof, 1953) standardized for design codes, proposing the effective contact width rule to estimate linearly the eccentric factor as shown in Equation (3). The numerical results show a good correlation when e/B is inferior to 0.2. Beyond that, the Meyerhof method underestimates considerably the bearing capacity under eccentric effect.

$$i_e = 1 - 2 \frac{e}{B} \quad (3)$$

Moreover, (Michalowski & You, 1998) presented their results using the kinematic approach of limit analysis with different assumptions for the soil-footing interface mechanism. The no-separation or perfect-adhesion models can approve the FDM findings. However, the tension cut-off model provides results close to the Meyerhof study.

Based on the no-separation model of Michalowski & You study, a slight modification is introduced for estimating the eccentric factor in this study. Therefore, the Equation (4) is proposed, which shows the best fit for the numerical results:

$$i_e = \frac{1}{1 + 2.5 \frac{e}{B}} \quad (4)$$

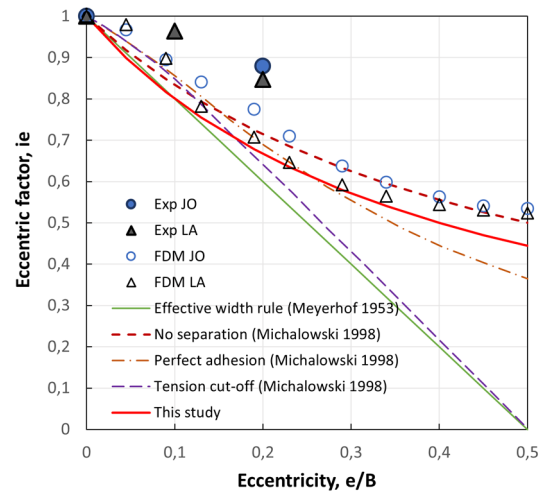


Figure 8: Footing design methods comparison for the eccentric effect

4.2 Inclination effect

In the same way, Figure 9 presents the parametric study of the relationship between the inclination angle, δ and the inclination factor, i_δ . The inclination angle ratio $\delta/180^\circ$ varies from 0 to 0.5 corresponding to the centric load orientation from a pure vertical to a pure horizontal one. It exists the difference between numerical and experimental data, as discussed in section 3.3. Especially for Labenne, they are initially closed, then the measured points suddenly decline when $\delta = 20^\circ$ and 30° . However, they still show relatively similar trends: both decline slightly at first, followed by a significant decrease. The numerical results also begin to diverge when the inclination angle exceeds 30° (or the ratio $\delta/180^\circ$ equals to 0.167).

Moreover, it is observed that the three components of inclination factor presented by (Vesic, 1975): $i_{\delta c}$, $i_{\delta q}$ and $i_{\delta \gamma}$ corresponding respectively to the effect of cohesion, the embedment and unit weight of soil, commonly used for the c and ϕ method (Terzaghi, 1943) and (Skempton, 1951), as well as the inclination factors, standardized in Eurocode 7 or the French standard (AFNOR, 2013), commonly applied for the direct design methods, all show conservative estimates of bearing capacity in comparison with the FDM method, as their curves fall quickly and below the numerical results.

$$i_\delta = \cos^2\left(\pi \frac{\delta}{180^\circ}\right) \quad (5)$$

Based on the numerical findings, this study introduces a simple formula for the inclination factor, i_δ detailed in Equation (5) following. Notably, this equation is also similar to the one used

in numerical and analytical methods of (Van Baars, 2014) with $\varphi = 0$.

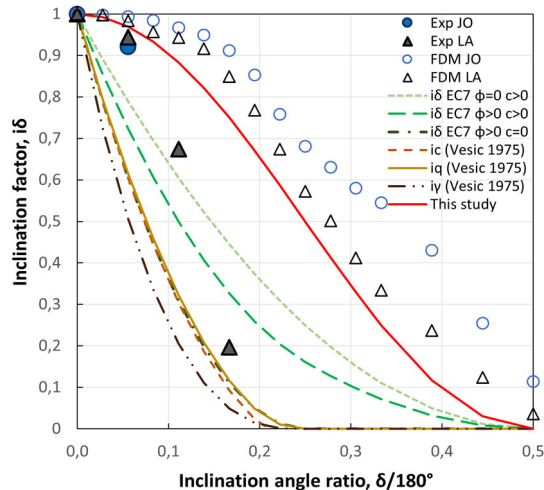


Figure 9: Footing design methods comparison for the inclined effect

5 CONCLUSIONS

By using the Finite Difference Method (FDM), authors presented firstly in this communication the modeling of footing loading test performed on sandy and silty soils. Hereafter, the numerical simulations of footings under simple as well as complex loads (account for the planar eccentric and inclined effects) are conducted, which show acceptable results in comparison with the experimental databases.

Several insights can be then withdrawn in the paper: 1) the contact stress under a rigid footing loaded vertically are highest at the corners, decreasing towards the edges, and lowest at the center; 2) the method based on Meyerhof's effective width rule with a linear function significantly underestimate the eccentric bearing capacity by comparing to the numerical results when $e > 0.2$; 3) for inclination effect, existing methods suggest a conservative design method for shallow foundation. Therefore, based on the numerical findings, two formulas have been proposed to improve the existing eccentric and inclined factors that govern the footing behavior.

Current design codes tend to limit the bearing capacity, then leading to conservative design results and material overuse. Consequently, this study introduced a new method that contributes to a better understanding of shallow foundation behavior and design optimization under eccentric/inclined loading. Nevertheless, the universality of the presented method is currently limited by the lack of variety in soil properties. At this point, only footings with simple configurations (symmetrical forms with no embedment) are studied. These then open perspectives for future research.

6 ACKNOWLEDGEMENTS

This paper is also completed thanks to the financial support provided by ANRT funding agency.

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