

Cyclic simple shear strength of a liquefiable sand under combinations of relative densities and vertical stresses by a critical state interpretation

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ABSTRACT: Soil liquefaction is a highly complex phenomenon, characterised by the rapid loss of soil strength and stiffness under cyclic loading. To advance the understanding of the cyclic strength of sands, this paper presents the results of an experimental programme conducted on a liquefiable sand by simple shear testing. The sand used in this research is of alluvial origin, obtained from the Lower Tagus River region in Portugal, which has historically been affected by earthquake-induced liquefaction following the 1755 Lisbon earthquake. The experimental tests involved various combinations of relative densities and vertical stress levels. In addition, the cyclic simple shear tests applied multiple cyclic stress ratios under constant volume conditions, allowing for a detailed characterisation of the cyclic behaviour of such granular soil. These tests provide valuable insights into the influence of relative density and vertical stress on the cyclic strength of sands. The study enhances the existing understanding of these effects by refining the equation for cyclic strength using the state parameter concept to capture the effects of relative density and stress state, which incorporates both relative density and vertical stress. This approach offers a robust and reliable method for predicting the cyclic strength of sands based on simple shear testing, contributing to a deeper understanding of liquefaction behaviour in sandy soils.

KEYWORDS: Liquefaction, Laboratory testing, critical state soil mechanics.

1. INTRODUCTION

Liquefaction is a complex phenomenon of cohesionless soils, which is characterised by the rapid loss of the strength and stiffness of soil. The generation of pore pressure excess causes this during cyclic action. Due to this generation, effective stress significantly decreases, showing values near zero during liquefaction (Ishihara, 1993). Recently, around the world, liquefaction has seriously damaged several infrastructures after earthquakes with magnitude M_w larger than 6, such as Christchurch (Cubrinovski et al., 2011), Italy (Minarelli et al., 2022), Indonesia (Jaya et al., 2019) and Turkey (Flora et al., 2024).

In Portugal, local earthquake-induced liquefaction dates from 1344, 1531, 1755 and 1909 when distant and local intraplate earthquakes (with a magnitude between 6.0 and 8.5) struck the Portuguese onshore mainland, mainly affecting the greater Lisbon area (Jorge, 1993). The 1755 earthquake, with an M_w of 8.5, is probably the greatest seismic disaster in Western Europe since it almost destroyed the Lisbon centre (Oliveira, 2008). Therefore, characterising the liquefaction resistance of cohesionless soils in areas with moderate to high seismic risk is essential to prevent damage and possible collapse of buildings and infrastructure.

This paper aims to characterise the liquefaction resistance of an alluvial natural sand by cyclic simple shear (CSS) testing. The investigated sand corresponds to TP-Lisbon sand, a historically liquefiable soil collected in the heart of the historical centre of Lisbon, specifically at the ‘Terreiro do Paço’ area (Molina-Gómez and Viana da Fonseca, 2021). The experimental program addresses a series of CSS tests comprising combinations of initial vertical stress and relative densities. CSS results to characterise the cyclic strength of TP-Lisbon sand. This approach captures the effects of stress and the state of soil on liquefaction resistance employing the critical state framework based on the state parameter.

2. MATERIALS AND METHODS

2.1. Tested soil

The soil used in this paper is TP-Lisbon sand, an alluvial sandy soil originating in the late Quaternary. This soil has settled by the Tagus River, specifically in the historical centre of Lisbon, at a site locally referred to as ‘Terreiro do Paço’ (which in English means ‘Court of the Palace’). The soil profile at Terreiro do Paço consists of marine and fluvial materials transported by the Tagus River, resting over glacial Miocene deposits of stiff clays. The shallow layers are made up of fine, clean sands and sandy silts, with intercalations of plastic soils (Gouveia et al., 2018). The presence of these intercalations suggests that the valley was filled through a heterogeneous process, likely influenced by tidal variations in the Tagus River. At depths of 0 to 8 meters, remnants of buildings that collapsed after the 1755 earthquake can still be observed. Today, the structures surrounding this historical site are comprised of old Pombalino masonry buildings, which are part of the Portuguese heritage (Couto et al., 2020).

The samples of TP-Lisbon sand were collected as part of the geotechnical exploration for constructing the Lisbon blue line tunnel for ‘Metropolitano de Lisboa’. These samples were obtained by Keller Groundbau GmbH using a double rotary probe sampler. This device penetrates the soil through an external tube, allowing the sampler to extract vertically via an attached Shelby tube with a diameter of 110 mm. The sampler successfully recovered samples of alluvial granular soils, including fine, clean sands, with an approximate success rate of 80% (Viana da Fonseca et al., 2023). The samples were collected during a site investigation at the coordinates 38°42'26.6"N, 9°8'13.4"W. The collected samples are representative of granular soils composing the liquefiable layer identified at depths ranging from 13 m to 23 m (Viana da Fonseca et al., 2023).

TP-Lisbon sand is composed of subangular particles (Molina-Gómez and Viana da Fonseca, 2021). The specific gravity of solid particles (G_s), the minimum void ratio (e_{min}) and the maximum void ratio (e_{max}) are shown in Table 1. The parameters of the particle size distribution are also exposed in Table 2, including the mean diameter (D_{50}), the coefficient of curvature (C_c), the coefficient of uniformity (C_u) and the fines content (FC). The particle size distribution of TP-Lisbon sand is depicted in Figure 1. The Unified Soil Classification System (USCS) classifies the TP-Lisbon sand as poorly graded (SP), with low values for C_c and C_u .

Table 1. Physical properties of TP-Lisbon sand

Parameter	Value
G_s	2.66
e_{min}	0.64
e_{max}	1.01
D_{50} (mm)	0.21
C_u	1.69
C_c	1.13
FC (%)	2.21

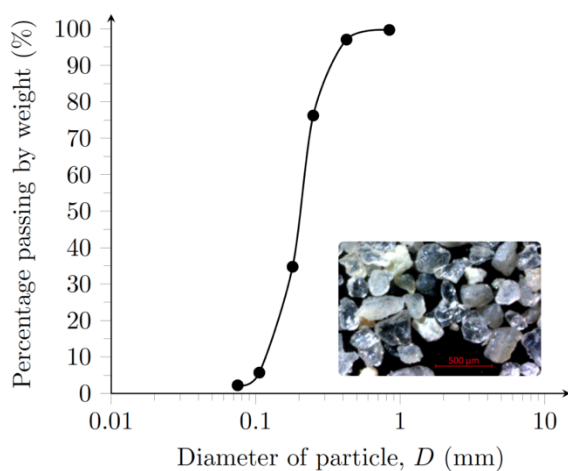


Figure 1. Particle size distribution of TP-Lisbon sand.

2.2. Experimentation

The experimental programme for assessing the liquefaction resistance of TP-Lisbon comprised an advanced laboratory testing programme using cyclic simple shear (CSS) tests. The CSS apparatus enables the rotation of principal stresses under harmonic sinusoidal loading conditions. This feature allows for a realistic representation of the shear stresses generated in the ground during seismic events (Mele, 2024).

This paper utilised an advanced and fully automatic simple shear apparatus to conduct advanced tests under stress control using an SGI configuration, consisting of a series of stacked brass rings that maintain a constant radial area throughout all testing stages, particularly during cyclic loading. Vertical and horizontal loads are applied using two high-quality servomotors, which are measured with a multi-axis load cell. Additionally, the vertical and horizontal displacements are recorded using two local LVDT transducers, which enable the accurate estimation of shear strain and the active control of constant volume conditions. Constant volume tests are equivalent to undrained shear tests (Dyvik et al., 1987).

CSS tests were conducted on reconstituted samples using the air pluviation method to achieve various values of relative density (Dr) ranging from 40% to 60%. This range of Dr was selected based on the *in situ* state of TP-Lisbon sand derived from CPTu soundings, as reported by Viana da Fonseca et al. (2023). All samples were saturated by circulating carbon

dioxide for 30 minutes and flushed with 500 mL of de-aired water (Viana da Fonseca et al., 2021). The degree of saturation (S_r) was estimated by monitoring the sample deformations during all test phases and measuring the water content at the end of each test (Cappellaro et al., 2021). In all tests, S_r levels higher than 99% were reported.

After the saturation phase, the samples were consolidated under initial effective vertical stress (σ'_0) values of 80 kPa and 150 kPa. These σ'_0 values represent the in-situ stress conditions at the Terreiro do Paço site. Following the consolidation, cyclic loading was applied using a continuous sinusoidal signal, maintained at a constant amplitude of shear stress (τ). This amplitude varied between tests to create different combinations of cyclic stress ratios ($CSR = \tau/\sigma'_0$). All tests were conducted at a frequency of 0.1 Hz to ensure precise control of the equipment during cyclic loading. The tests were terminated automatically once a shear strain (γ) higher than 7.5% was recorded.

3. RESULTS AND DISCUSSION

The liquefaction resistance of TP-Lisbon sand was characterised by 24 CSS tests. Figure 2 exemplifies the typical results obtained from CSS testing, displaying the hysteresis loops of the stress-strain response and the stress path during cyclic loading from two tests conducted at a CSR of 0.13. These tests comprised different Dr and σ'_0 values, which highlight the variations in soil behaviour reported in this study.

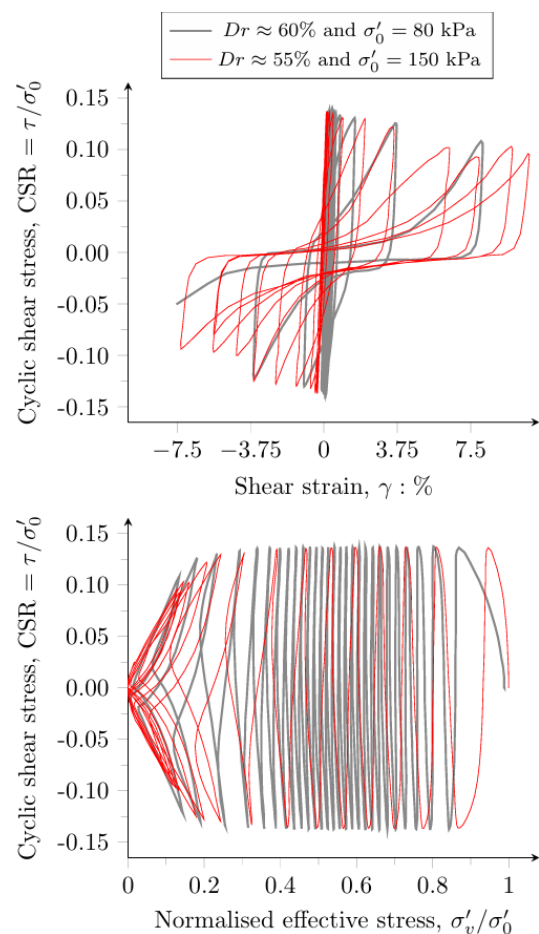


Figure 2. Typical results of CSS tests in TP-Lisbon sand

From this Figure, it can be observed that during the first loading cycles, the development of shear strain (γ) remains in the elastic range. When the shear strain exceeds 1%, the soil

experiences progressive deformation over a few cycles, resulting in significant changes when γ exceeds 3.75% and reaches the limit set to conclude the test. This phenomenon is attributed to the degradation of soil stiffness resulting from cyclic loading.

On the other hand, Figure 2 illustrates a continuous decrease in the effective vertical stress (σ') due to the application of cyclic loading (represented by τ), with values approaching zero. This situation indicates the onset of liquefaction. However, after liquefaction triggering, further reversal loading results in a partial recovery of the strength and stiffness of the soil. This recovery is attributed to the phase transformation effects of the sand, which are reflected in the σ' during cyclic loading. From the CSS test, a series of liquefaction resistance curves were derived. These curves follow the general power fitting:

$$CSR = a(N_L)^b \quad (1)$$

where N_L corresponds to the number of cycles for liquefaction onset, and a and b are fitting coefficients defining the power law describing the liquefaction resistance curve (LRC). Figure 3 shows the LRC of the CSS tests and the coefficients a and b obtained for the best data fitting, with a coefficient of determination greater than 0.90.

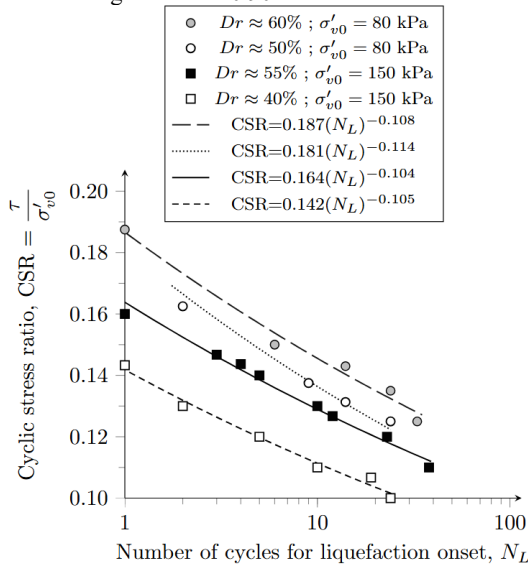


Figure 3. Typical results of CSS tests in TP-Lisbon sand

The results indicate a series of power fittings that exhibit parallel tendencies, which signify distinct intercepts (a coefficient values) but notably similar slopes (b coefficient values). These tendencies highlight the effects of density and vertical stress on the liquefaction resistance during CSS tests. Given that the testing conditions varied concerning these parameters, a critical state approach was implemented to interpret the effects of Dr and σ'_0 . This critical state approach utilised the state parameter (ψ). Wroth and Bassett (1965) introduced originally ψ to describe the state of soil. Been and Jefferies (1985) termed the vertical distance between the soil state and the critical state, defined as follows:

$$\psi = e_0 - e_{cs} \quad (2)$$

where e_0 represents the void ratio at the beginning of cyclic loading, and e_{cs} is the void ratio at the critical state. Both void ratios are at the same mean effective stress (p'_0), which was estimated by combining σ'_0 and the effective friction angle (ϕ'):

$$p'_0 = \frac{\sigma'_0(1 + 2(1 - \sin \phi'))}{3} \quad (3)$$

This procedure was applied using a ϕ' of 34° , as suggested by Viana da Fonseca et al. (2023). In addition, the critical state locus (CSL) derived by Molina-Gómez & Viana da Fonseca (2021) was used to compute e_{cs} . Figure 4 presents the CSL and the stress state prior to CSS testing.

All CSS tests were conducted under ψ with negative values, specifically in the range $-0.17 < \psi < -0.06$. The results indicate that all samples exhibit a dilative behaviour. This behaviour explains the phase transformation that leads to a partial recovery of soil strength when normalised effective stress approaches zero (see Figure 2). Table 2 summarises the state parameters of each testing condition employed to derive the LRC of this study.

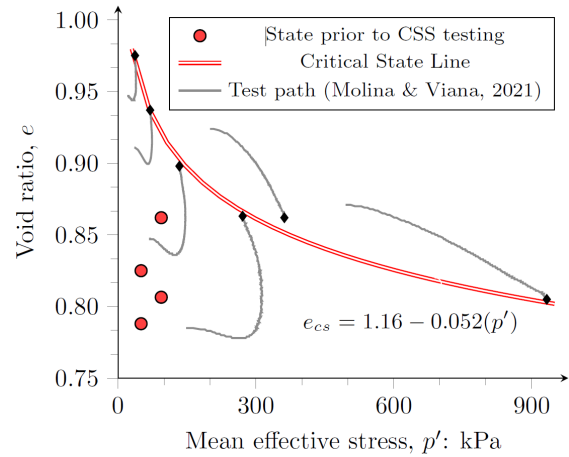


Figure 4. CSL of TP-Lisbon sand and the state prior to CSS

Table 2. State parameters for each LRC

Dr (%)	σ'_0 (kPa)	e_0 (-)	p'_0 (kPa)	e_{cs} (-)	ψ (-)
60	80	0.79	50	0.95	-0.17
50	80	0.83	50	0.95	-0.13
55	150	0.81	94	0.92	-0.12
40	150	0.86	94	0.92	-0.06

Experimental investigations compiled by Jefferies and Been (2015) reveal a consistent relationship between cyclic resistance and the state parameter. These correlations remain valid across various confining pressures, relative densities, and fines contents (Tomasello and Porcino, 2024). Figure 5 illustrates the link between the cyclic stress ratio required to trigger liquefaction at 15 loading cycles (CSR_{15}) and the state parameter. The results obtained from TP-Lisbon sand are contrasted against a database compiled by Jefferies and Been (2015).

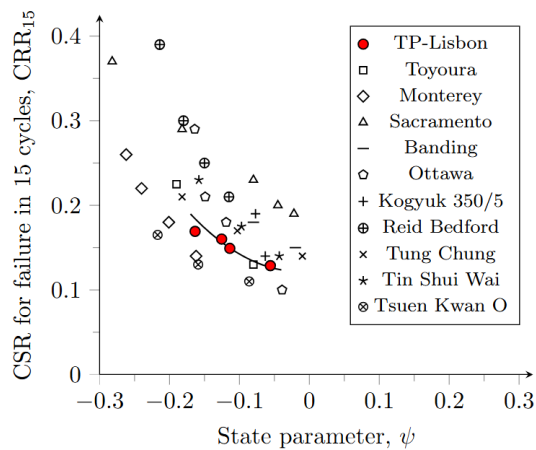


Figure 5. Relationship between the state parameter and CSR_s for TP-Lisbon sand and its comparison against other sands.

Figure 5 shows that liquefaction resistance increases with more negative ψ values, a behaviour commonly associated with the increment in soil strength caused by soil dilatancy since the CSR₁₅ increases as ψ exhibits more negative values. These findings indicate that no universal relationship exists between the state parameter and earthquake-induced liquefaction resistance. Instead, such relationships should be established individually for each type of sand. The critical state framework can provide accurate predictions of CSS tests.

4. CONCLUSIONS

The CSS tests conducted on TP-Lisbon sand provided key insights into its liquefaction resistance and state-dependent behaviour. The experimental results confirmed that:

- Liquefaction resistance is strongly influenced by D_r and σ'_{v0} , as reflected in the parallel trends of LRCs.
- All tested samples exhibited dilative behaviour, with $-0.17 < \psi < -0.06$. This dilatancy explains the observed partial recovery in stiffness and strength after liquefaction onset due to phase transformation effects.
- A consistent relationship between the CSR₁₅ and ψ was observed, in agreement with trends reported in the literature. However, results reinforce that this relationship is not universal and must be calibrated for each specific sand and sample preparation method.
- The critical state framework was shown to be a reliable approach for interpreting the cyclic response of TP-Lisbon sand in CSS testing.

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