

Application of Volume Averaging Technique for geotechnical problems on soft clays

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ABSTRACT: Due to increasing urbanisation, there is increasing need to construct in densely populated urban areas on marginal quality subsoils. Therefore, there is a need for ground improvement techniques to ensure that the buildings, essential infrastructure and utilities have a robust, yet environmentally friendly, foundation. Ground improvement with rigid inclusions, such as stone columns and deep mix columns, is an economical and sustainable solution. Deep mixing with dry method is adopted in Scandinavia due to its flexibility: the columns can be arranged as individual columns into a regular grid and by overlapping the columns it is possible to form panels, walls or grids with ever increasing area replacement ratio. This periodic nature enables to use homogenisation technique, stemming from modelling composite materials, to rigorously map the 3D system configuration into 2D plane strain analyses. With appropriate static and kinematic boundary conditions, it is possible to theoretically derive the stress-strain matrix for the homogenised material yet modelling the two constituents (soft clay and the columns) separately, with their respective (different) constitutive models. The static equilibrium and kinematics constraints assumed for deriving the homogenised stress-strain matrix are discussed for two typical geotechnical problems involving deep mixing: an embankment on soft clay and a deep excavation. The method, referred to as Volume Averaging Technique (VAT), is then applied to two examples, with comparison to 3D analyses and field data, demonstrating that VAT is a computationally efficient technique to capture the time-dependent response of these 3D geotechnical systems.

KEYWORDS: soft clay, ground improvement, numerical modelling

1 INTRODUCTION

The properties of very soft clays, silts and organic soils can be improved by rigid inclusions that are unreinforced columns, installed either as individual columns (deep mixing or stone columns) or overlapping columns forming panels of grids (deep mixing or jet grout columns). In Scandinavia, where the *in situ* water content of natural clays typically exceeds the liquid limit, dry soil mixing with a combination of lime, cement and other additives as a binder mix is common.

Deep mixed columns are extensively used to reduce deformations and to improve the overall stability of embankments and foundations on soft soils. Whilst the most typical applications use columns under road and railway embankments, increasingly the method has been used under foundations (pioneered in Poland), as an environmentally friendly (low CO₂) and cost-effective alternative for piling. More recently, the dry soil mixing has been applied in deep excavations, where the technique has two functions:

1. Stabilising the clay to be excavated (with a low binder content) eases the excavation and mass handling.
2. Stabilisation of the bottom of the excavation (with an adequately high binder content) provides support against the bottom heave and hinders excessive wall movements.

In all the applications above, the aim is to substantially reduce the risk of unwanted deformations and to ensure that the geostructure is safe against failure in Ultimate Limit State. Under railway embankments, deep mixed columns are used not only to control deformations, but also to control the vibrations in the proximity of the track caused by the passing trains.

The commonly used design methods for deep mixing tend to start with ensuring the stability against failure (Ultimate Limit State) based on the undrained shear strength of the clay and assumed (design) target design undrained strength of the columns. The latter rarely has any relation to the reality, as often columns are found to be much stronger and stiffer than assumed in design. In case of embankments, stability analyses are followed by settlements analyses, imposing the condition of equal vertical settlements, where the stiffness of the columns is accounted for based on an assumed ratio between the stiffness of the columns and the design undrained shears strength. The resulting predictions of time-dependent settlements movements are thus highly suspect. This is enhanced by the common

misconception that the deep mixed columns act as drains, even though research is rather suggesting that when cement is used, the hydraulic conductivity of the columns is of the same order of magnitude or even less than the surrounding natural clay. The misconception stems from not understanding that due to the presence of the columns, the stiffness of the system is much higher than that of untreated clay, thus increasing the coefficient of consolidation.

When simulating movements of geostructures, such as embankments and excavation involving highly non-linear materials, numerical analyses with finite elements (FE) is an attractive alternative to conventional method of analyses. FE analyses are particularly suitable when considering the Serviceability Limit State (SLS), i.e. performing deformation analyses under working loads involving non-linear material response. FE analyses allow using advanced constitutive models that take account of the complex stress-strain behaviour of natural soil and stabilized columns, respectively, as well as coupling deformations and pore pressure changes.

The challenge is that problems with a grid of circular columns under an embankment or at the bottom of excavation are truly 3D problems. 3D analyses are very laborious and expensive. Thus, in this paper an enhanced 2D technique using the so-called volume averaging technique (VAT) is utilised. The basic idea is to describe the column-improved ground as a homogenised composite material, with an averaged stiffness matrix derived using simple mathematics, to map the true 3D problem into 2D. Once the constitutive relations for both constituents are defined, the time-dependent response of the column-improved ground can be simulated in 2D subject to arbitrary loading and boundary conditions.

First, an embankment on soft soil is simulated, using VAT for embankments developed by Vogler & Karstunen (2007, 2009). Second, an excavation, using VAT for excavations developed by Bozkurt et al. (2025) is discussed. In both cases, the properties for the columns and the natural clay, represented by the MNhard model (Benz 2007) and the SCLAY1S model (Karstunen et al. 2005), are adopted, with model parameters based on experimental results. Because of the different static and kinematic conditions, the equation for the homogenised stiffness matrix will be different for these two cases.

2 VOLUME AVERAGING TECHNIQUE

2.1 Introduction to Volume Averaging Technique

The origin of homogenisation techniques stems from the early works by Voigt and Reuss on the estimation of the elastic moduli for two-phased homogenised material. Natural clays, however, are highly non-linear materials, and thus cannot be modelled as elastic materials. The same applies to e.g. stone columns and deep mixed columns. Furthermore, the stiffnesses of the soft natural clay and the columns differ by several orders of magnitude, resulting in complex soil-structure interaction. Naturally, as the columns are stiffer than the clay, the stresses tend to concentrate on the columns. If the shear capacity of the columns is exceeded, i.e. the columns start to yield, the stresses are redistributed to the surrounding soft natural clay.

To assess the effectiveness of rigid inclusions in reducing settlements, and to realistically simulate the hydromechanical response of the system, an elastoplastic analysis needs to be adopted, as the results are inaccurate if only elastic perfectly plastic behaviour is considered (Canetta and Nova 1989).

In Volume Averaging Technique (VAT) any representative constitutive model can be adopted. A periodic distribution of the columns in the natural soil is assumed. The column improved ground will be idealised as a homogeneous medium and the volume ratios for column Ω_c and soil Ω_s are defined as:

$$\Omega_c = \frac{A_c}{A} \quad ; \quad \Omega_s = \frac{A_s}{A} \quad (1)$$

where A_c and A_s are the cross-sectional areas covered by column and soil, respectively, and the total area $A = A_s + A_c$. The sub-superscript s and c will be used in the following for soil and columns, respectively.

Fundamental assumptions of VAT are based on averaging the stress and strain rates based on volume ratios as given in Equations (2) and (3). The coordinate system assumes the x - and y -axes as horizontal and the z -axis as vertical.

$$\Delta\sigma^{eq} = \Omega_s\Delta\sigma^s + \Omega_c\Delta\sigma^c \quad (2)$$

$$\Delta\varepsilon^{eq} = \Omega_s\Delta\varepsilon^s + \Omega_c\Delta\varepsilon^c \quad (3)$$

The stiffness matrix for the equivalent material can be derived by analytical means forming to Equation (4), provided adequate number of static and kinematic constraints are assumed (in total six are needed for generalised 3D case). In Equation (4) the S_I matrices for the soil and the columns, respectively, are a function of the stiffnesses matrices and fractions of the constituents.

$$D^{eq} = \Omega_s D^s S_1^s + \Omega_c D^c S_1^c \quad (4)$$

Figures 1 and 2 illustrate the typical application of VAT for embankments and excavations, respectively. As the loading is different for these two cases, i.e. mainly vertical loading for the embankment and a combination of vertical unloading and horizontal loading for the excavation problem, the S_I matrices will be different for the two cases.

2.2 Volume averaging for embankment loading

VAT for embankments by Vogler (2008) assumed Voigt assumption in the axial direction and Reuss assumption in the radial direction, similarly to Lee and Pande (1998). The local equilibrium of an embankment on rigid inclusions can thus be formulated as:

$$\Delta\sigma_{xx}^{eq} = \Delta\sigma_{xx}^c = \Delta\sigma_{xx}^s \quad (5)$$

$$\Delta\sigma_{zz}^{eq} = \Delta\sigma_{zz}^c = \Delta\sigma_{zz}^s \quad (6)$$

$$\Delta\tau_{xy}^{eq} = \Delta\tau_{xy}^c = \Delta\tau_{xy}^s \quad (7)$$

$$\Delta\tau_{yz}^{eq} = \Delta\tau_{yz}^c = \Delta\tau_{yz}^s \quad (8)$$

In parallel, the following kinematic constraints are applied, ensuring that there is no split or slip between the soil and the columns:

$$\Delta\varepsilon_{yy}^{eq} = \Delta\varepsilon_{yy}^c = \Delta\varepsilon_{yy}^s \quad (9)$$

$$\Delta\gamma_{zx}^{eq} = \Delta\gamma_{zx}^c = \Delta\gamma_{zx}^s \quad (10)$$

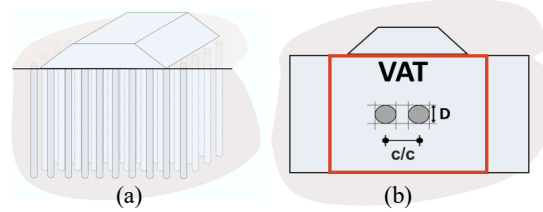


Figure 1. Numerical analyses of an embankment. (a) 3D, (b) 2D-VAT.

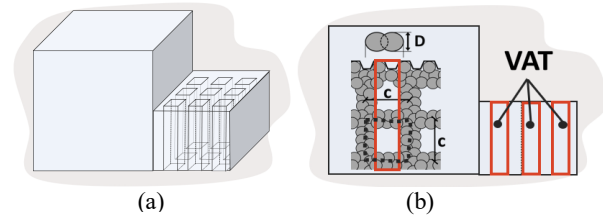


Figure 2. Numerical analyses of an excavation. (a) 3D, (b) 2D-VAT.

Based on these assumptions, it is possible to derive the S_I matrices (see Vogler 2009 for full details). The validation of these assumptions, by systematic comparison between 3D simulation with equivalent VAT simulations, was done in Vogler et al. (2007, 2009). In order to have representative material properties, the model parameters of Vanttila clay (Koskinen & Karstunen, 2004) and triaxial tests result of exhumed cement columns by Aalto (2003) were adopted (see Vogler 2009 for full details). Becker and Karstunen (2013, 2014) highlighted the accuracy and the limitations of the method in terms of the number of columns and the stiffness ratio between columns and soft soil.

In the equivalent 3D simulations of a representative “slice” of an embankment (as presented in Figure 3) is used. A new mesh needs to be created for each centre to centre (c/c) spacing, which is time-consuming. In contrast, with VAT different c/c spacings can effectively be investigated with the same mesh, with the area with columns represented by VAT material (Figure 3b), just changing the input for volume ratio. This gives a powerful tool for optimisation of c/c spacing and column length for a given SLS requirement, as recently demonstrated by Bozkurt et al. (2025b).

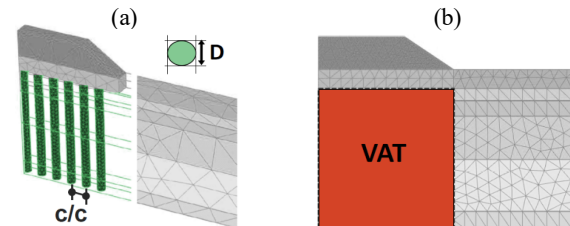


Figure 3. Numerical analyses of an embankment. (a) 3D slice with an out of plane thickness defined by the c/c spacing, (b) the corresponding 2D-VAT.

2.3 Volume averaging for excavations

As the loading condition in an excavation supported by a sheet pile wall and a grid of overlapping columns on the passive side of excavation is more complex, Bozkurt et. al (2025a) performed a series of 3D FE analyses to investigate the representative static equilibrium and kinematic constraints for this particular problem. Again, for the model parameters to be representative of real site conditions, the values for the model parameters were taken from a fully validated case, i.e. the back-analyses of a sheet-pile supported deep excavation with lime-cement columns Central Station in Gothenburg (see Bozkurt et al. (2023) for details) were adopted. Based on 3D simulations, the following local equilibrium conditions were adopted:

$$\Delta\sigma_{zz}^{eq} = \Delta\sigma_{zz}^c = \Delta\sigma_{zz}^s \quad (11)$$

$$\Delta\tau_{yz}^{eq} = \Delta\tau_{yz}^c = \Delta\tau_{yz}^s \quad (12)$$

$$\Delta\tau_{zx}^{eq} = \Delta\tau_{zx}^c = \Delta\tau_{zx}^s \quad (13)$$

In the case of a deep excavation, the following kinematic conditions between the individual materials were assumed:

$$\Delta\varepsilon_{xx}^{eq} = \Delta\varepsilon_{xx}^c = \Delta\varepsilon_{xx}^s \quad (14)$$

$$\Delta\varepsilon_{yy}^{eq} = \Delta\varepsilon_{yy}^c = \Delta\varepsilon_{yy}^s \quad (15)$$

$$\Delta\gamma_{xy}^{eq} = \Delta\gamma_{xy}^c = \Delta\gamma_{xy}^s \quad (16)$$

Details of the \mathcal{S}_I matrices and a systematic validation of VAT for excavations based on the case illustrated in Figure 4 can be found in Bozkurt et al. (2025a). In line with Vogler et al. (2007, 2009), the response of the columns was represented by the MNhard model (Benz 2007), which is a simple deviatoric hardening model with Matsuoka and Nakai failure condition, and the natural clay was represented with the SCLAY1S model developed for modelling soft sensitive clays (Karstunen et al. 2005).

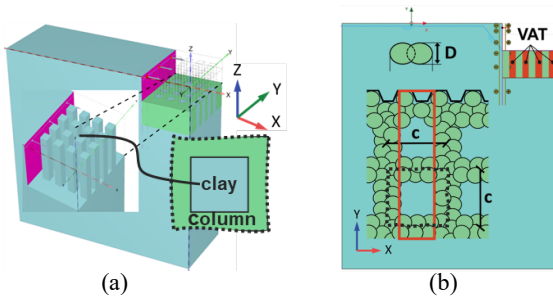


Figure 4. Numerical analyses of a braced excavation. (a) 3D, (b) 2D-VAT.

Figure 5 illustrates how well the 2D simulations of this deep excavation with VAT can reproduce the movements of a sheet pile wall (SPW) during two excavation stages.

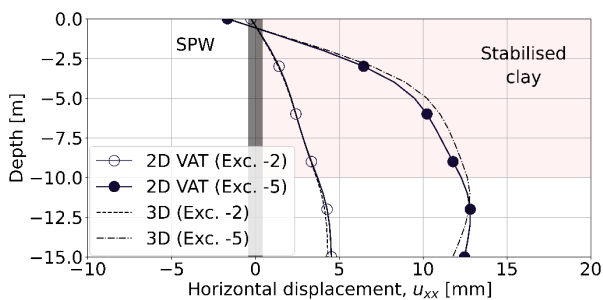


Figure 5. Comparison of the displacement profile of the SPW between the 2D and 3D simulations.

3 APPLICATION TO PAIMIO TEST EMBANKMENTS

Trial embankments were constructed by Finnish Road Administration in 1989 in Paimio located at about 30 km east of Turku in Finland (Vepsäläinen and Arkima, 1992). The embankments were made of slightly compacted well-draining sand with an average unit weight $\gamma = 18.5 \text{ kN/m}^3$. One of these embankments one was constructed on natural soil without any improvement, and the other on a clay improved deep-mixed columns using cement as a binder. The columns had 1.0, 1.2 and 1.4 m c/c distances (see Figure 6.). The thickness of the soft clay at the test site varied between 8-13 m. All columns were end-bearing, reaching the permeable stiffer stratum at the bottom. The average diameter of the columns was 0.6 m. Due to the variations in the clay depth and the inclination of the stiff stratum, the columns lengths and the height of the embankments were varying, which was important to consider in the modelling (see Table 1).

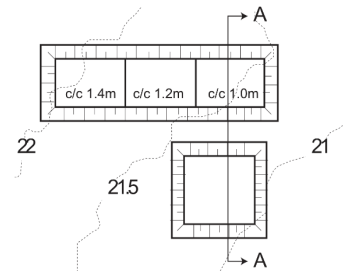


Figure 6. Paimio test embankments (from Abed et al. *in press*).

Table 1. The variations in the geometry of the trial embankments on deep mixed columns with different c/c spacings.

c/c [m]	Ω_c [-]	Emb. height [m]	Emb. width [m]	L [m]	3D strip width [m]
1.0	0.283	2.1	9.6	9.6	1.0
1.2	0.196	1.9	10.0	9.2	1.2
1.4	0.144	1.7	10.0	7.5	1.4

The embankments were instrumented for measuring displacements, porewater pressures and earth pressures in the soil and the columns etc., and the monitoring was conducted over two years (1989-1991). The full set of field measurements can be found in Vepsäläinen and Arkima (1992). The Modified Cam Clay model parameters of the clay layers were derived by Vepsäläinen et al. (1991) and Vepsäläinen and Arkima (1992) based on 1D compression tests and triaxial tests conducted at Helsinki University of Technology. These were utilised to derive the parameters for the SCLAY1S model. The additional parameters for anisotropy and bonding were calibrated using the procedures suggested by Koskinen et al. (2002) and Wheeler et al. (2003).

First, the embankment without ground improvement was simulated, using both axisymmetric and plane strain analyses (see Abed et al, *in press*). The latter enabled direct comparison with the embankment with deep mixed columns. The 2D calculations with VAT were carried out for the three different cases of column spacings considering the geometrical differences, as listed Table 1. The predicted vertical settlements at a point directly under the centreline of the embankment, calculated using VAT, are compared to the measured values in Figure 7. Just for context, the predicted and measured greenfield settlements had a value of about 80 mm after 800 days. Thus, with all column configurations, there was a significant improvement in terms of settlements already in the

very short term. The difference between the VAT predictions and 3D predictions were only a few percent for c/c from 1.0 m and 1.4 m, and therefore negligible. Overall, the VAT predictions match well with the field measurements (Figure 7). The clear outlier was the c/c spacing of 1.4m, where VAT predictions (and 3D simulations, see Abed et al, *in press*) for the lowest embankment height underpredict the settlements during the first year. Furthermore, due to extensive piling to the stiff stratum for the actual motorway, located only 25 m away from the ramp where the test embankments were situated, the measured pore pressures started to increase at depths greater than 6 m after 400-600 days. Consequently, the embankment with deep mixed columns started to lift, while this was not “felt” in the same way by the greenfield embankment. As this pore pressure increase was not considered in the numerical simulations, there are some discrepancies between the VAT predictions and field measurements from 600 days onwards.

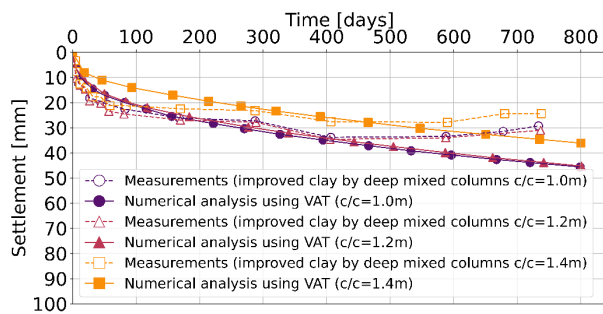


Figure 7. Time-dependent settlements for test embankment in Paimio with various column spacing (data from Abed et al. *in press*).

4 CONCLUSIONS

An enhanced 2D technique using so-called Volume Averaging Technique (VAT) is utilised for system-level modelling of geostructures on deep mixed columns. VAT enables to describe the column-improved ground as a homogenised composite material, with an averaged stiffness matrix derived analytically based on the given boundary conditions to map the true 3D problem into 2D. Once the constitutive relations for both constituents are defined, the time-dependent response of the column-improved ground can be simulated in 2D plane strain conditions subject to arbitrary loading and boundary conditions. Two problems were considered, an embankment on soft clay stabilised with deep mixed columns and an excavation supported by sheet pile wall and overlapping columns in the passive side of the excavation. The results demonstrate that VAT is a powerful tool for system-level predictions and optimisation, as various columns configurations can be accounted for by simply changing just one input parameter, the volume fraction Ω_c .

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