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Mathematical development of stress influence factors for rectangular foundations based on the Schmertmann method (1978)

Christian Camacho
Laboratory of Geotechnics, San Simon University, Bolivia, ch.camacho@umss.edu

ABSTRACT: In 1970, Schmertmann proposed a settlement calculation procedure for continuous foundations placed on granular soils, based on Cone Penetration Tests (CPT) results. Later, in 1978, he presented the distribution of stress influence factors to depth, highlighting two distinct conditions: one for a square footing (L/B = 1) and another for a continuous footing (L/B = 10). The stress influence depth for the square condition is 2B, and for the continuous condition, it is 4B. The equations to calculate the stress influence factors at any depth for both cases are clearly identified. In a rectangular condition (L/B ratio greater than 1 and less than 10), Schmertmann suggests using a linear interpolation for the distribution of the stress influence factor. However, no specific details are provided, such as the depth at which the influence of stress is zero. Recent studies, such as those by Terzaghi *et. al.* (1996), Lee *et. al.* (2008) and Salgado (2008), present equations that address the rectangular situation based on approximations to those proposed by Schmertmann. This article mathematically develops the interpolation method proposed by Schmertmann (1978) for rectangular conditions. It establishes the distribution of stress influence factors with depth, revealing two peaks, and provides equations for the depth of influence as well as the stress influence factors at varying depths.

KEYWORDS: Schmertmann method, stress factor influence, rectangular foundation.

1 INTRODUCTION

Settlement estimation es a fundamental aspect of foundation design. The Schmertmann method is widely used today in practice to estimate settlements under the center of the footing. The Schmertmann method (1978) is the most widely used method for calculating immediate settlements in granular soils. In square or continuous conditions, the method provides a complete and clear procedure. However, for rectangular foundations, the method only provides a linear interpolation equation, without specifying the depth at which the stress influence factor becomes zero.

This detail led several authors to propose improvements or procedures to address the calculation of settlements in rectangular foundations. Some of these authors were Terzaghi *et. al.* (1996), Lee *et. al.* (2008) and Salgado (2008).

This article presents the mathematical development of the influence factors for rectangular foundations based on the Schmertmann method.

2 SCHMERTMANN-HARTMAN-BROWN METHOD

The Schmertmann-Hartman-Brown method is commonly used for estimation of elastic (immediate) settlement of shallow foundations. In this method, immediate settlement (S_i) of shallow foundations is calculated by Equation (1) as seen below.

$$S_i = C_1 C_2 C_3 \left(q - \sigma'_{zD}\right) \sum_{i=1}^{n} \frac{\Delta z_i I_{zi}}{E_{si}} \tag{1}$$

where

$$C_1 = 1 - 0.5\left(\frac{\sigma'_{zD}}{q - \sigma'_{zD}}\right) \tag{2}$$

Correction for strain relief due to excavation,

$$C_2 = 1 + 0.2\left(\frac{t}{0.1}\right) \tag{3}$$

Correction for creep,

$$C_3 = 1.03 - 0.03\left(\frac{L}{B}\right) \geq 0.73 \tag{4}$$

where q is the gross contact pressure of footing; σ'_{zD} is the effective stress at the base level of footing before the construction; Δz_i is the thickness of soil layer i; I_{zi} is the strain influence factor of layer i; E_{si} is the modulus of elasticity of layer i; B is the width of foundation; L is the length of foundation. According to Figure 1, the exact value of I_{zi} at any depth can be determined as follows: for square and circular footings (L/B=1):

$$I_{zs} = 0.1 + \frac{z}{B}\left(2I_{ps} - 0.2\right) \qquad 0 \leq z \leq \frac{B}{2} \tag{5}$$

$$I_{zs} = \frac{2}{3} I_{ps}\left(2 - \frac{z}{B}\right) \qquad \frac{B}{2} \leq z \leq 2B \tag{6}$$

For continuous footings (L/B≥10):

$$I_{zc} = 0.2 + \frac{z}{B}\left(I_{pc} - 0.2\right) \qquad 0 \leq z \leq B \tag{7}$$

$$I_{zc} = \frac{1}{3} I_{pc}\left(4 - \frac{z}{B}\right) \qquad B \leq z \leq 4B \tag{8}$$

For the rectangular foundations in which the length is less than or equal to ten times the width, a linear interpolation between the axisymmetric and plane strain cases is performed, dependent on the length to width ratio. For the rectangular foundations ($1 < L/B \leq 10$):

$$I_{zr} = I_{zs} + \frac{1}{9}\left(I_{zc} - I_{zs}\right)\left(\frac{L}{B} - 1\right) \tag{9}$$

The procedure for determining I_{zr} consists of calculating the deformation influence factors for both square footings (I_s) and continuous footings (I_c). Once these values have been calculated, they must be combined in Equation (9). However, no specific details are provided, such as the depth at which the influence of stress is zero.

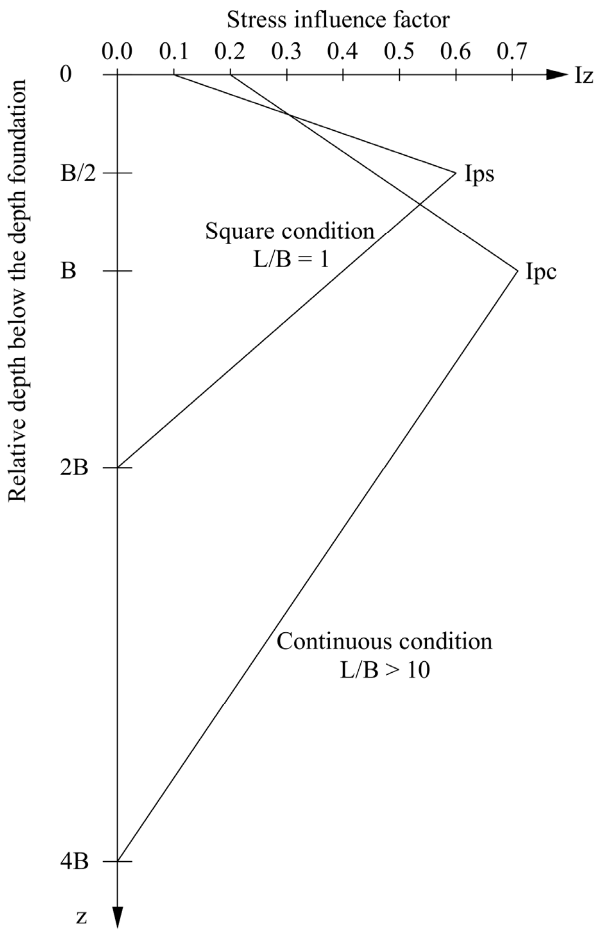

Figure 1. Variation of stress influence factors for square/circular and continuous foundations (Schmertmann et al., 1978).

3 MATHEMATICAL DEVELOPMENT

Mathematical development is based on interpolating the graphs for square conditions and continuous conditions according to Schmertmann's original relationship. From this formulation, expressions can be derived to determine the depth at which the influence factor becomes zero.

For the procedure, the ratio factor is defined as:

$$R = \left({}^{L}/_{B} - 1\right)/9 \tag{10}$$

Equation (9) can then be rewritten as:

$$I_{zr} = I_{zs} + (I_{zc} - I_{zs})R \tag{11}$$

This equation separates Schmertmann's original graph into sections, as shown in Figure 2.

3.1 *Section A-B (0 ≤ z ≤ B/2)*

As shown in Figure 3, for point A, the stress influence factor for rectangular condition at depth z=0 ($I_{zr\text{-}A}$) is:

$$I_{zr-A} = I_{zs} + (I_{zc} - I_{zs})R \tag{12}$$

$$I_{zr-A} = 0.1 + (0.2 - 0.1)R = 0.1 + 0.1R \tag{13}$$

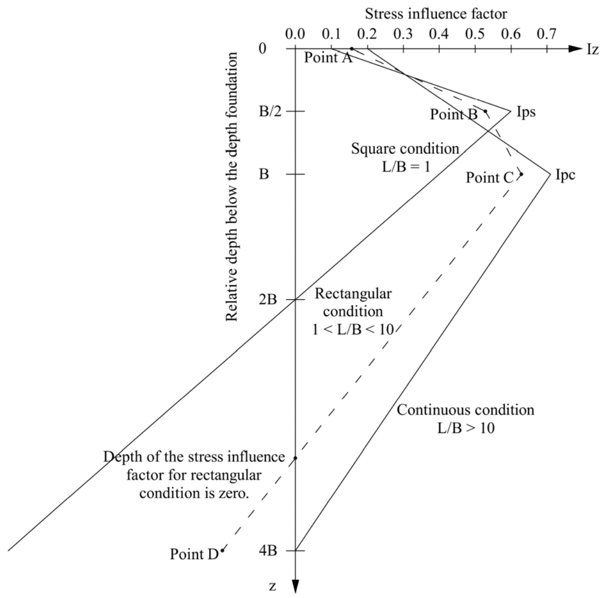

Figure 2. Diagram of stress influence factors for square, rectangular and continuous foundations based on Schmertmann et al. (1978).

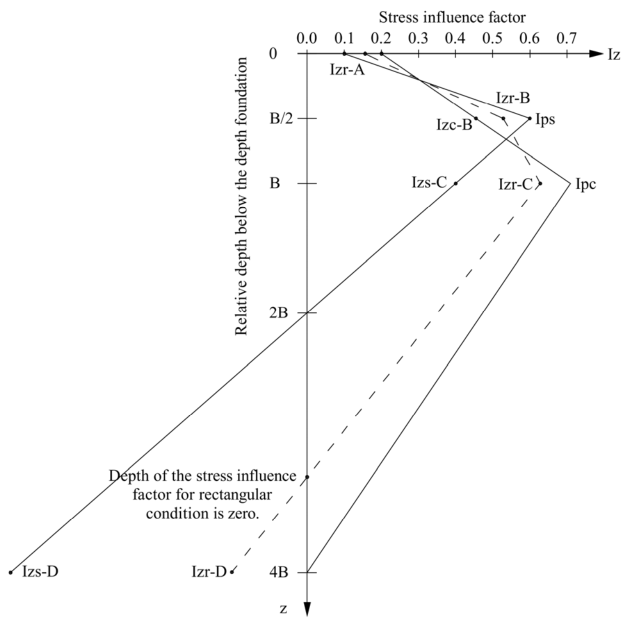

Figure 3. Detail of stress influence factors of section A-B.

For point B, the stress influence factor for continuous condition at depth z=B/2 is:

$$I_{zc-B} = 0.2 + \frac{(B/2)}{B}\left(I_{pc} - 0.2\right) = 0.1 + \frac{I_{pc}}{2} \tag{14}$$

From this, the stress influence factor for rectangular condition is obtained as ($I_{zr\text{-}B}$) using equation (14) in equation (11).

$$I_{zr-B} = I_{ps} + \left(I_{zc-B} - I_{ps}\right)R \tag{15}$$

$$I_{zr-B} = I_{ps} + \left(0.1 + \frac{I_{pc}}{2} - I_{ps}\right)R \tag{16}$$

Then, the equation that represents the stress influence factor line for section A-B (0 ≤ z ≤ B/2) has been deduced:

$$I_{zr} = 0.1 + 0.1R + \left(2I_{ps} + \left(I_{pc} - 2I_{ps}\right)R - 0.2\right)(z/B) \quad (17)$$

3.2 *Section B-C (B/2 ≤ z ≤ B)*

According to Figure 3, for point C, the stress influence factor for square condition at depth z=B ($I_{zs\text{-}C}$) is:

$$I_{zs-C} = \frac{2}{3}I_{ps}\left(2 - \frac{B}{B}\right) = \frac{2}{3}I_{ps} \quad (18)$$

From which it obtains, the stress influence factor for rectangular condition ($I_{zr\text{-}C}$) using equation (18) in equation (11).

$$I_{zr-C} = I_{zs-C} + \left(I_{pc} - I_{zs-C}\right)R \quad (19)$$

$$I_{zr-C} = \frac{2}{3}I_{ps} + \left(I_{pc} - \frac{2}{3}I_{ps}\right)R \quad (20)$$

Then, the equation that represents the stress influence factor line for section B-C (B/2 ≤ z ≤ B) has been deduced:

$$I_{zr} = \frac{4}{3}I_{ps} + \left(0.2 - \frac{4}{3}I_{ps}\right)R + \left(\left(I_{pc} + \frac{2}{3}I_{ps} - 0.2\right)R - \frac{2}{3}I_{ps}\right)(z/B) \quad (21)$$

3.3 *Section C-D (B ≤ z ≤ 4B)*

According to Figure 3, for point D, the stress influence for square condition factor at depth z=4B ($I_{zs\text{-}D}$) is:

$$I_{zs-D} = \frac{2}{3}I_{ps}\left(2 - \frac{4B}{B}\right) = -\frac{4}{3}I_{ps} \quad (22)$$

From which it obtains, the stress influence factor for rectangular condition ($I_{zr\text{-}D}$) using equation (22) in equation (11).

$$I_{zr-D} = I_{zs-D} + \left(0 - I_{zs-D}\right)R \quad (23)$$

$$I_{zr-D} = -\frac{4}{3}I_{ps} + \frac{4}{3}I_{ps}R \quad (24)$$

Then, the equation that represents the stress influence factor line for section C-D (B ≤ z ≤ 4B) has been deduced:

$$I_{zr} = \frac{4}{3}\left(I_{ps} + \left(I_{pc} - I_{ps}\right)R\right) + \left(\left(I_{ps} - \frac{I_{pc}}{2}\right)R - I_{ps}\right)(2z/3B) \quad (25)$$

3.4 *Depth I_{zr}=0*

By setting equation (25) equal to 0, the depth where the stress influence factor for rectangular condition is zero is determined.

$$0 = \frac{4}{3}\left(I_{ps} + \left(I_{pc} - I_{ps}\right)R\right) + \left(\left(I_{ps} - \frac{I_{pc}}{2}\right)R - I_{ps}\right)(2z/3B) \quad (26)$$

$$-4B\left(I_{ps} + \left(I_{pc} - I_{ps}\right)R\right) = \left(\left(I_{ps} - \frac{I_{pc}}{2}\right)R - I_{ps}\right)2z \quad (27)$$

$$z = \frac{4B\left(I_{ps} + \left(I_{pc} - I_{ps}\right)R\right)}{2I_{ps} + \left(I_{pc} - 2I_{ps}\right)R} \quad (28)$$

For rectangular condition, Equation 28 represents the depth at which the stress influence factor ends.

4 COMPARISON WITH OTHER METHODS

4.1 *Terzaghi et al (1996)*

Terzaghi et. al. (1996) presents the following relationships for the interpolation of Iz_0 at z=0, Z_1 and Z_2 for rectangular conditions.

$$Iz_0 = 0.2 \quad (29)$$

$$Z_1 = B/2 \quad (30)$$

$$Z_2 = 2B[1 + \log(L/B)] \qquad L/B \leq 10 \quad (31)$$

4.2 *Salgado (2008)*

Salgado (2008) presents the following relationships for the interpolation of Iz_0 at z=0, Z_1 and Z_2 for rectangular conditions.

$$Iz_0 = 0.1 + 0.0111(L/B - 1) \leq 0.2 \quad (32)$$

$$\frac{Z_1}{B} = 0.5 + 0.0555(L/B - 1) \leq 1 \quad (33)$$

$$\frac{Z_2}{B} = 2 + 0.222(L/B - 1) \leq 4 \quad (34)$$

4.3 *Lee et al (2008)*

Lee *et. al.* (1996) presents the following relationships for the interpolation of Iz_0 at z=0, Z_1 and Z_2 for rectangular conditions.

$$Iz_0 = 0.1 + 0.0111(L/B - 1) \leq 0.2 \quad (35)$$

$$\frac{Z_1}{B} = 0.5 + 0.11(L/B - 1) \quad (36)$$

$$\frac{Z_2}{B} = 0.95cos\left\{\left[\frac{\pi}{5}\left(\frac{L}{B} - 1\right)\right] - \pi\right\} + 3 \quad L/B \leq 6 \quad (37)$$

5 HYPOTHETICAL CASE

For the hypothetical case, the foundation and soil profile seen in Figure 4 are employed.

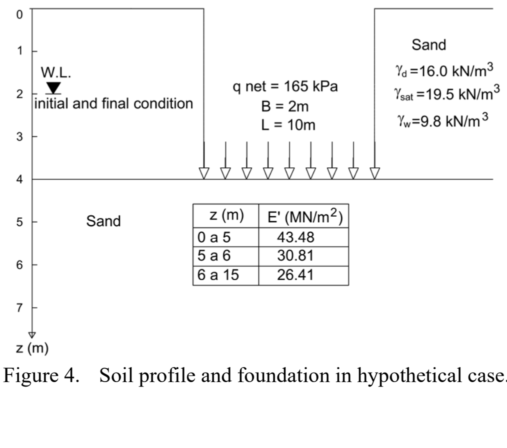

Figure 4. Soil profile and foundation in hypothetical case.

Figure 5 summarizes the variation of the stress influence factor for all methods evaluated. The Table 1 presents a summary of the results.

Table 1. Summary of results.

Method	Iz_0 (-)	Z_1 (m)	Ip (-)	Z_2 (m)	Si (m)
Terzaghi et. al. (1996)	0.200	1.00	0.644	6.79	9.38
Salgado (2008)	0.322	1.44	0.658	5.77	8.29
Lee et. al. (2008)	0.322	1.88	0.653	7.89	12.20
Mathematical development (2025)	0.351	1.00 2.00	0.554 0.535	5.13	7.04

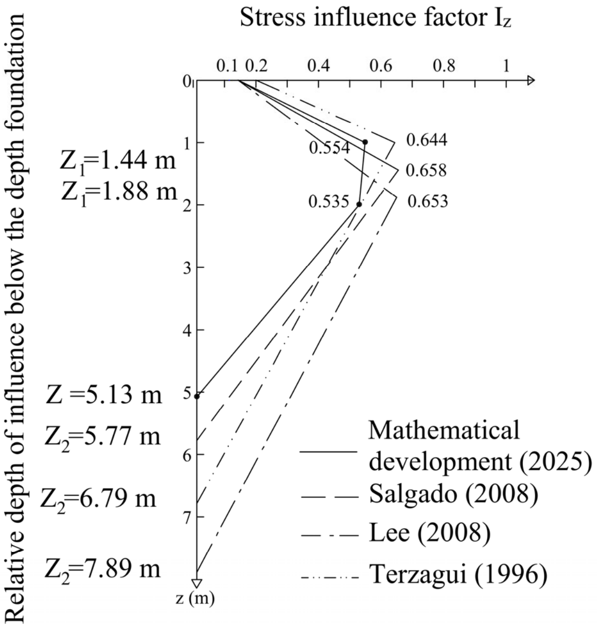

Figure 5. Summary of stress influence factors for rectangular condition.

6 CONCLUSIONS

The mathematical development presented in this study provides an explicit formulation for calculating the stress influence factor for rectangular foundations, extending the procedure originally proposed by Schmertmann (1978). The resulting expressions allow the stress influence distribution to be defined continuously with depth and make it possible to determine the depth at which the influence factor becomes zero.

Application of the formulation to a hypothetical soil profile demonstrates that the predicted influence depth (Z_2) and corresponding settlement values differ noticeably from those obtained using existing empirical approaches. These differences arise precisely from the explicit definition of Z_2, which governs the extent of the stress bulb in the rectangular condition.

Overall, the proposed development offers a more consistent and theoretically grounded alternative for estimating stress influence factors in rectangular foundations, while maintaining compatibility with the classical square and continuous footing diagrams. Further validation using numerical analyses and experimental data is recommended to reinforce the practical applicability of the method.

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