

A multi-criteria framework for p - y curve calibration from lateral load test data

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ABSTRACT: Interpreting the full pile foundation response from lateral load tests is challenging due to noise in field measurements and the amplification thereof throughout the numerical differentiation process. This study presents an integrated approach that improves the stability and physical plausibility of pile profiles and p - y curves derived from inclinometer and strain gauge data. The method combines both datasets through Levenberg–Marquardt nonlinear regression, incorporates measured pile head boundary conditions into the residual formulation, and applies Savitzky–Golay smoothing to reduce unrealistic oscillations without distorting underlying trends. Results from field tests on laterally loaded steel piles in sand show that the combination of datasets through the Levenberg–Marquardt approach yields profiles that represent both measurement types with a single curve; that boundary-informed fitting adjusts the profiles to better match the known pile head loads; and that smoothing can effectively dampen profile spikes when present. The resulting p - y curves exhibit characteristics more consistent with soil mechanics expectations, when compared to profiles and p - y curves generated from traditional least-squares fits.

KEYWORDS: Piles, deep foundations, lateral loading, p - y curves, nonlinear regression

1 INTRODUCTION

1.1 Background theory

Understanding the internal bending and shear forces in a pile foundation, as well as the p - y response along the pile-soil interface, is essential for interpreting lateral load test behavior. Accordingly, Euler-Bernoulli beam theory is applied to the test data to calculate these loads and reactions. Under this framework, the pile deflection (y), slope (S), curvature (ϕ), internal moment (M), internal shear (V), and external load or soil reaction (p) are related through the following equations:

$$S(x) = \frac{dy}{dx} \quad (1)$$

$$\phi(x) = \frac{d^2y}{dx^2} \quad (2)$$

$$M(x) = EI * \frac{d^2y}{dx^2} \quad (3)$$

$$V(x) = EI * \frac{d^3y}{dx^3} \quad (4)$$

$$p(x) = EI * \frac{d^4y}{dx^4} \quad (5)$$

Where x = distance along the beam and EI = the bending stiffness. Because it is difficult to measure these internal loads and p - y relationships directly, reverse analysis is performed through measurements of pile displacement and curvature obtained from inclinometers and strain gauges, respectively. After fitting smooth curves to these measurements, the Euler-Bernoulli beam theory is applied to reconstruct deflection, rotation, curvature, bending moment, shear, and soil reaction profiles. Subsequent pairing of the soil reaction with deflection yields site-specific p - y curves.

While the Euler-Bernoulli beam theory is well-established, the inclinometer and strain gauge data are often noisy, and up to four derivations need to be performed to obtain all pile profiles. Numerical differentiation is a noise-amplifying process, and when the nonlinear bending stiffness relationship is combined with data noise, seemingly insignificant fluctuations in the data/fitted curve can lead to large, unrealistic oscillations in pile profiles downstream of the derivation. Therefore, methods for containing the pile profiles within known boundaries, or smoothing the profiles within reasonable accuracy are desirable tools to produce a lateral load test analysis based on reality.

1.2 Literature review

A variety of approaches have been developed to fit functions to test data and mitigate noise amplification in the differentiation process. When full pile profiles – displacement, slope, curvature, moment, shear, and soil reaction – are not required, some researchers have fit piecewise low-order polynomials using the linear least squares (LLS) method to strain gauge to calculate p and y at discrete depths corresponding to the center of the polynomial segments (Matlock & Ripperger 1956, Morrison & Reese 1986, Little & Briaud 1987). Others have fit LLS high-order polynomials (Pathak et al. 2011, Haiderali & Madabhushi 2016, Li et al. 2019) or various spline formulations (Georgiadis et al. 1992, Dou & Byrne 1996, Soltani & Muraleetharan 2015, Haiderali & Madabhushi 2016) to represent lateral load test data, and then derived or integrated these functions to obtain full pile profiles. Ting 1987 fit a high-order polynomial to strain gauge data while applying constraints to the derivatives of the polynomial. Some authors have applied the strong smoothing effect of fitting a LLS cubic polynomial to strain gauge data, producing a linear p profile upon the second derivative (El Naggar & Wei 1999, Rovithis et al. 2009) while others have applied simple finite-difference slopes between strain gauge points, yielding midpoint estimates rather than continuous profiles (Komolafe 2024). Sinnreich & Ayithi 2014 proposed a combined use of inclinometer and strain gauge data when both are available. In their method, the inclinometer data are differentiated twice to obtain curvature, and a high-order polynomial is fit jointly to both datasets. An exponential decay coefficient is applied to suppress derivative-induced oscillations at depth.

Various researchers have employed computational techniques rooted in finite element (FE) formulations – particularly weak-form approaches such as the variational and weighted residual methods – to interpret lateral load test data and develop p - y curves. Wilson (1998) applied the Galerkin method of weighted residuals using piecewise linear basis functions to strain gauge data obtained from centrifuge tests on aluminum piles. Numerical differentiation and integration were used to obtain pile displacement, slope, shear, and soil reaction profiles. Brandenburg et al. (2010) extended Wilson’s weighted residual approach by introducing a Fourier series expansion to assess how accurately derivatives are computed at different spatial frequencies of the moment profile, quantify data uncertainty, and evaluate error propagation. Lin and Liao (2006) employed a Rayleigh-Ritz variational method to

minimize the total potential energy of the pile-soil system using a Fourier series as the basis function to describe inclinometer data from full scale load tests on steel and concrete piles. Displacement data were numerically differentiated using the Cesaro summation technique to obtain complete pile profiles. De Sousa Coutinho (2006) used a Galerkin-based weighted least squares method to fit B-spline basis functions to strain gauge data from full scale load tests on concrete piles and pile groups. The B-spline functions were analytically integrated to obtain displacement and slope profiles, and a Volterra integral was solved to relate bending moment and soil reaction profiles.

1.3 Objectives

To reduce the propagation of noise-induced effects through multiple derivatives, this study introduces an integrated approach – comprising of three complementary techniques – aimed at improving the reliability of pile profile interpretation from lateral load test data. The first technique combines inclinometer and strain gauge data into a single representative curve using nonlinear regression via Levenberg-Marquardt optimization. Thereafter, boundary conditions, such as measured pile head moment and shear are imposed through constrained nonlinear regression to guide the fitted curve toward known pile end conditions. The third step applies Savitzky-Golay smoothing filters to pile moment-curvature and internal shear and moment profiles to dampen unrealistic oscillations and increase the reliability of the profiles. These techniques, and their application, are described in detail in the following section, and are intended to constrain the pile response within physically plausible bounds and extract more stable p - y curves.

2 METHODOLOGY

2.1 Combination of Inclinometer & strain gauge data

When both strain gauge and inclinometer data are available, it can be beneficial to use both datasets simultaneously. However, the datasets exist in separate parameter spaces – pile displacement (y) for inclinometer data and pile curvature (ϕ) for strain gauge data. As a compromise between the two datasets, this study implements the Levenberg-Marquardt (LM) nonlinear regression algorithm (Levenberg 1944; Marquardt 1963), which solves for the parameter vector, θ , by minimizing the sum of squared residuals, $[\varepsilon(\theta)]^2$, using an iterative Gauss-Newton-style parameter update with a damping factor. For the compromise between the inclinometer and strain gauge data, the residual vector is defined as:

$$\varepsilon(\theta) = w_1(\tilde{y} - f(\theta)) + w_2\left(\tilde{\phi} - \frac{d^2f(\theta)}{dx^2}\right) \quad (6)$$

Where $\varepsilon(\theta)$ = the residual vector to be optimized in the LM algorithm, w_1 and w_2 = weights for the respective residual component, \tilde{y} = the measured pile displacement data, $\tilde{\phi}$ = the measured pile curvature data, and $f(\theta)$ = a pile displacement function modeled as a polynomial (typically of order ≥ 6) with coefficients θ . Instead of using repeated, randomized starting points for the LM optimization of θ , which is inefficient and difficult in producing reasonable results because the polynomial coefficients do not have any physical meaning, this approach uses the linear least squares (LLS) solution of the pile displacement data as the starting point. This provides an initial curve that fits the displacement data well, and the LM algorithm adjusts that curve to better match the available pile curvature data, thus creating a combination between the two data sets. The weights are adjusted to produce the best compromise. Because optimization begins from a displacement-based fit, the final

result will naturally tend to favor the pile displacement measurements if residual components are weighted evenly.

Alternatively, this process can begin using the LLS solution of the pile curvature data as the start point. In this case, the curvature-based function is twice integrated with known boundary conditions/integration constants to compare against measured displacements. Similarly, this approach will then favor the pile curvature measurements if residual components are weighted evenly.

2.2 Pile end boundary-informed curve fitting

During pile foundation lateral load testing, boundary conditions are often known at both the pile head and toe. These include shear and moment loads at the pile head, and displacement and slope at the pile toe. Between the load application point and the ground surface, the pile shear equals the applied lateral load, and the pile moment varies linearly from zero at the load application point to the product of the applied load and the distance to the ground surface. If the pile foundation behaves as a long or flexible pile, rather than a short or rigid pile, zero displacement and zero slope can be assumed at the pile toe or fixity point. These known boundary conditions can be entered into the LM framework described in section 2.1, modifying the residual vector as follows when fitting a displacement function to the inclinometer data:

$$\begin{aligned} \varepsilon(\theta) = & w_1(\tilde{y} - f(\theta)) + \\ & w_2\left(\tilde{\delta}_{\text{toe}} - \frac{df(\theta)}{dx}\bigg|_{\text{toe}}\right) + \\ & w_3\left(\tilde{V}_{\text{head}} - EI * \frac{d^2f(\theta)}{dx^2}\bigg|_{\text{head}}\right) + \\ & w_4\left(\tilde{M}_{\text{head}} - EI * \frac{d^3f(\theta)}{dx^3}\bigg|_{\text{head}}\right) \end{aligned} \quad (7)$$

Where w_1 , w_2 , w_3 , and w_4 = the residual weights; $\tilde{\delta}_{\text{toe}}$ = the pile slope at the toe (equal to 0); $f(\theta_{\text{toe}})$ = the pile displacement function evaluated at the pile toe; \tilde{V}_{head} = the shear at the pile head (equal to the applied load); and \tilde{M}_{head} = the corresponding moment at the pile head. The weights can initially be set equal to 1.0 and adjusted to emphasize or de-emphasize specific boundary conditions during fitting. This boundary-informed framework can be applied independently to a pile displacement function (as shown above) or curvature function, or the framework can be applied in combination with the inclinometer / strain gauge combination approach described in Section 2.1. Once again, the LLS solution of the pile displacement data is used as the start point, and the LM algorithm iteratively modifies the curve to satisfy both the measured data and imposed boundary conditions.

2.3 Savitzky-Golay curve smoothing

As stated previously, smooth profiles are desirable for differentiation within the Euler-Bernoulli beam framework. However, these profiles are often not smooth due to data noise and nonlinear M - ϕ relationships. To address this, a Savitzky-Golay filter (Savitzky and Golay 1964) is employed to smooth the data while preserving the original signal. The filter operates by performing successive, local LLS fits of a low-order polynomial over a defined frame length (number of data points) and replacing each central point with the fitted value. The authors propose applying a Savitzky-Golay filter to the M - ϕ relationship as well as the calculated M and V profiles. The frame length and polynomial order should be selected to preserve the shape and magnitude of the original while reducing noise and limiting rapid fluctuations in profile slope.

3 CASE STUDY

3.1 Experimental program

A large-scale lateral load test was conducted on a steel pipe pile measuring 16 inches in diameter, 16 feet in length, and 0.25 inches in wall thickness. The pile was fabricated from ASTM A992 Grade 50 steel and installed in the Structural Engineering Testing Hall at the University of California, Irvine. The test pile was secured inside the 20 x 30 x 14-foot-deep reinforced concrete “Soil Pit” with an 8-inch-thick reinforced concrete base at the pile toe. Poorly graded sand was then pluviated around the test pile with a target relative density of approximately 20%.

The pile was subjected to a displacement-controlled reverse cyclic (loaded in the push and pull directions) lateral load test. A total of 30 pile head displacement increments were imposed, ranging from 0.01 inches up to 10 inches, with typically three cycles per increment in each direction. Figure 1 presents an engineering drawing of the test setup, and Figure 2 shows photographs taken during construction. While some photographs show a total of six piles, this paper focuses exclusively on the steel pipe pile.

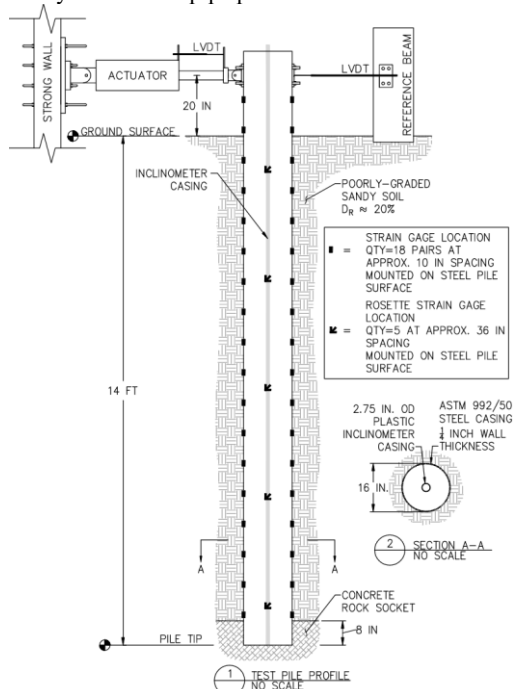


Figure 1. Test configuration.



Figure 2. Construction photos (left: test pile secured in the Soil Pit; top right: sand pluviation; bottom right: during testing).

3.2 Instrumentation and measurements

Instrumentation included a centrally-located inclinometer with readings at 6-inch intervals, and pairs of electric resistance strain gauges installed every 10 inches along the pile. A linear variable differential transducer (LVDT) was positioned 20 inches above the ground surface at the lateral load application point to measure displacement. A load cell was mounted on the hydraulic actuator to record the applied lateral load. Figure 1 includes a detailed schematic of the test pile with the instrumentation layout.

4 ANALYSIS OF TEST DATA

4.1 Methodology implementation and parameter selection

A sixth-order polynomial LLS fit was applied to both the displacement and curvature data, and all subsequent profiles and p - y curves were calculated using Euler-Bernoulli beam theory. This initial set of profiles serves as a baseline for comparison with the two modified approaches – LLS with pile end boundary-informed curve fitting (Section 2.2) and LLS with pile end boundary-informed curve fitting combined with Savitzky-Golay smoothing (Section 2.3). For each approach, three sets of profiles were generated: one using only inclinometer data, one using only strain gauge data, and one using the inclinometer / strain gauge combination (Section 2.1).

4.1.1 Inclinometer / strain gauge combination parameters

The Levenberg-Marquardt algorithm was evaluated in MATLAB using the built-in `lsqnonlin()` function with the residual vector defined in Section 2.1. The finite difference step size was iteratively selected to balance the contributions of the inclinometer and strain gauge data. Due to the small magnitude of the polynomial coefficients – often as low as $1.0E-20$ in this study and in lateral load testing more broadly – a finite difference step size of $1.0E-10$ was chosen to encourage algorithm convergence and numerical precision. Step sizes that are too large produce a poor Jacobian approximation, resulting in inaccurate search directions, while step sizes that are too small introduce numerical noise, causing slow or unstable convergence. All other `lsqnonlin()` settings were left at their default values.

A weight of 1.0 was applied to the residual between the fitted displacement profile and the inclinometer data, and a weight of $1.0E+6$ was applied to the residual between the second derivative of the fitted displacement profile and the curvature derived from strain gauge data. Because the nonlinear regression was initialized with the LLS fit to the inclinometer data, the large curvature weight was necessary to escape the local minimum associated with that starting point and to ensure that the strain gauge data meaningfully influenced the resulting pile profiles.

4.1.2 Pile end boundary-informed curve fitting parameters

In the pile end boundary-informed curve fitting procedure (Section 2.2), only the pile head conditions were input to the residual equation. The pile toe was embedded in 8 inches of reinforced concrete, resulting in erratic behavior at the base that is inconsistent with the rest of the profile. Applying pile toe constraints in this case only increased the nonlinearity of the objective function without improving the resulting profiles. The authors note that, for a pile with a fixed toe in an all-soil profile, including toe conditions in the residual vector could be beneficial. A uniform weight of 1.0 was applied to all residual vector components, except for the inclinometer / strain gauge combination term, which was assigned a weight of $1.0E+6$ as outlined in Section 4.1.1.

4.1.3 Savitzky-Golay curve smoothing parameters

Savitzky-Golay smoothing parameters – polynomial order and frame length – were iteratively selected to minimize distortion of the original profiles while reducing spikes, oscillations, and abrupt changes in slope. For the M and V profiles, 2nd- and 1st-order polynomials were used with a frame length of 13 for both. For the M - ϕ relationship, a 7th-order polynomial and frame length of 13 were applied. The smoothed M - ϕ curve closely matched the original, with negligible differences when plotted together. The original M - ϕ relationship was developed using the computer program SAP2000 (version 16, Computers and Structures, Inc). The smoothing effects of the polynomial order and frame length are interdependent, but in general, a frame length that is too large can over-smooth the data, while a frame length that is too small may not sufficiently reduce noise. Similarly, a polynomial order that is too high can introduce oscillations, while an order that is too low may fail to capture the key features of the data.

4.2 Results

Figures 3 and 4 present the complete pile profiles – pile deflection (y), slope (S), curvature (ϕ), internal moment (M), internal shear (V), and lateral soil reaction (p) – arranged from left to right across each column for 1-inch and 4-inch pile head

displacement, respectively. Each figure shows results from three data sources and three curve-fitting procedures. Blue symbols and lines represent inclinometer measurements and their derived profiles, red symbols and lines represent strain gauge data and their derived profiles, and purple lines represent the inclinometer / strain gauge combination profiles obtained via nonlinear regression (Section 2.1). In each figure, the top row shows profiles generated from a sixth-order polynomial LLS fit applied to the inclinometer (blue) and strain gauge (red) data, with the combination profiles (purple) obtained from the LLS starting point followed by nonlinear regression. The middle row uses the same starting profiles as the top row, but with pile head boundary-informed curve fitting applied via nonlinear regression (Section 2.2). The profiles in the bottom row build on the middle row by applying a Savitzky-Golay smoothing filter (Section 2.3). Together, these figures illustrate the step-by-step refinement of pile profiles from the initial LLS fit to the application of boundary conditions, and finally smoothing, across multiple pile head displacement levels.

Figure 5 shows p - y curves up to 4 inches of pile head displacement at depths of 2, 4, 6, 8, and 10 feet (lighter to darker blue) for three cases: (top) a 6th-order polynomial LLS fit to strain gauge data, (middle) a 6th-order polynomial LLS fit to inclinometer data, and (bottom) the 6th-order polynomial LLS inclinometer fit used as the starting point for Levenberg–

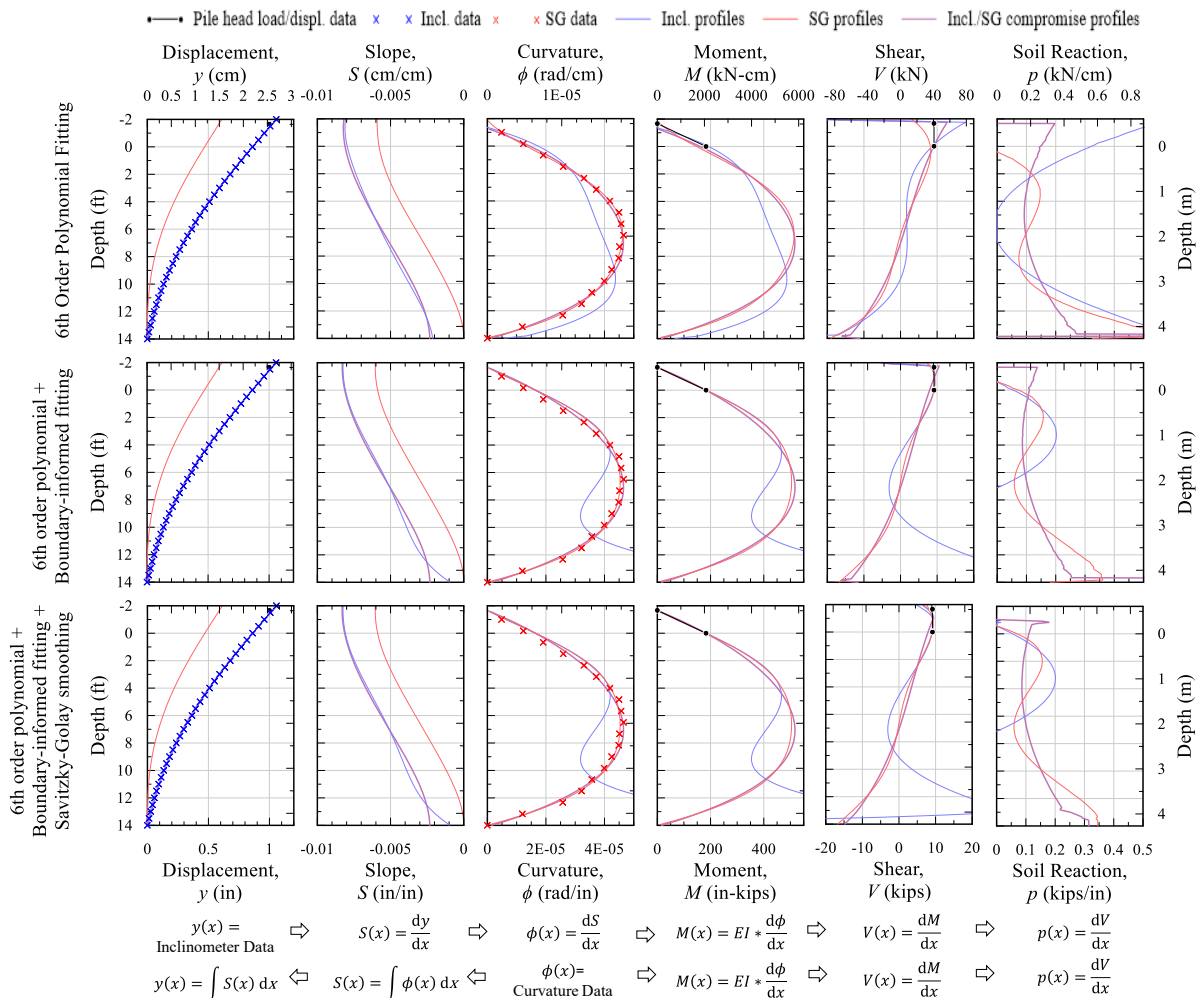


Figure 3. Pile profiles at 1.0-inch pile head displacement. Top row: 6th order polynomial fitting. Middle row: 6th order polynomial, boundary condition-informed nonlinear regression. Bottom row: 6th order polynomial, boundary condition-informed nonlinear regression with a Savitzky-Golay smoothing filter.

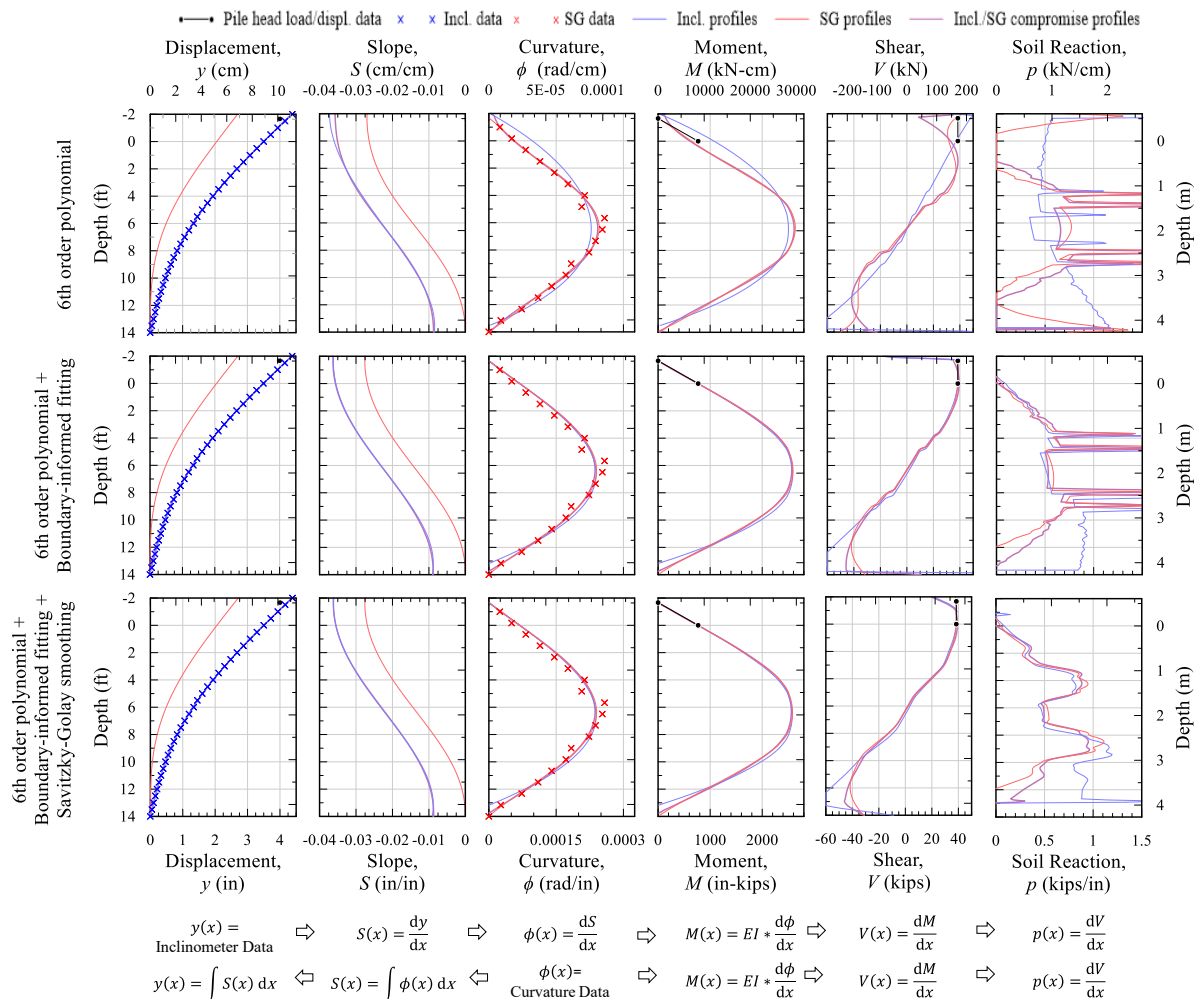


Figure 4. Pile profiles at 4.0-inch pile head displacement. Top row: 6th order polynomial fitting. Middle row: 6th order polynomial, boundary condition-informed nonlinear regression. Bottom row: 6th order polynomial, boundary condition-informed nonlinear regression with Savitzky-Golay smoothing filter

Marquardt nonlinear regression combining inclinometer and strain gauge data (Section 2.1), with pile head boundary-informed fitting (Section 2.2) and Savitzky-Golay filter smoothing (Section 2.3). Collectively, these plots illustrate a progression from the most elementary, LLS curve fitting (top and middle) to the fully integrated approach described in this paper.

5 DISCUSSION

As shown in Figures 3 and 4, the methods introduced in Section 2 successfully generate pile profiles from lateral load test data. The inclinometer / strain gauge combination profiles (purple lines) capture both datasets well, demonstrating that a single curve can represent both measurements when using this approach. The boundary-informed curve fitting approach modifies all investigated curve fitting approaches to satisfy the known pile head boundary conditions (black lines and circles), producing more realistic profile shapes. Figure 3 illustrates that when profiles are smooth and free of abrupt slope changes, the Savitzky-Golay smoothing filter has little effect – when no smoothing is needed, no smoothing is applied. In contrast, Figure 4 shows that when large spikes are present, the filter effectively dampens those spikes while preserving the underlying profile shape, further improving the pile profiles.

Figure 5 illustrates the results of applying the complete methodology outlined in Section 2. A polynomial LLS fit of the

inclinometer data served as the initial guess for a Levenberg-Marquardt nonlinear regression that combined inclinometer and strain gauge measurements with pile head boundary conditions in the error function. The resulting profiles were then smoothed using a Savitzky-Golay filter. This boundary-informed and profile smoothing approach addresses inconsistencies with soil mechanics principles that appear when p - y curves are derived from minimally processed measurements. For example, the p - y curves from only the LLS fits contain localized oscillations where a shallower p - y curve appears stiffer than a deeper curve. Such behavior is not expected in a homogeneous sand profile where lateral resistance and shear strength increase with depth. Similarly, the minimally processed curves exhibit negative p -values rather than strictly positive soil reactions. The boundary-informed and profile smoothing fitting procedure eliminates these inconsistencies, producing p - y curves that adhere to trends consistent with soil mechanics principles. In addition to the more logically consistent p - y curves, the inclinometer / strain gauge combination yields a single set of pile profiles consistent with the full dataset. Enforcing the pile-head boundary conditions improves the shallow-depth response by acknowledging the known applied loads, and the Savitzky-Golay filter suppresses high-frequency noise in the p -profiles, allowing more load steps to be used for meaningful p - y interpretation.

6 CONCLUSIONS

This study applies established mathematical techniques – Levenberg-Marquardt nonlinear regression and Savitzky-Golay filtering – to lateral load test data, combining inclinometer and strain gauge data within a boundary condition-informed framework. These techniques are applied to a large scale case study showing the resulting pile profiles are smoother, more physically consistent, and more representative of both inclinometer and strain gauge datasets. The derived p - y curves better reflect expected soil-structure interaction behavior. Compared with profiles derived from linear least squares polynomial curve fitting, the approach reduces noise-induced artifacts without suppressing pile profile details, offering a practical and adaptable toolset for analyzing instrumented pile lateral load tests.

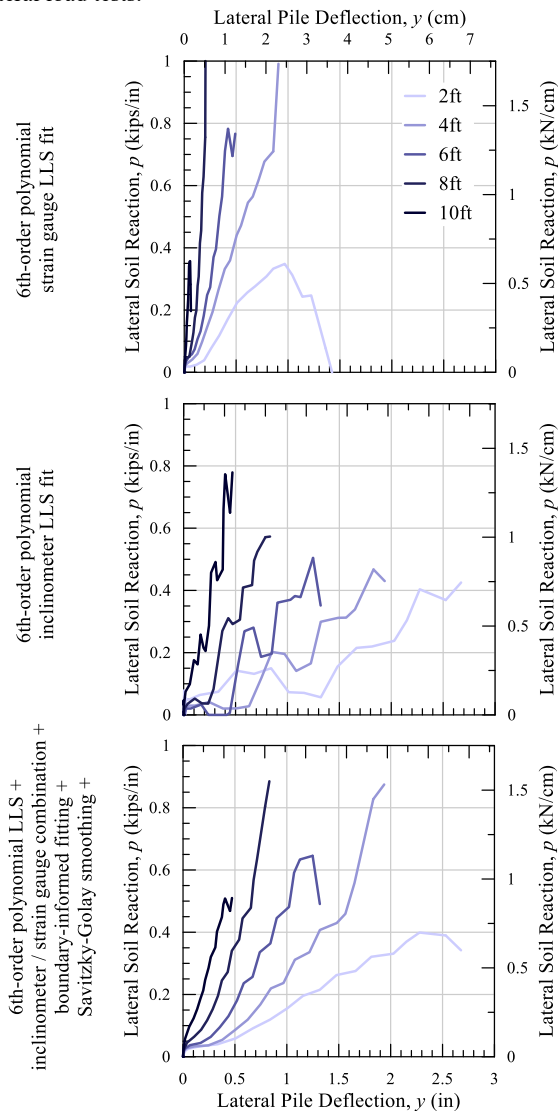


Figure 5. p - y curves up to 4 inches of pile head displacement at 2, 4, 6, 8, and 10 feet depth. Top: 6th-order polynomial LLS fit of strain gauge data. Middle: 6th-order polynomial LLS fit of inclinometer data. Bottom: Inclinometer / strain gauge combination, pile head boundary-informed curve fitting with Savitzky-Golay smoothing filter.

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