

Chartering a new era of embedded finite elements for pile simulations

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ABSTRACT: Due to their ability to capture the behaviour of slender structures with reasonable accuracy and moderate computational expense, embedded finite elements (FE) have become a main workhorse for modelling pile-type structures. In what appears to be the most recent research stream concerning the development of formulations belonging to this structural FE type—often referred to as embedded pile or embedded beam in geotechnical applications—researchers from Seequent, The Bentley Surface Company, and Graz University of Technology have expanded its ground-structure coupling domain from an implicit interaction line to an implicit interaction surface. In this way, the ground-structure interaction behavior can be numerically described at the physical contact geometry, rather than along the centerline. To date, insights into the relative merits of related versions of this augmented FE type have primarily been shared within the research community, leaving some ambiguity regarding its deployment in engineering practice. To bridge this gap, the present contribution aims to serve as a single-source reference for the disseminated publications, which are briefly discussed in a practice-oriented manner. In addition, a comparative FE study concerning a capped pile group provides insight into the performance of the developed embedded FE formulation. Overall, this contribution marks the concluding part of a multi-year research project and aims to herald a new era for embedded FEs within the widely used finite element code PLAXIS 3D.

KEYWORDS: pile, beam-solid coupling, embedded finite element, interaction surface, application.

1 INTRODUCTION

Deep foundation systems comprising a large number of piles are often the only practical option for supporting large-scale structures founded on soft soils and subjected to high loads (Poulos and Davis, 1980), such as high-rise buildings or wind turbines (Tschuchnigg and Schweiger, 2013; Ravichandran *et al.*, 2018). A key challenge in the design of related projects is the credible representation of the superstructure–pile–ground interaction, which requires robust analysis methods capable of accurately capturing the mechanics at reasonable computational costs. While simplified approaches based on empirical or closed-form solutions remain widely used during preliminary design phases, their applicability is inherently limited. These limitations become particularly apparent in projects featuring non-symmetric geometries, heterogeneous ground conditions, complex loading scenarios, or demanding structural design criteria (Granitzer *et al.*, 2024c).

To improve the accuracy of geotechnical predictions on a site-specific scale, three-dimensional (3D) finite element (FE) simulations are increasingly adopted in both academic and practical settings. A major factor influencing the computational effort of such simulations is the selection of the pile modelling

approach (Granitzer *et al.*, 2024a). The conventional method involves fully discretizing both, piles and surrounding ground units with 3D solid FEs, which leads to mesh topologies and computational costs that are impractical for many real-world applications (Figure 1). A viable solution to circumvent obstacles associated with this discrete modelling approach is the use of embedded FE models (EBFs), which have gained popularity due to their efficient meshing and reduced runtimes.

To the best of our knowledge, however, EBFs incorporated in commercial 3D FE codes are constrained to the assumption that the ground-pile interaction is described along the centreline geometry; related formulations are referred to as embedded FE models with implicit interaction line (EB-Ls). While the assumption underlying EB-Ls may be acceptable for pile-type structures with high length-to-diameter slenderness ratios (e.g. micropiles), it may oversimplify the geometrical relations in foundation layouts and involves numerical singularities, with potentially detrimental effects on the numerical fidelity (e.g. mesh-size dependent pile response). This has motivated the development of the embedded FE with implicit interaction surface (EB-I) that expands the interaction geometry from the centreline to the physical ground-structure contact (Granitzer, 2024).

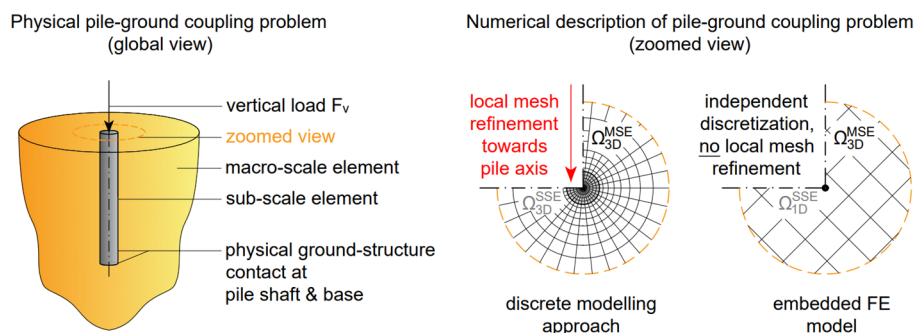


Figure 1. 3D FE representation of physical pile-ground coupling problem through discrete and embedded FE model (Granitzer *et al.*, 2024c).

In view of the upcoming EB-I release in the commercial FE code PLAXIS 3D, this work aims to consolidate the recent research efforts and serve as a practical reference for engineers who may need to assess its applicability. In this context, Section 2 outlines practical characteristics and inherent limitations of state-of-the-art EBFs (Tschuchnigg and Schweiger, 2015; Granitzer, 2024), which may support an informed evaluation of potential application scenarios. Section 3 gives an overview of reported advantages associated with the developed EB-I, followed by its application to a piled cap problem in section 4. Section 5 closes with the conclusions of this work.

2 EMBEDDED FINITE ELEMENT FORMULATIONS IN GEOTECHNICS

From a geometrical point of view, EBFs consist of 1D beam FEs that are coupled at discrete coupling points \mathbf{x}_i to 3D solid FEs representing the ground. The union of \mathbf{x}_i spans the implicit coupling domain Γ_c , which is either a line (Γ_c^{1D} , EB-L) or a surface (Γ_c^{2D} , EB-I); cf. Figure 2. Regardless of the coupling domain geometry, the interface constitutive behaviour is numerically described by 1D (non-linear) relationships that have the ability to account for the stress state of the ground as well as tangential frictional slip (Granitzer *et al.*, 2024b). The employed coupling technique is classified as a displacement coupling method (Steinbrecher *et al.*, 2022).

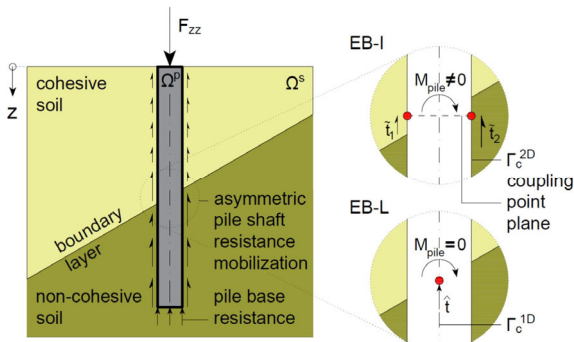


Figure 2. Comparison of point-wise traction system under vertical loading mobilized with EB-L and EB-I, adopted in modified form from Granitzer *et al.*, 2024d.

A key assumption of EBFs is that the ground and the modelled pile-type structures overlap. As such, it has been suggested that reducing the EBF unit weight may help maintain mass balance across the overlapping domains (Tschuchnigg and Schweiger, 2015) and better captures global ground-structure interaction effects within the analysis domain (Franza and Marshall, 2019). This approach, however, introduces layer-dependent parameter variations in stratified ground conditions where the ground unit weight differs across layers. Additionally, it is important to note that lowering the EBF unit weight may lead to a significant underestimation of the so-called “residual loads” (Fellenius and Alataee 1995)—that is, loads inherently present in the pile prior to any analysis or measurement—which can compromise the design. In this regard, Granitzer *et al.* (2024a) emphasizes that the EBF unit weight shall be carefully selected, with particular consideration given to the specific analysis aims.

Mesh-refinement studies indicate that the mesh topology may notably govern the numerical response of EBFs (Granitzer and Tschuchnigg, 2021). Unlike discrete modelling approaches, further mesh refinement does not necessarily yield spatial convergence to the exact solution. This unwanted behaviour stems from the static nature of the coupling problem, where concentrated coupling forces at \mathbf{x}_i lead to a boundary

value problem (BVP) analogous to a 3D point load acting on an infinite solid, also referred to as Kelvin problem (Podio-Guidugli and Favata, 2014). This results in pronounced local singularities in the predicted field quantities (e.g. stresses) (Granitzer and Tschuchnigg, 2022), particularly if the ground is modelled by means of non-linear constitutive models.

To mitigate adverse force localization effects on the calculation accuracy and convergence, EBFs are equipped with a spatially constrained elastic zone within the physical domain of the sub-scale element (SSE); cf. Figure 1. In this zone, stress points of the ground units are constrained to remain elastic. While this approach may produce locally unrealistic field quantities, it allows for a significant improvement in numerical robustness, runtime and prevention of premature failure (Engin *et al.*, 2007). For applications prioritizing global system behaviour over local stress accuracy in the SSE vicinity, the elastic zone offers an effective compromise between numerical robustness and fidelity. Moreover, since EBF stress resultants can be directly extracted from the beam FEs, EBFs allow for a seamless integration of geotechnical and structural design tasks.

Given the broad range of potential applications for EBFs (Granitzer and Tschuchnigg, 2021), it is essential to draw on insights from validation studies to assess their suitability in practice. Most validations studies focus on axially loaded vertical piles, assuming negligible installation effects. In this scenario, Tschuchnigg and Schweiger (2015) demonstrates that EBFs can realistically reproduce the load–settlement behaviour and mobilization of skin resistance. This renders EBFs particularly suitable for parametric FE analyses (FEAs), where the geometry and arrangement of pile-type structures shall be optimized or considered as random variables to reliability based design tasks (Sudret, 2014). Guidance on their applicability under more complex loading, however, remains limited. This is partly due to the high number of influencing factors, such as the slenderness and the stiffness of SSE representing the pile-type structure, the constitutive model and parameters of the macro-scale element representing the ground (Figure 1), as well as the mesh topology and boundary conditions. The numerical EBF behaviour may further vary with its actual implementation (cf. Section 3). In this light, and in the absence of benchmark cases, it is recommended to carry out case-specific preliminary studies, ideally by comparing EBF results with those from the discrete modelling approach (Granitzer and Tschuchnigg, 2021; Tschuchnigg and Schweiger, 2015).

3 EMBEDDED FINITE ELEMENT WITH IMPLICIT INTERACTION SURFACE

A key development of the EB-I is that it expands Γ_c from an implicit interaction line (Γ_c^{1D}) to an implicit interaction surface (Γ_c^{2D}). Mechanically, this means that the EB-I induces a point-wise traction system over the ground–structure contact at the shaft and base, rather than along the centreline. Interested readers may refer to Granitzer *et al.* (2024d) for mathematical details of the underlying mapping procedure.

Essentially, this feature provides advantages of practical relevance. In scenarios that lead to an eccentric mobilization of skin resistance, they become particularly evident. For instance, the demonstration case shown in Figure 2 involves an axially loaded pile in layered ground with a non-horizontal boundary layer, leading to an asymmetric mobilization of skin resistance and non-zero bending moments (M_{pile}). While EB-Is capture this behaviour realistically, the EB-L yields zero-valued M_{pile} due to its centreline coupling, overlooking critical eccentric effects. Similar limitations of EB-Ls may arise in scenarios with pile-pile interaction effects or torsional loads, potentially

resulting in unsafe designs in terms of structural pile capacities (Franza and Sheil, 2021; Granitzer and Tschuchnigg, 2022).

Due to the distributed mobilization of point-wise traction forces (with reduced amplitude), the EB-I is less sensitive to singularities triggered by the Kelvin problem. This feature proves beneficial with respect to the mesh-size effect on the numerical response of pile-type structures, and suppresses the development of unwanted oscillations in the predicted skin traction profiles (Granitzer and Tschuchnigg, 2023). Moreover, the EB-I tends to produce an improved conditioning of the global stiffness matrix in the linearized system of equations, as evidenced by Granitzer *et al.* (2024d) by means of eigenvalue analyses. This observation is practically significant, as it governs the simulation runtime; for example, comparative studies with respect to a deep foundation supported by 170 piles presented in Granitzer *et al.* (2024a) show that the EB-I use allows for FEAs that are not only less dependent on the mesh topology, but it also tends to decrease the simulation runtime compared to both, the discrete modelling approach and the EB-L. From this perspective, it can be argued that the EB-I offers advantages of practical relevance that may increase the application range of EBFs in engineering design tasks.

4 DEMONSTRATION CASE

This section examines the EB-I performance by means of comparative FEAs of a 3×3 capped pile group subjected to a vertical point load placed at a varying distance from the cap centre; see Figure 3. The reference scenario, including the Hardening Soil Small parameter set representing Vienna fine sand, phase calculation sequence and loading procedure, has been adopted from Franza and Sheil (2021) and Granitzer *et al.* (2024a). The piles are additionally modelled employing the EB-L (Tschuchnigg and Schweiger, 2015) and the standard FE approach (SFEA), representing a discrete modelling approach with zero-thickness interface FEs (Day and Potts, 1994) attached to the ground-structure contacts along the shaft and at the base. The SFEA is therefore considered as numerical reference solution. It is pointed out that the cap is not in contact with the underlying ground, meaning the foundation behaviour is entirely governed by pile-soil-pile interaction. This geometric configuration is therefore especially well-suited for validation purposes.

The first set of FEAs explores the influence of the load eccentricity (e/D_{pile}) on the global roto-translational behaviour of the pile cap, whereas the normalized settlement (u_z/D_{pile}) and rotation around the y-axis (θ_y) are evaluated at the intersection of the central pile with the ground surface (Figure 4). As can be observed from the results, the initial load-settlement response is hardly affected by the load eccentricity. At large settlements, however, an increasing load eccentricity decreases the pile resistance but increases θ_y -values. These observations can be attributed to the mobilization of a lateral ground resistance induced by external bending moments, that is, non-zero vertical load eccentricities are equivalent to the application of an additional horizontal load component at the pile cap centre; cf. Franza and Sheil (2021). In view of the EBF performance, the EB-I shows a remarkable agreement with the numerical SFEA benchmark, while EB-Ls overestimate the roto-translational stiffness. Relative merits of the EB-I compared to the EB-L tend to increase with increasing vertical load magnitude.

Figure 5 compares the distribution of the normal force (N), bending moment (M_x, M_y) and skin traction (T_{skin}) along the embedment length as function of the pile modelling approach. The results are analysed at the corner pile. Principally, the results obtained with the EB-I are in good agreement with the

SFEA benchmark. While the EB-I also captures the general trend $M_y > M_x$, the predicted values are slightly higher compared to the numerical SFEA benchmark. A comparison with the EB-L results, however, yields two notable deviations. On the one hand, the EB-L overestimates the N-values, which can be attributed to an overestimation of the pile base stiffness. On the other hand, the EB-L shows considerable skin traction oscillations. Both limitations align with the results of previous studies (Granitzer and Tschuchnigg, 2021, 2023) and underscore selected merits of the EB-I compared to the EB-L.

5 CONCLUSIONS

This work aims to serve as a practical reference for engineers seeking to exploit the capabilities of the novel embedded finite element (FE) model with implicit interaction surface (EB-I). This FE type has been primarily designed for the analysis of BVPs involving numerous pile-type structures with insignificant installation effects, typically deep foundations supported by bored piles, and has been incorporated in the commercial FE code PLAXIS 3D. The EB-I has been developed and validated as part of a collaborative, multi-year research project between Graz University of Technology and Sequent, The Bentley Subsurface company (Granitzer, 2024).

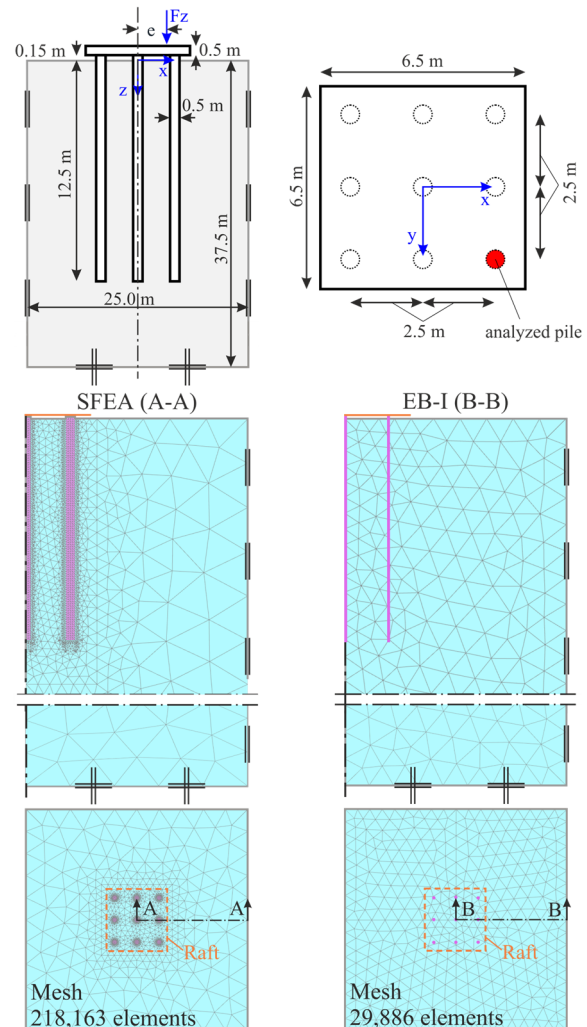


Figure 3. Geometry and mesh topologies of 3×3 capped pile group reference scenario adopted from Franza and Sheil (2021) and Granitzer *et al.* (2024a).

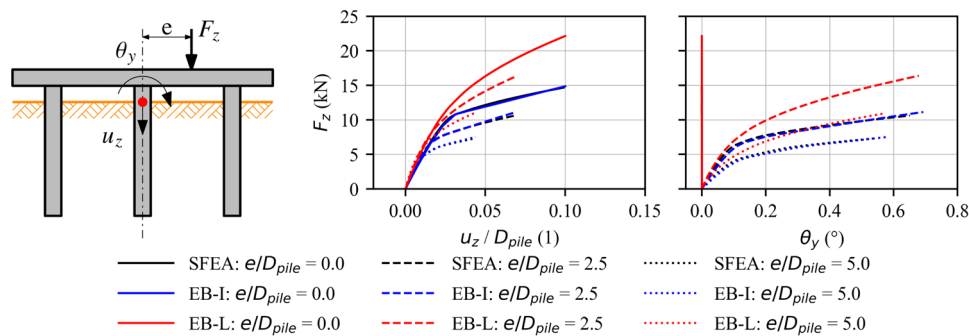


Figure 4. Global capped pile group response as function of vertical load eccentricity and pile modeling technique.

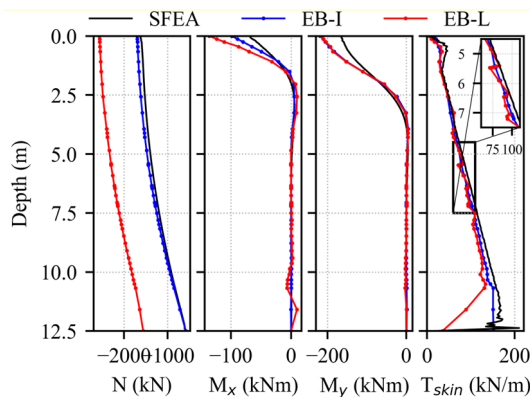


Figure 5. Distribution of stress resultants and skin traction along embedment length of corner pile ($u_z = 0.1 \times D_{pile}$ and $e/D_{pile} = 5.0$).

In anticipation of its upcoming software release, this work provides an intuitive overview of key characteristics and inherent limitations of embedded FE models. It then concisely outlines relative advantages of the developed EB-I. These are partially demonstrated through application to a 3×3 capped pile group. Focus is placed on the global roto-translational behaviour under eccentric vertical loading and the distribution of stress resultants, with comparisons made to both the state-of-the-art embedded FE model with implicit interaction line (EB-L) implemented in PLAXIS 3D, as well as a numerical benchmark solution. The results highlight relative merits of the EB-I compared to the EB-L and may support its more confident adoption in future applications. Interested readers may refer to case studies reported in Granitzer et al. (2022; 2024a) where the EB-I has been successfully validated based on single-pile problems and large-scale multiple-pile foundation systems.

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