

# Constitutive modelling of multi-directional cyclic ratcheting

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**ABSTRACT:** This paper presents a macro-model designed to simulate the response of systems under multi-directional cyclic loading. The model, called CLAP (Cyclic Loading Analysis of Piles), fully couples horizontal forces and moments ( $H_x$ ,  $H_y$ ,  $M_x$ ,  $M_y$ ) to capture non-linear kinematic hardening behaviour during loading, employing the same principles as the REDWIN model (Page et al., 2019). Consequently, CLAP reproduces hysteretic behaviour during unloading and reloading. The model also simulates the ratcheting phenomenon—the accumulation of permanent deformation over numerous cycles—using principles derived from the HARM framework (Abadie, 2015; Houlsby et al., 2017). The capabilities of CLAP are demonstrated through simulations of 1,000 cycles of a representative multi-directional cyclic loading scenario known as the fan test. These simulations demonstrate the model’s effectiveness in capturing the four-degree-of-freedom coupling of kinematic hardening yield surfaces, while enabling user-defined tuning of ratcheting for large cycle numbers with minimal computational cost. Originally developed to predict the macro-behaviour of monopiles and shared anchor piles under complex, multi-directional cyclic loads (Abadie and Page, 2025), CLAP could also be adapted to define the coupling of soil reaction and moment curves in a 1-D PISA-type model (Burd et al., 2020). Looking forward, the model holds potential to enhance modelling capabilities for practical industry applications, particularly in the design and advancement of mooring and anchoring systems for floating offshore wind projects.

**KEYWORDS:** Multi-directional cyclic loading, constitutive modelling, yield surfaces, kinematic hardening, macro-model, foundation, shared anchor piles, monopiles.

## 1 INTRODUCTION

In geotechnical engineering, many problems involve systems (e.g., soil elements, foundations, rigid inclusions) subjected to multi-directional cyclic stresses or loads. The response in such cases often exhibits ratcheting in different directions. Ratcheting refers to the progressive accumulation of permanent deformation under repeated, cyclic loading. A common approach to model cyclic behaviour, due to its simplicity, is kinematic hardening, which follows Masing’s rules (Masing, 1926) and does not generate ratcheting. In soil–structure interaction problems, 1D models are frequently applied, presenting two main challenges: (i) capturing the coupling between different degrees of freedom under multi-directional loading, and (ii) efficiently modelling ratcheting effects over very large cycle numbers without excessive computational cost.

The purpose of this paper is to present the modelling capabilities of a new theoretical framework—CLAP (Cyclic Loading Analysis of Piles, Abadie and Page, 2025, Figure 1)—designed to address these two challenges. CLAP integrates the key features of two modelling approaches developed over the past decade: (i) the REDWIN model (Page et al., 2019), which enables full coupling of four degrees of freedom with a kinematic hardening formulation, and (ii) the HARM framework (Houlsby-Abadie Ratcheting Model, also referred to as the Hyperplastic Accelerated Ratcheting Model; Abadie, 2015; Houlsby et al., 2017), which captures 1D ratcheting effects while maintaining hysteretic behaviour through kinematic hardening.

This work was initially motivated by the analysis of piles for offshore wind energy applications (Abadie and Page, 2025;

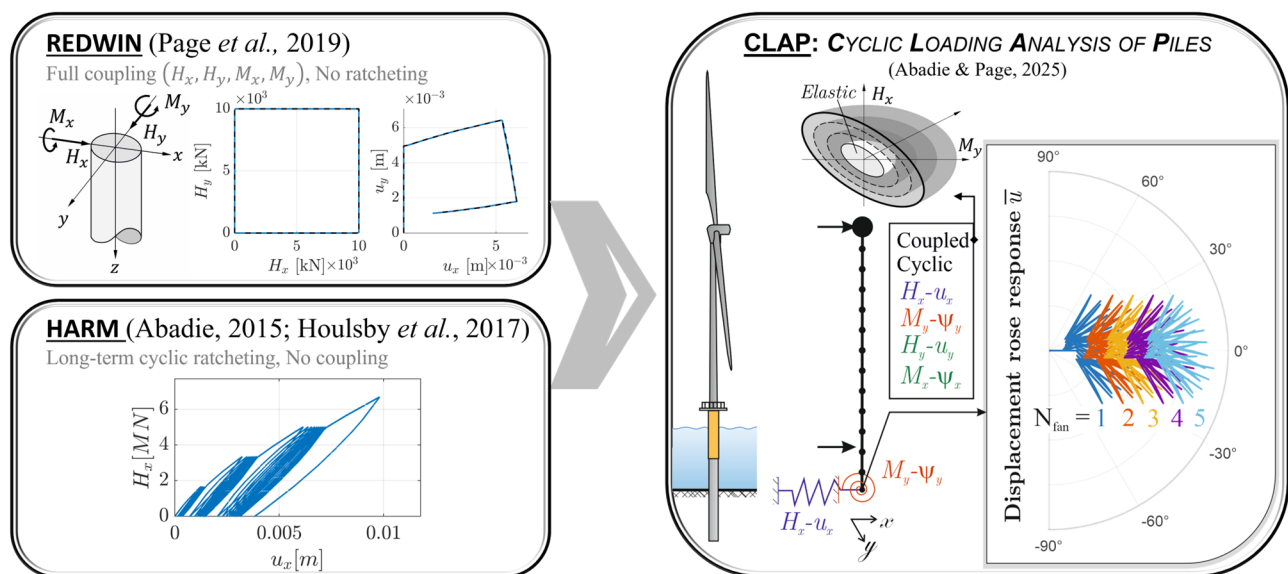


Figure 1. Key modelling principles of the CLAP model (graphs from Abadie and Page, 2025)

Abadie, 2025). Offshore wind foundations must be designed to withstand large numbers of load cycles of varying directions and amplitudes, where both ratcheting and load multi-directionality can become critical issues (Rudolph et al., 2016; Richards et al., 2019; Abadie, 2025; Zabatta et al., 2025; Chalhoub et al., 2025; Ifeobu et al., 2025). Typically, over the lifetime of an offshore turbine, the foundation can experience several  $10^8$  cycles (Abadie et al., 2023) of varying magnitude and direction. For monopiles, (unidirectional) cyclic loading is already recognised as a key design consideration (e.g., Leblanc et al., 2010; Peralta, 2010; Cuellar, 2011; Klinkvort, 2012; Abadie et al., 2019a), and the HARM model has been applied for this purpose (Abadie et al., 2023). However, the combined effects of multi-directional cyclic loading have yet to be rigorously captured in modelling (Richards, 2019).

In current design practice, the lack of a comprehensive understanding of pile behaviour under multi-directional cyclic loads necessitates the use of empirical safety factors, which can result in over-conservative designs and increased costs. There is a clear need for modelling approaches that can benchmark foundation response to multi-directional cyclic loading with both robustness and computational efficiency.

The CLAP model was therefore developed with this motivation in mind, adopting a macro-element approach (Abadie and Page, 2025). This strategy, first introduced by Schotman (1989) and Nova and Montrasio (1991), offers significant advantages in terms of computational efficiency and accuracy when representing foundation behaviour in structural analyses. In recent years, macro-elements have been applied successfully to monopile modelling, avoiding the need to explicitly simulate soil–pile interaction along the entire pile shaft (e.g., Page et al., 2018; Houlsby et al., 2017; Abadie et al., 2019b, Bergua et al., 2022), resulting in much faster modelling of the pile for design applications (Abadie et al., 2023). In the present work, the formulation developed for pile design is adopted, focusing on the relationship between the governing lateral loads ( $H_x, H_y, M_x, M_y$ ) and the corresponding pile head displacements and rotations at ground level ( $u_x, u_y, \psi_x, \psi_y$ ).

The paper illustrates the key features of the model and its behaviour under a representative multi-directional cyclic load fan—an established loading scheme for investigating pile response to multi-directional cyclic loading (Rudolph et al., 2016; Richards et al., 2019; Zabatta et al., 2025). While the offshore monopile problem provides the primary motivation for this study, the model itself has much broader applicability to the analysis of soils and other materials subjected to multi-directional cyclic stress conditions.

## 2 CLAP MODEL

### 2.1 Key features and formulation

The CLAP model incorporates key features observed in piles used for offshore wind energy applications, including: (i) non-linear behaviour; (ii) coupling between the translational and rotational degrees of freedom that describe the lateral response of the pile; (iii) distinct loading and unloading responses, leading to hysteresis and energy dissipation through hysteretic damping; and (iv) ratcheting.

These features are enabled by the model formulation. CLAP is built within the multi-surface kinematic hardening (MSKH) framework and includes a ratcheting component. The MSKH approach, adopted from the REDWIN model (Page et al., 2019), simulates non-linear, path-dependent behaviour using multiple yield surfaces of fixed size that translate in the  $H_x$ - $H_y$ - $M_x$ - $M_y$  load space in response to the applied load history. This enables the model to capture stiffness degradation and the

development of coupled plastic displacements and rotations under multi-directional cyclic loading.

Coupling between translational and rotational responses is introduced through the shape of the yield surfaces, which also serve as potential surfaces due to the use of an associated flow rule. This ensures that loading in one degree of freedom influences the response in the others.

Ratcheting is incorporated using the HARM framework (Abadie, 2015, Houlsby et al., 2017) by accumulating a small fraction of permanent plastic displacements and rotations to the monotonic elastic and plastic components of the original REDWIN model.

### 2.2 4-D kinematic hardening and ratcheting

The evolution of plasticity and ratcheting in CLAP is governed by the geometry of the yield surfaces in a four-dimensional load space defined by  $H_x, H_y, M_x,$  and  $M_y$ . These surfaces are shaped by a quadratic yield criterion, identical to that used in REDWIN (Page et al., 2019), resulting in 4D ellipsoidal yield surfaces. In this space, the yield surface cross-sections are circular in the  $H_x - H_y$  and  $M_x - M_y$  planes, elliptical in the  $H_x - M_x$  and  $H_y - M_y$  planes, and rotated ellipses in the  $H_x - M_y$  and  $H_y - M_x$  planes.

To simplify the macro-element implementation, a linear coordinate transformation is applied to both loads and displacements (Page et al., 2019). This maps the original ellipsoidal yield surfaces into 4D spheres, converting all elliptical cross-sections into circles in the transformed space. This transformation enables the model to be formulated as a Von Mises model with kinematic hardening and an associated flow rule. Ratcheting is implemented assuming that the displacement increment can be decomposed into elastic, plastic and ratcheting components.

### 2.3 Calibration

CLAP is implemented through initialisation of the system states, followed by iterative updates of trial stresses and backstresses using a displacement-controlled explicit correction algorithm, as described in Page et al. (2018) and Abadie and Page (2025). Load-controlled simulations are solved using a Newton–Raphson scheme.

Model calibration of the backbone response in four dimensions involves direct input of backbone curves for pure horizontal loading ( $H - u - \psi$ ) and pure moment loading ( $M - u - \psi$ ), from which the elastic stiffness is obtained (Page et al., 2019). In this study, the stiffness is derived directly from Page et al. (2019).

The magnitude of the accumulated quantities is governed by the ratcheting rate,  $R_n$ , defined by Equation (1) and based on methods in Abadie (2015), Abadie et al. (2019b), and Richards (2019), with variables provided in Table 1 and values of parameters from Abadie and Page (2025) and Abadie (2025).

$$R_n = R_0 \left( \frac{k_n}{k_u} \right) \left( 1 + \frac{\beta}{\beta_0} \left( \left( \frac{k_u}{J_z} \right)^{\frac{m_s}{m_r}} - 1 \right) \right)^{-m_r} \quad (1)$$

## 3 MULTI-DIRECTIONAL CYCLIC FAN LOADING

### 3.1 Macro-response

In the following, the macro-response of the pile is analysed in terms of the norms of the load  $\bar{H}$  and displacement  $\bar{u}$ :

$$\bar{H} = \sqrt{H_x^2 + H_y^2} \quad (2)$$

$$\bar{u} = \sqrt{u_x^2 + u_y^2} \quad (3)$$

Table 1. Ratcheting parameters

Property	Definition	Value(s) / Equations
$R_n$	Ratcheting rate	Equation (1)
$R_\beta = R_0 \times \beta_0^{-m_r}$	Initial ratcheting rate	0   1E-5   1E-4
$k_n$	Radius of each yield surface $n$	Derived from monotonic curve
$k_U$	Radius of the largest yield surface – corresponding to ultimate capacity	Derived from monotonic curve
$\beta$	Accumulated ratcheting strain calculated from the norm of the ratcheting strain increment at each increment	Computed at each iteration
$\beta_0$	Initial hardening strain	1E-4
$J_2$	Second invariant of the deviatoric load vector	Computed from the load vector
$m_s$	Empirical exponent controlling the ratcheting rate with history	2.2
$m_r$	Empirical exponent controlling the ratcheting rate with load level	2.2

The performance of CLAP is demonstrated through simulations of a fan test, a loading scenario often used to explore the response of foundations to multidirectional cyclic loading (Rudolph et al., 2014; Richards et al., 2020; Abadie, 2025; Abadie and Page, 2025). In this test, lateral loads are applied in a rotating pattern to represent offshore cyclic forces such as wind and waves with changing direction. Figure 2(a) shows the normalised horizontal load  $\bar{H}/H_R$ , highlighting the variation in loading direction from +60° to -60° with increments of 12°. Each fan consists of 20 sinusoidal cycles, and the total number

of fan repetitions,  $N_{fan}$ , is set to 50, totalling 1,000 multidirectional cycles. The computational time for this load sequence was approximately 300 seconds on a standard laptop (13th Gen Intel® Core™ i7-13800H, 2.50 GHz, 32 GB RAM, 64-bit operating system). These results show the potential of the model for applications involving very large cycle numbers, particularly if future acceleration methods are implemented within the HARM framework (see Abadie, 2015; Houlsby et al., 2017), which could potentially extend the model’s applicability to the full operational lifetime of wind farms (Abadie et al., 2023).

Figure 2 also presents the total displacement response obtained with CLAP for (b) no ratcheting, and for an arbitrary small (c) and large (d) ratcheting rate. The no-ratcheting case matches REDWIN results, showing a coupled response, different stiffnesses during loading and unloading, and no displacement accumulation. The spread angle of the displacement response is similar to the load variation. The ratcheting cases show accumulated displacement in the main loading direction, with increasing intensity. The first fan (in black) shows the most pronounced response, while subsequent fans resemble the REDWIN response but are offset by the accumulated displacement. The rate of accumulated displacement decreases with the fan number  $N_{fan}$ . Although the spread angle of the displacement response narrows due to ratcheting, the overall shape of the displacement response remains quite similar after the completion of the first fan.

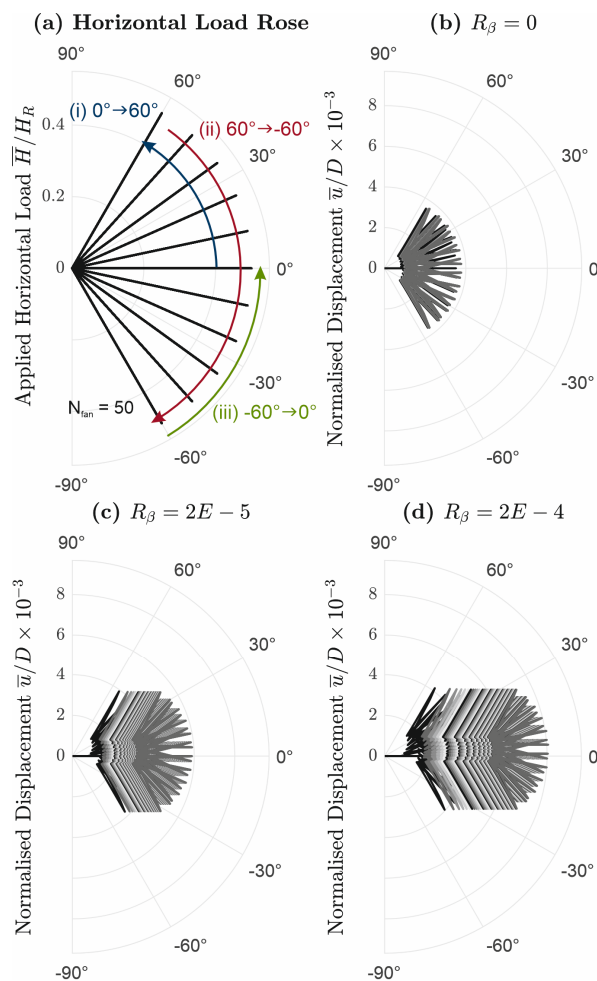


Figure 2. (a) Input fan load. Global displacement response obtained with the CLAP model for (b) no ratcheting ( $R_\beta = 0$  – REDWIN results only), (c) an arbitrary small value of ratcheting ( $R_\beta = 2E-5$ ) and (d) an arbitrary large value of ratcheting ( $R_\beta = 2E-4$ )

### 3.2 Evolution of the kinematic hardening yield surfaces

The behaviour of CLAP can be explained by the translation of the yield surfaces in the load space. Figure 3 shows 12 different positions of the yield surfaces in the  $H_x$ – $H_y$  cross section, each corresponding to a different stage of the fan test—from the first cycle in the first fan to one of the last cycles in the final fan. These snapshots help illustrate how the yield surfaces move in load space and how this movement leads to the development of elastic, plastic, and accumulated displacements as the test progresses.

The plot for Fan number: 0 – Cycle No: 0 shows the initial position of the yield surfaces in the  $H_x$ – $H_y$  cross section, with normalised axes and four of the 40 surfaces displayed. During the first loading (Fan number: 1 – Cycle No: 1), the yield surfaces are translated horizontally, generating displacement in the same direction due to the alignment between the load and the gradient of the yield surface. As this is virgin loading, the displacement is larger than in subsequent fans, as also seen in Figure 2(c) and 2(d) (black line). Upon load reversal, the two inner surfaces yield while the outer ones do not, resulting in a stiffer unloading response.

The plot for Fan number: 1 – Cycle No: 6 shows the yield surfaces after six sinusoidal cycles with varying directions.

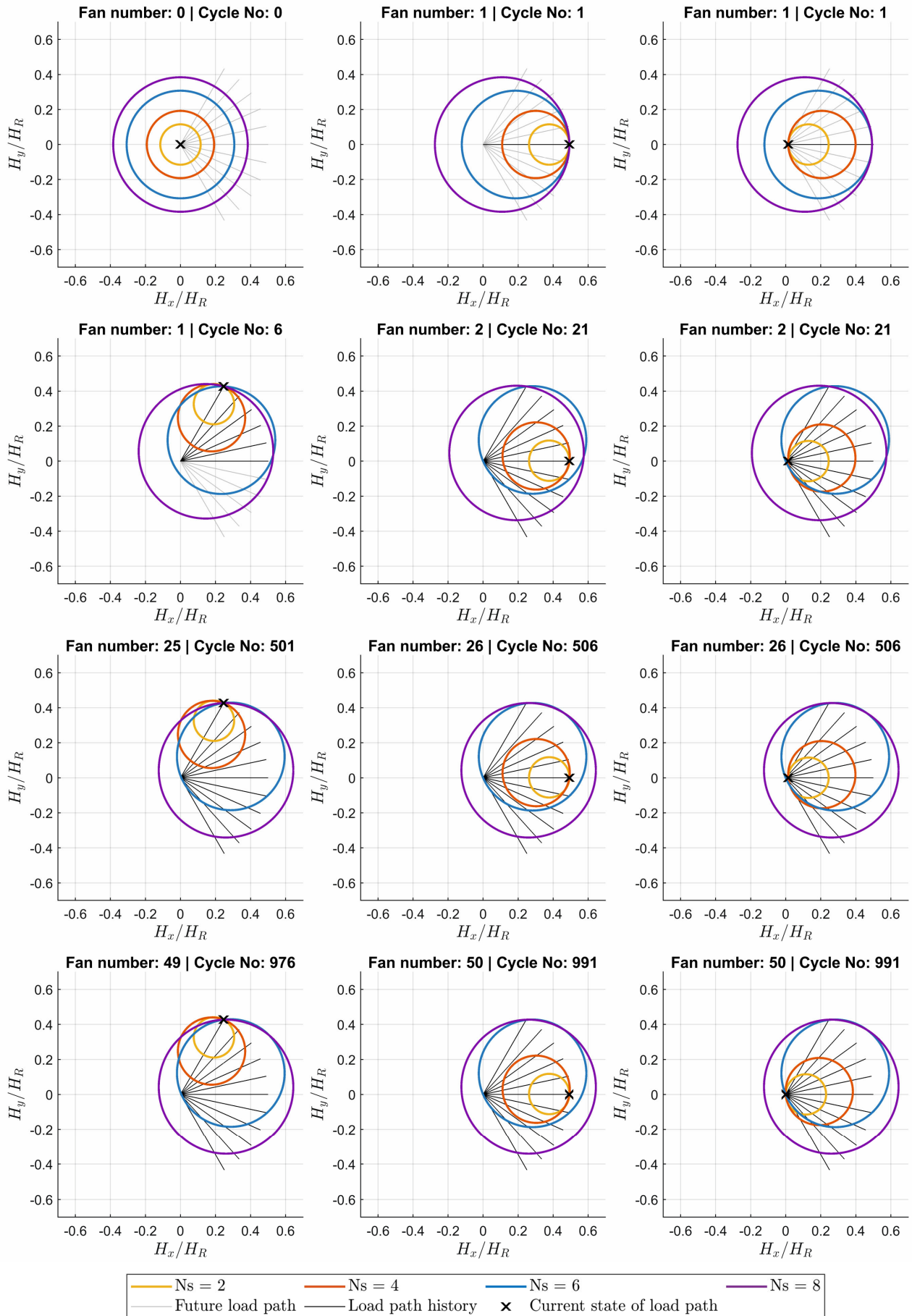


Figure 3. Evolution of the field surfaces during 1000 cycles and 50 fan loading paths modelled with the CLAP model

Compared to Cycle No: 1, all displayed surfaces have moved further, generating additional plastic and ratcheting displacements beyond what would have occurred under unidirectional loading. At this stage, the gradient of each yield surface at the loading point is no longer aligned with the loading direction, resulting in displacement components that deviate from the applied load direction. The chosen translation rule (Prager, 1949), which is identical to the Ziegler (1959) translation rule in this case, can lead to the intersection of the yield surfaces, as shown in Figure 3.

Subsequent loading and unloading along the horizontal axis (Fan number: 2 – Cycle No: 21) mobilise the yield surfaces differently than the initial virgin loading and unloading (Fan number: 1 – Cycle No: 1): only the two innermost surfaces yield, compared to four during the first cycle. The influence of loading history is evident from the shifted starting positions of the yield surfaces. After Fan 2, the movement of the yield surfaces becomes consistent across cycles, as seen in Fan 25 and Fan 49. Ratcheting continues to develop, proportional to the ratcheting rate (Figure 2c and 2d), as the yield surfaces translate - though it does not alter their trajectories.

### 3.3 Ratcheting

The accumulation of ratcheting deformation, as well as the influence of Equation (1) and the values of the ratcheting parameter  $R_\beta = R_0 \times \beta_0^{-m_r}$ , is best examined by plotting the load radius against the displacement radius and visualising the build-up of permanent deformation. This is illustrated in Figure 4(a-c) for increasing values of  $R_\beta$ , ranging from zero (no ratcheting) to  $10^{-4}$ . The figure shows the performance of the model both without ratcheting (i.e., REDWIN only) and with ratcheting, and demonstrates how the magnitude of the ratcheting rate can be controlled through the parameter  $R_\beta$ .

To illustrate this more clearly, a common practice (e.g., Long and Vanneste, 1994; Leblanc et al., 2010; Peralta, 2010; Klinkvort, 2012; Abadie et al., 2019a; Richards et al., 2020) is to plot the accumulation of ratcheting deformation at peak load against the cycle number:

$$\frac{\Delta \bar{u}_{max}}{D} = \frac{(\bar{u}_{max,N} - \bar{u}_{max,1})}{D} \quad (4)$$

This is shown in Figure 4(d). The irregular variations between successive cycles, compared with previously published results for unidirectional cyclic loading, arise purely from changes in the projected load magnitude within a fan cycle. This representation clearly highlights the influence of  $R_\beta$  on the ratcheting rate and the added value of combining HARM with the REDWIN model to capture multi-directional cyclic ratcheting. It also shows that  $R_\beta = 2 \times 10^{-4}$  and  $R_\beta = 2 \times 10^{-5}$  produce parallel responses, demonstrating that  $R_\beta$  does not influence the slope of the ratcheting rate, which is primarily governed by the parameter  $m_r$  (Table 1; Abadie, 2015; Abadie et al., 2019b; Abadie et al., 2023). Further studies will be performed to understand more thoroughly the influence of this parameter, as well as the parameter  $m_s$  and best formulations of Equation (1) (e.g. Abadie, 2015, Crispin et al., 2025).

## 4 CONCLUSIONS

This paper presents the inner workings of CLAP (Cyclic Loading Analysis of Piles, Abadie and Page, 2025) – a new constitutive model based on coupled kinematic hardening yield surfaces ( $H_x, H_y, M_x, M_y$ ), to which cyclic loading ratcheting is added to capture the accumulation of permanent deformation under long-term cyclic loading. The model merges two

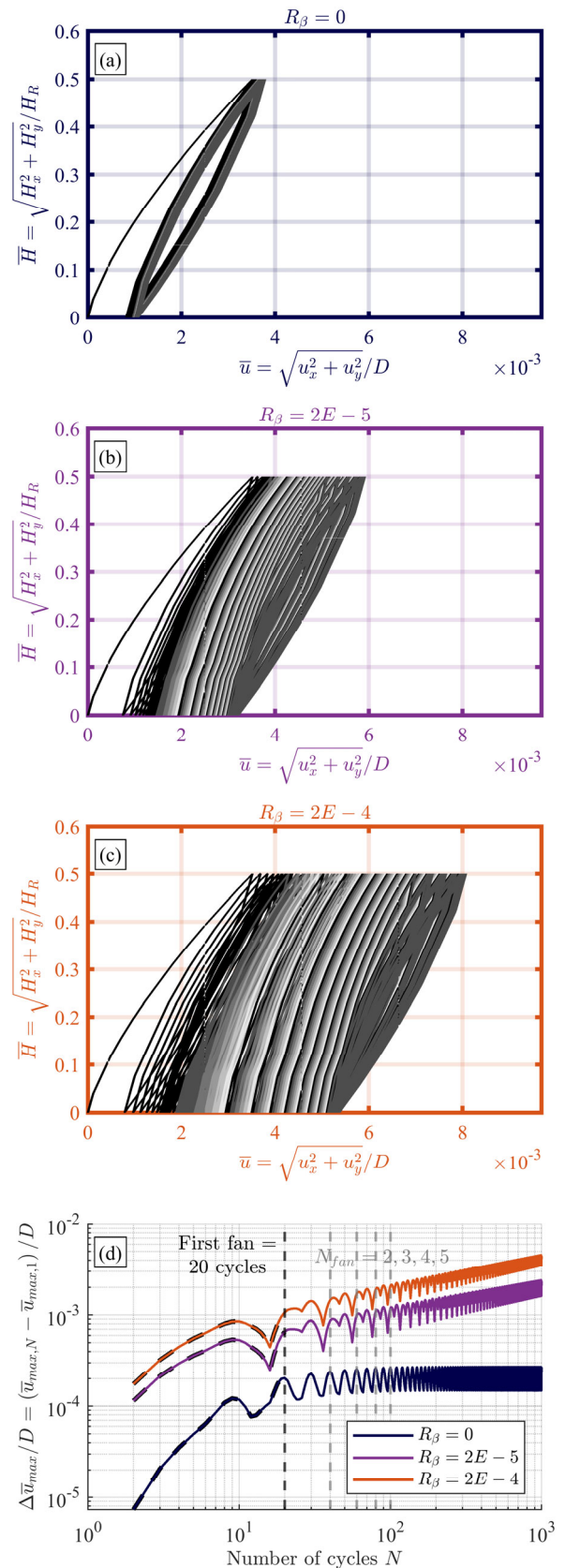


Figure 4. Cyclic Load-Displacement Response Using the CLAP Model with Varying Ratcheting Rates. (a) no ratcheting  $R_\beta = 0$ , (b)  $R_\beta = 2E-5$ , (c)  $R_\beta = 2E-4$ . Each fan is highlighted with progressively different colours. (d) Compared accumulated displacement for all 3 values of ratcheting rate (first fan highlighted in dashed black line)

modelling philosophies from offshore geotechnical engineering typically used for macro-modelling of monopile foundations: the REDWIN framework, which couples all relevant directions of lateral loading on a monopile ( $H_x$ ,  $H_y$ ,  $M_x$ ,  $M_y$ ) using kinematic hardening yield surfaces, and the HARM framework, which introduces the accumulation of permanent cyclic ratcheting into kinematic hardening plasticity.

The paper demonstrates the capabilities of the model through application to a typical multi-directional cyclic fan loading involving 1000 cycles. First, the model computes the response efficiently in approximately 300 seconds on a standard laptop, highlighting its potential for future design applications. It captures the pile macro-response in the 2D horizontal plane, enabling assessment of global pile displacements against design limitations. The results also illustrate the movement of the yield surfaces during cyclic loading, demonstrating the coupling between different degrees of freedom. Furthermore, the importance of ratcheting and its associated parameters is emphasised in achieving a realistic pile response, with calibration shown to control both the severity of the ratcheting rate and the magnitude of accumulated deformation.

Further research is required to advance model calibration and understanding of ratcheting parameters, particularly through in situ investigations (e.g. CPT) and laboratory element tests (e.g. static and cyclic direct simple shear (DSS) tests, triaxial tests), thereby reducing reliance on back-analysis. Nevertheless, this paper proposes a promising approach to enhance the modelling capabilities for multi-directional cyclic loading, with key applications to monopile design and shared anchors for floating offshore wind turbines. Although offshore monopiles provide the primary motivation for this study, the model is broadly applicable to analysing soils and other materials under multi-directional cyclic loading.

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