

Data assimilation for identifying hydro-mechanical parameters of an unsaturated slope during rainfall-induced seepage and deformation

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ABSTRACT: Data assimilation is a technique used to estimate uncertain parameters by integrating observed data. While it has been applied in geotechnical engineering for consolidation and slope seepage problems, its application to unsaturated slope seepage and deformation is still limited. This study explores the applicability of a particle filter (PF) for identifying hydro-mechanical parameters during seepage-induced deformation in unsaturated slopes. The PF algorithm employs a fully coupled three-phase hydromechanical model, integrating the Extended Modified Cam-Clay model and Van Genuchten model as mechanical and hydraulic constitutive models respectively. Observation data, including porewater pressure and displacement, were collected from centrifuge model experiments under rainfall and groundwater flow. Data assimilation was performed using pore water pressure and horizontal displacement as observational data. Hydraulic parameters (rainfall flux rate, permeability, SWCC parameters) and mechanical parameters (elastic modulus, compression index, swelling index) were identified. With the use of deformation data, particles that accurately reproduced the experimental results could be identified. This study highlights the need to consider the uncertainty in not only hydraulic information but also the mechanical parameters of the slope for predicting slope failure due to rainfall infiltration.

KEYWORDS: Data assimilation, hydro-mechanical parameters, coupled three-phase analysis, unsaturated slope.

1 INTRODUCTION

Data assimilation (DA) is a technique used to estimate uncertain parameters by integrating observed data. While it has been applied for consolidation and slope seepage problems in geotechnical engineering (Shuku et al. 2012; Murakami et al. 2013), its application to seepage-induced deformation of unsaturated slope is still limited. This study explores the applicability of a particle filter (PF) for identifying hydro-mechanical parameters of an unsaturated slope during rainfall-induced seepage and deformation. The PF algorithm employs a fully coupled three-phase hydromechanical model, integrating the Extended Modified Cam-Clay model and Van Genuchten model as mechanical and hydraulic constitutive models respectively. Observation data, including porewater pressure and displacement, were collected from centrifuge model experiments under rainfall and groundwater flow.

2 CENTRIFUGE MODEL TESTS AND FINITE ELEMENT MODELING

2.1 Centrifuge model tests

Centrifugal model experiments (Jayakody et al. 2024) were conducted to simulate groundwater and rainfall-induced seepage and deformation in an unsaturated slope. Figure 1 depicts the experimental setup with PWP transducers (PPTs) and markers for tracking the displacement. The experiment had two main steps, i) generation of the initial steady groundwater flow and ii) simulation of surcharged groundwater flow with rainfall infiltration. A rainfall with 25 mm/h in prototype scale was applied on the slope and pore water pressure (PWP) in the intermediate tank also increased simultaneously and maintained in the range of 95 to 105 kPa. PPTs 2, 3, and 4 are considered to discuss the development of PWP while markers 5, 6, 11, 12, 17, and 18 are selected to discuss the progress of displacement. The reason for selecting these monitoring locations is because these points are situated in the failure zone which is the interesting zone in this study. The experiment was conducted in 50g conditions and slope monitoring data has been recorded in 1-second interval. More details of the experiment steps, slope

materials and results are extensively discussed in Jayakody et al. (2024).

2.2 Finite element modeling

To validate the experimental results, three-phase 2-D coupled hydromechanical FEM software outlined by Uzuoka & Borja (2012) was employed in Jayakody et al. (2024). The in-house FEM software produces compelling results in both quasi-static and dynamic analysis exhibiting its versatility. The three

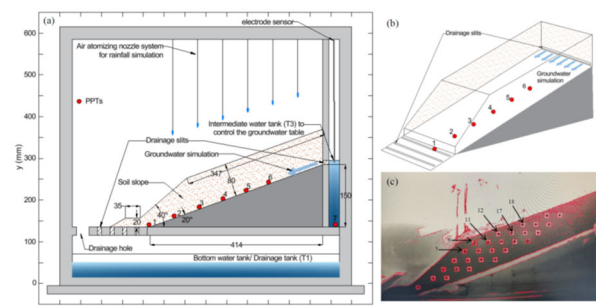


Figure 1. Centrifuge experimental model (a) The schematic diagram of the centrifuge apparatus, (b) Applying groundwater flow into the slope model and (c) Markers for tracking the displacement (Jayakody et al. 2024).

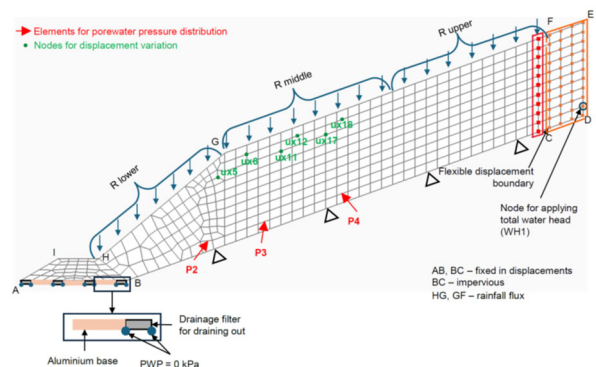


Figure 2. 2-D finite element model (Jayakody et al. 2024).

simplified governing equations derived by incorporating porous media theory at finite strain are the soil-water-air phase's momentum balance equation, pore-water, and pore-air's mass and momentum balance equations. Soil skeleton velocity, pore water pressure, and pore air pressure are the three primary variables of weak-form equations generated using the Galerkin method for quasi-static analysis. The water retention behavior of the soil was simulated using the Van Genuchten (VG) model (van Genuchten, 1980) in this simulation. The Extended Modified Cam-Clay model (Uzuoka et al. 2011) was utilized to describe the mechanical behavior of unsaturated soils by incorporating the effect of suction on the yield function.

Figure 2 shows the finite element model used for 2D coupled analysis with boundary conditions. The material parameters were determined by laboratory tests and preliminary analysis conducted in Jayakody et al. (2024). Two hydraulic boundary conditions are imposed in the analysis considering the seepage simulation and rainfall infiltration. Node WH1 is for applying the total head applied and during the simulation the data recorded from the PPT7 (which placed inside the intermediate tank) was used. However, rainfall was simulated as a flux and the flux values applied on the entire slope were not the same as shown, R_{lower} , R_{middle} and R_{upper} in Figure 2. Surface runoff was observed during the experiment and the middle segment had a high probability of receiving more infiltration compared to the upper part and lower part. Therefore, the actual rainfall flux cannot be determined from the experiments, and it should be calibrated to reproduce the experimental results. In this study several representative elements and nodes are selected for further analysis. P2, P3, and P4 are the elements parallel to PPT locations and u5, u6, u11, u12, u17, and u18 are nodes parallel to markers positions. The calculated data of the pore water pressure and horizontal displacement can be used as forward simulations to identify uncertain parameters using particle filter.

3 PARTICLE FILTER (PF) FRAMEWORK

The principle of PF concentrates on representing the required posterior density function by a set of random samples called 'particles', and their associated weights (Gordon et al. 1993; Doucet et al. 2000). Each particle represents a potential state of the system (numerical simulation), and the associated weight represents the likelihood of that state. In this study, an elastoplastic constitutive model is used to simulate the mechanical behavior of soil in the coupled FEM which is affected not only by the current stress state but also its history of loading. Therefore, it is preferable to use SIS (sequential importance sampling) (Doucet et al. 2000) without resampling since SIR (sequential importance resampling) (Gordon et al. 1993) creates new particles ignoring the history experienced by each particle (Shuku et al. 2012; Murakami et al. 2013).

The SIS algorithm of particle filter DA integrated into time-dependent geotechnical problems follows four steps (Initialization, Prediction, Filtering, and Weight update) as shown in Figure 3. To briefly explain the methodology behind the PF let's consider that α and β are two parameters or boundary conditions with some uncertainty. The prior probability density function of uncertain parameters can be assumed to be a log-normal or uniform distribution depending on the available information. The ensemble size (N) can be determined considering the number of uncertain parameters, parameters' sensitivity, and credibility of prior distribution etc. An ensemble of $\{\alpha_1, \alpha_2, \dots, \alpha_N\}$, and $\{\beta_1, \beta_2, \dots, \beta_N\}$ is generated and values are assigned randomly to parameters reflected in each particle. Forward simulation is conducted using N number of unique input files generated to predict the

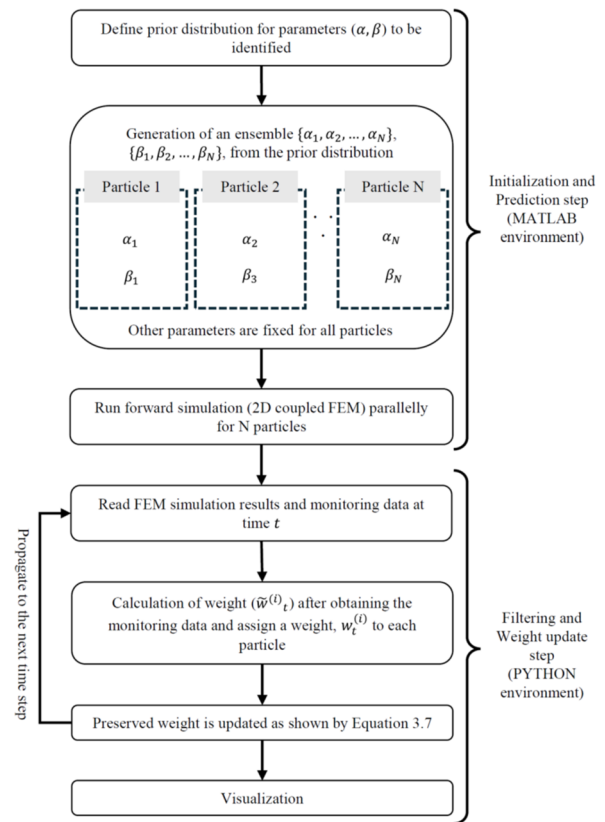


Figure 3. Particle filter algorithm developed in this study.

geotechnical response of the problem up to the desired time step. During this simulation parameters other than the uncertain parameters are kept constant. Displacements of selected nodes and PWP of selected elements are extracted at the end of the forward simulation. Once the monitoring data is realized for the respective time step, weight will be assigned to the particles via likelihood calculation. Subsequently, the updated posterior distribution is estimated. During the likelihood calculation, the observation noise is set by trial-and-error method. During the analysis, there is simulation data and monitoring data for each 1-second time step. Therefore, the weight of each particle will be updated and propagated in every arbitrary time step. At the end of the analysis graphs to visualize the output such as weight distribution, time history of parameters, etc., will be generated.

4 IDENTIFICATION OF HYDRO-MECHANICAL PARAMETERS

Data assimilation with PF was performed using pore water pressure and horizontal displacement as observational data. First hydraulic parameters (permeability, rainfall flux rate, etc.) were identified using PWP observation data. Second elastoplastic material parameters (elastic parameters, compression index, swelling index) were identified with the fixed identified hydraulic parameters using displacement observation data.

4.1 Identification of hydraulic parameters using pore water pressure as observation data

Table 1 shows the settings of prior distribution for candidate hydraulic parameters. Saturated water permeability coefficient (k), rainfall flux rate (ratio of rainfall infiltration into the ground to the specified rainfall intensity of 25 mm/h) along the slope (R_{lower} , R_{middle} and R_{upper}), and SWCC parameters (θ_r , θ_s , α , n) are candidate parameters and boundary conditions for this

analysis. The number of particles was set to 2000. This setting was made because previous analyses (Jayakody et al. 2024) and trail analyses conducted have shown that using values outside this range would basically make it impossible to reproduce the experiment in the analysis. DA was performed using only three PPTs (P2, P3, and P4) as observation points. The DA interval was set to 150 s~190 s, the weight update interval was every 5 s, and the standard deviation of the observed noise was commonly set to 2 kPa.

Figure 4 shows the time histories of the experimental (observed) results (black line) and analysis results (grey lines) for particles with higher weights (up to a total weight of 0.95) after DA up to 190s. P2 is the representative result of PWP and ux6 is that of horizontal displacement in Figure 2. The red dashed line indicates the final DA time. This illustrated that particles with higher weights reproduce the experimental PWP behavior relatively acceptable limits. However, since displacement data was not used as observed data and elastoplastic parameters were not included in the PF estimation, the displacement was not reproduced to satisfactory levels.

Figure 5 shows the weight distribution of particles at 175s for the permeability coefficient k and the SWCC parameter α . To avoid overly restrictive narrowing, the weight distribution is based on the results at the assimilation point of 175s. The weighted mean values of each estimated parameter are shown on the weight distributions for reference. The weight distribution shows that the permeability coefficient k is very sensitive, with particles with high weights distributed in a narrow range. On the other hand, the weight distribution of the SWCC parameter α becomes relatively broad and exhibits lower sensitivity compared to sensitivity of k . The parameters determined by laboratory testing are found to lie within the range of particles with high weights. This indicates that the weight distribution of the SWCC parameters is too broad to be effectively narrowed down. Furthermore, the test values do not show any significant deviation from acceptable ranges.

4.2 Identification of mechanical parameters using displacements as observation data

After identification of hydraulic parameters in the previous section, the remaining mechanical parameters identification were performed. Table 2 shows the settings of prior distribution for candidate mechanical parameters. Elastic parameters (μ_0 , μ_1), compression index (λ) and swelling index (κ) are candidate mechanical parameters for this analysis. The μ_0 is constant elastic shear modulus and, μ_1 is dependency coefficient of elastic shear modulus on confining pressure. The hydraulic parameters and the boundary conditions were fixed as the predetermined values in this case. A total of 1500 particles were used in the analysis. DA was performed using six horizontal displacement points (ux5, ux6, ux11, ux12, ux17, and ux18) near the slope front as observation points. DA started in the 170s, and weights were updated every 2 s. The standard deviation of the observation noise at ux5, ux6, ux11, and ux12 was set to 0.001 m. The standard deviation of the observation noise at ux17 and ux18, where almost no deformation occurred in the experiment, was set to 0.0002 m, which is relatively small.

Figure 6 shows a plot of the experimental (observed) results (black line) and analysis results (grey lines) for particles with higher weights (up to a total weight of 0.95) after DA up to 192s. The red dashed line indicates the final DA time. This illustrated that particles with higher weights reproduce the experimental PWP and displacement behavior relatively acceptable limits since displacement data was used as observed data in this case. When compared with Figure 4, the simulated PWP slightly overestimated the observed PWP at P2 because

Table 1. Prior distribution conditions for candidate hydraulic parameters.

Parameter	Distribution	μ	σ
Saturated water permeability (k , m/s)	log-normal	2×10^{-5}	1.5×10^{-5}
	Distribution	Min.	Max.
R_{lower}	uniform	0.0	0.7
R_{middle}	uniform	0.3	0.7
R_{upper}	uniform	0.0	0.7
	Distribution	μ	σ
Minimum water saturation (θ_r)	normal	0.25	0.03
Maximum water saturation (θ_s)	normal	0.91	0.03
VG model parameter (α)	normal	0.41	0.15
VG model parameter (n)	normal	2.2	0.30

μ : mean value, σ : standard deviation

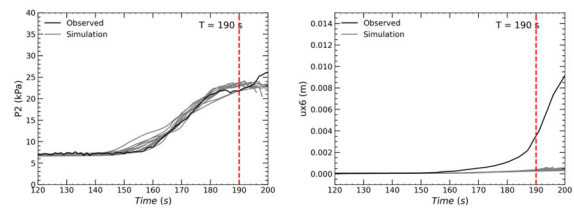


Figure 4. DA results of hydraulic parameters with higher weight particles (up to 0.95 total weights) at 190s and experimental result (Observation points: PWP only) (left: PWP at P2, right: Horizontal displacement at ux6).

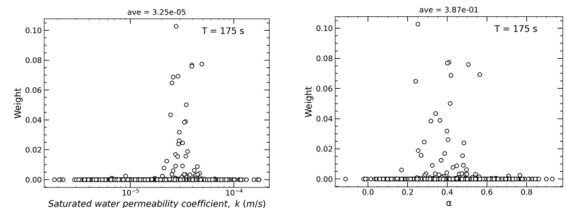


Figure 5. Weight distribution of permeability at 175s (Observation points: PWP only) (left: permeability coefficient k , right: SWCC parameter α).

the hydraulic parameters were predetermined in this case. This indicates that we need further study with full DA for both hydraulic parameters and mechanical parameters as simultaneous estimation targets.

Figure 7 shows the weight distribution of particles at 182s for the elastic parameters μ_1 and swelling index κ . The weight distribution shows that the elastic parameters μ_1 is biased while the weight distribution of the swelling index κ becomes relatively broad and exhibits lower sensitivity compared to sensitivity of μ_1 . The value (0.025) of κ determined by laboratory testing is found to lie within the range of particles with high weights.

5 CONCLUSIONS

Hydraulic parameters (rainfall flux rate, permeability, SWCC parameters) and elasto-plastic parameters (elastic modulus, compression index, swelling index) were identified using PF. The PF algorithm employs a fully coupled three-phase hydromechanical model, integrating the Extended Modified Cam-Clay model and Van Genuchten model as mechanical and hydraulic constitutive models respectively. Observation data,

including porewater pressure and displacement, were collected from centrifuge model experiments under rainfall and groundwater flow. With the use of deformation data, particles that accurately reproduced the experimental results could be identified. This study highlights the need to consider the uncertainty in not only hydraulic information but also the elasto-plastic parameters of the slope for predicting slope failure due to rainfall infiltration.

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Table 2. Prior distribution conditions for candidate mechanical parameters.

Parameter	Distribution	Min.	Max.
Elastic parameters (μ_0)	uniform	5	50
Elastic parameters (μ_1)	uniform	70	140
Compression index (λ)	uniform	0.025	0.20
Swelling index (κ)	uniform	0.010	0.050

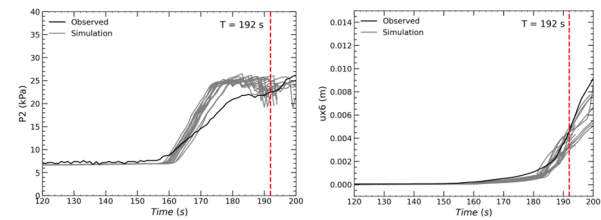


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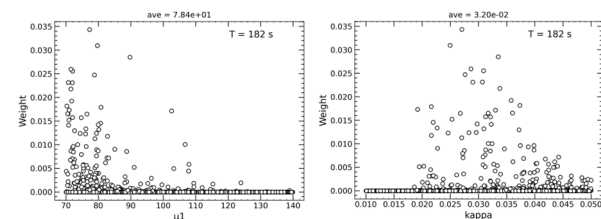


Figure 7. Weight distribution of mechanical parameters at 182s (Observation points: displacement only) (left: elastic parameters μ_1 , right: swelling index κ).