

Parallel spring constitutive modeling of expansive soft rock describing degradation of cementation due to wetting-drying cycles

Yoshifumi Kondo, Hiroyuki Kyokawa
 Nagoya Institute of Technology, Nagoya, Japan, konyoshi@kajima.com

Mamoru Kikumoto
 Kyoto University, Kyoto, Japan

Ying Cui
 Yokohama National University, Yokohama, Japan

ABSTRACT: The objective of this paper is to propose a new constitutive model to accurately represent the behavior of expansive soft rock during wetting-drying cycles. In the proposed model, the expansive soft rocks are assumed as a composite stiffness structure combining the matrix part and the soil part, namely, which is “parallel spring model”. In addition, the damage model, which plays a role in controlling the composite stiffness with damage of the matrix part, is modified to consider the damage progressing irreversibly and smoothly in response to cyclic external conditions like wetting-drying cycles. The validity of the proposed model is demonstrated by comparison with experimental results on non-expansive soil with cementation, and the expansive soft rock. Furthermore, through the simulation considering the wetting-drying cycles, it has been successfully demonstrated that the proposed model can uniquely represent the manifestation of swelling behavior and strength reduction with cycle progression.

KEYWORDS: Constitutive model, expansive soft rock, swelling, wetting-drying cycles.

1 INTRODUCTION

Uplifting of a roadbed and extruding a sidewall in a tunnel during its construction and service period have become a problem in expansive soft rock ground containing expansive clay minerals such as smectite. In current tunnel design in practice, although the deformation potential of such ground is comprehensively evaluated using macroscopic indicators such as strength and smectite content, a quantitative assessment for such deformation of such ground has not been established.

The causes of such deformation of expansive soft rock ground are considered to be the shearing of the ground, so-called squeezing and the swelling of the soil, and it is appropriate to deal with both squeezing and swelling (Einstein, 1996). Thus, it would be natural to think that both swelling and shearing would occur simultaneously and/or influence each other, and it results in progressive deformation due to wetting and drying cycles in the service period of a soil structure like a tunnel. In this study, aiming to precisely predict the magnitude and timing of such deformation in an expansive soft rock ground, parallel spring modeling is proposed to consider the interaction between the microscopic behavior in clay mineral scale and plastic deformation due to weathering. The proposed model can express the suppression of swelling due to cementation and the manifestation of swelling behavior as damage progresses under wetting-drying cycles (interaction between swelling and cementation), which could not be considered in other models.

2 CONSTITUTIVE MODELING OF EXPANSIVE SOFT ROCK CONSIDERING AN INTERACTION BETWEEN SWELLING NATURE AND CEMENTATION

2.1 Micro and macro characteristics of expansive soft rock

Soft rock consists of two phases: soil parts like clayey soils, and hard matrix parts that cementation developed through diagenesis. In addition, the cementation is lost due to the swelling and shrinkage of clay minerals during wetting-drying cycles, which leads to a strength degradation (Cai et al., 2020). Moreover, in the swelling test in oedometer using an intact sample of expansive soft rock and its remolded sample that

artificially lost their cementation (refer to Figure 6), the swelling amount of the intact sample was smaller than that of the remolded one. This suggests that the cementation suppresses swelling, and as the cementation is lost, the swelling potential becomes apparent. Furthermore, in the reconsolidation process after water immersion, it was observed that the consolidation curve of the intact sample approached that of the remolded sample.

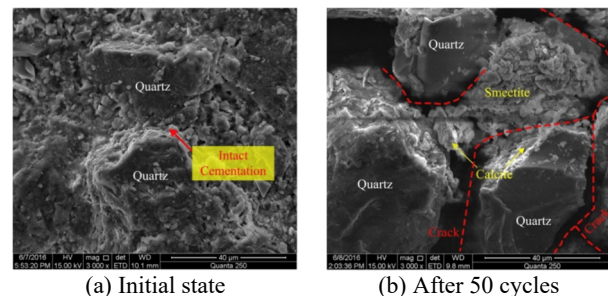


Figure 1. SEM image of sandstone (Cai et al., 2020).

2.2 A parallel spring-damage model of expansive soft rock considering cyclic deformation

The proposed model extends the previous model for expansive soft rock which considers cementation and swelling nature (Kyokawa et al., 2023) by taking into account damage history, so that the damage evolves irreversibly and smoothly under cyclic external condition including wetting and drying cycles.

With reference to the microscopic structure of soft rock, as shown in Figure 1, a composite stiffness structure, namely a parallel spring model, is assumed to be responsible for expansive soft rock in this study. It should be noted that the matrix part and the soil part undergo the same strain, with the stiffness of each component influencing the behavior of the other. It would be reasonable to represent the interaction between swelling potential and cementation.

The stress partition in the parallel spring model is expressed by using the scalar variable ξ as follows:

$$\sigma_{ij} = (1 - \xi)\sigma_{ij}^{\text{net M}} + \xi\sigma_{ij}^{\text{net S}} \quad \text{where } 0 \leq \xi \leq 1 \quad (1)$$

where the subscripts M and S denote the matrix part and the soil part, respectively, and the direction of compression is assumed to be positive unless otherwise noted in the following. Unsaturated conditions are incorporated using Bishop's effective stress (Equation (2)), where the degree of saturation of each component is assumed to be equal for simplicity and uniquely determined by suction via the van Genuchten model (Equation (3)).

$$\sigma_{ij}^{\text{net M}} = \sigma''_{ij}^{\text{M}} - S_r^{\text{M}} s \delta_{ij}, \quad \sigma_{ij}^{\text{net S}} = \sigma''_{ij}^{\text{S}} - S_r^{\text{SS}} s \delta_{ij} \quad (2)$$

$$S_r^{\text{M}} = S_r^{\text{SS}} = (1 + (\alpha_{ss} s)^{n_{ss}})^{-m_{ss}} \quad (3)$$

The stress partition ratio ξ in Equation (1) is considered equivalent to the damage variable in continuum damage mechanics and is given as a function of the boundary equivalent strain κ_{max} , developed with reference to the isotropic damage model (Kurumatani et al., 2016):

$$\xi = \xi_0 + (1 - \xi_0) \left(1 - \frac{\kappa_0}{\kappa_{\text{max}}}\right) \exp(-\beta(\kappa_{\text{max}} - \kappa_0)) \quad (4)$$

$$\beta = \frac{E_M \kappa_0 h_e}{G_f}$$

where E_M denotes the Young's modulus of the matrix part, κ_0 represents the initial value of equivalent strain, h_e is the representative element length, and G_f is the fracture energy. The boundary equivalent strain κ_{max} means the largest value of the equivalent strain κ ever. Furthermore, the κ is a scalar variable and given by the following monotonically increasing function depending on the invariants I_1 and J_2 of the total strain tensor:

$$\kappa = \frac{1}{2k_M} \left(\gamma I_1 + \sqrt{(\gamma I_1)^2 + \frac{12k_M}{(1 + \nu_M)^2} J_2} \right) \quad (5)$$

$$\gamma = \frac{k_M - 1}{1 - 2\nu_M}$$

where ν_M is Poisson's ratio of the matrix part, and k_M represents the ratio of uniaxial compressive strength to uniaxial tensile strength.

In this study, the boundary equivalent strain κ_{max} is redefined to describe the irreversible damage evolution for non-monotonic deformations even in small deformation. And then, the κ_{max} develops by following the evolution rule based on the past deformation history:

$$L = 1 - \exp(-(\kappa_{\text{max}} - \kappa)) \quad 0 \leq L \leq 1 \quad (6)$$

$$\delta L = -\frac{1}{a} L |L| \delta \kappa_{\text{max}} \quad (7)$$

$$\begin{cases} \delta \kappa_{\text{max}} > 0 \text{ when } \delta \kappa = \frac{\partial \kappa}{\partial \boldsymbol{\varepsilon}} \cdot \delta \boldsymbol{\varepsilon} > 0 \\ \delta \kappa_{\text{max}} = 0 \text{ when } \delta \kappa = \frac{\partial \kappa}{\partial \boldsymbol{\varepsilon}} \cdot \delta \boldsymbol{\varepsilon} \leq 0 \end{cases} \quad (8)$$

where L is the relative magnitude of the current strain variable κ with respect to its maximum value, and a is the material parameter related to damage evolution. Figure 3 shows the typical behavior of the stress partition ratio ξ , which is equal to the damage variable, given by Equations (4) through (8). During cyclic deformation, the value of ξ increases, namely damage progress, when $\delta \kappa_{\text{max}} > 0$ ($\delta \kappa > 0$).

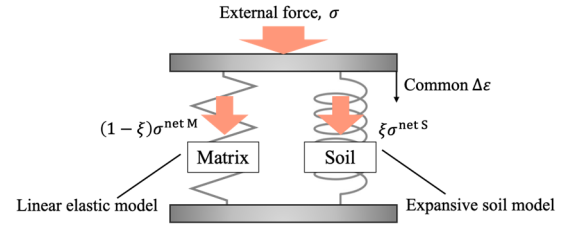


Figure 2. Diagram of stress partition in a parallel spring.

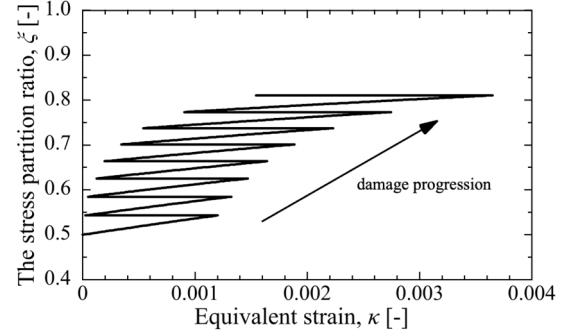


Figure 3. Change in the stress partition ratio ξ during cyclic deformation.

The proposed model is formulated by individually assigning suitable constitutive law to each component of parallel spring structure. The matrix part is assumed to be a linear elastic material, and its constitutive relation is expressed by the following equation.

$$\sigma''_{ij}^{\text{M}} = D_{ijkl}^{\text{eM}} \varepsilon_{kl} \quad (9)$$

In addition, the soil part is modeled by the expansive soil model (Kyokawa, 2021), which considers the hydro-mechanical-chemical interactions on the crystal surface in order to reproduce the swelling nature of expansive soft rock. In this model, the overall void space is assumed to have a double-porous structure consisting of soil skeleton voids and interlaminar voids within the clay crystals. Using the subscripts ss and il to represent soil skeleton and interlaminar components, respectively, the total strain of the soil component $\varepsilon_{ij}^{\text{S}}$ is expressed by the following equation.

$$\varepsilon_{ij}^{\text{S}} = \varepsilon_{ij}^{\text{SS}} + \varepsilon_{ij}^{\text{il}}(d) \quad (10)$$

For interlaminar strain $\varepsilon_{ij}^{\text{il}}$, its behavior is determined by calculating the interlaminar distance d between layered mineral crystals from the equilibrium equation for the four forces acting on the crystal surface, namely van der Waals force f_a and mean effective stress of soil part p''^{S} as attractive forces, and osmotic force f_h and hydration force f_r as repulsive forces:

$$F^*(d, c_\alpha, \sigma''_{ij}^{\text{S}}, S_r^{\text{SS}}) = f_a - f_r - f_h + p''^{\text{S}} = 0 \quad (11)$$

where c_α is the concentration of α -cation. During soaking process, the osmotic force f_h and the hydration force f_r increase leading to an expansion of the interlaminar distance. This process represents macroscopic swelling due to osmotic and crystalline swelling.

The change in soil skeleton strain $\varepsilon_{ij}^{\text{SS}}$ in Equation (10) is represented by a general elasto-plastic constitutive model. The yield function of its model is expressed as in Equation (12), and the effective stress of the soil σ''_{ij}^{S} is required for $\varepsilon_{ij}^{\text{SS}}$ and $\varepsilon_{ij}^{\text{il}}$.

$$f = \ln \frac{p'^{rS}}{p'^{sS}_0} + \ln \left(1 + \left(\frac{\eta'^{rS}}{M} \right)^2 \right) - \frac{1 + e_0^S}{\lambda - \kappa_{swe}} \varepsilon_V^{ss} p + \frac{1 + e_0^S}{\lambda - \kappa_{swe}} \left(\frac{\rho - \rho_0}{1 + e_0^S} \right) - \frac{1 + e_0^S}{\lambda - \kappa_{swe}} \left(\frac{\psi - \psi_0}{1 + e_0^S} \right) = 0 \quad (12)$$

Here, M is the stress ratio at the critical state, η'^{rS} is the stress ratio of the soil part, λ is the compression index, and κ_{swe} is the swelling index. The variables ρ and ψ are the state variables for density and unsaturation effects respectively; thus, this model can represent typical mechanical and hydraulic behaviors of unsaturated soil such as soaking collapse and its stress level dependency.

3 VALIDATION OF PROPOSED MODEL FOR NON-EXPANSIVE SOIL WITH CEMENTATION

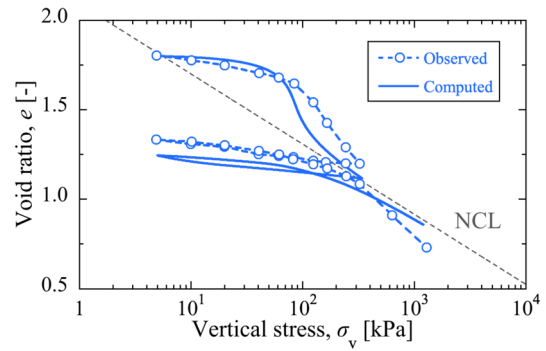
As a first step in the validation of the proposed model, the behavior of non-expansive soil with cementation, namely the structured soil, is simulated. In the model, by applying the interlaminar void ratio $e^{il} = 0$, the soil part of the parallel spring structure represents the behavior of the non-expansive soil, namely $\varepsilon_{ij}^S = \varepsilon_{ij}^{ss}$ in Equation (10).

As shown in Figure 4(a) and (b), the model generally reproduces the typical behavior of the structured soil, which exhibits a higher void ratio than the normally consolidation line (NCL) under the same confining pressure and behavior leading to critical state with the strain softening during shear. These results indicate that the proposed parallel spring-damage model can adequately represent the behavior of geomaterials with cementation.

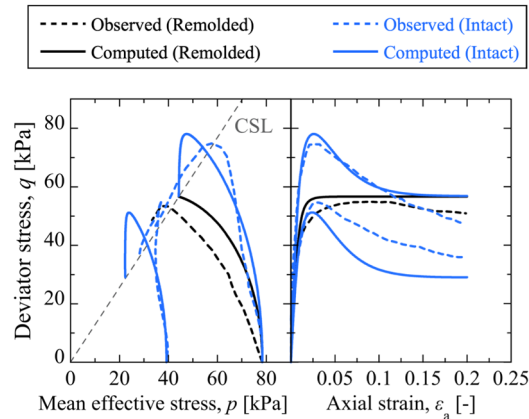
The difference in stress between the intact and remolded samples shown in Figure 4(b) represents the stress borne due to cementation, and the overall behavior of the intact soil approaches that of the remolded soil with the loss of cementation. In the previous modeling of strain softening behavior of the soil with cementation (Adachi and Oka, 1995), it is assumed that the material strength is composed of frictional strength and others due to cementation (and/or the cohesion), and the cementation effect on strength decreases with deformation, which eventually frictional strength develops. In the proposed model, the stress contribution of the matrix part decreases as the damage progresses, and the soil part eventually bears the stress, which is based on a concept similar to the previous study.

Figure 5 shows the simulation results of uniaxial compression and tension tests, which indicates that the tensile strength is weaker than compression strength because the damage evolves more easily in tension than in compression due to the characteristic of the damage model in Equation (5).

Some previous Cam clay type models (e.g. Asaoka et al., 2000) can reproduce the typical behaviors of structured soil like Figure 4. However, such a model could not consider the tensile stress state in general. On the other hand, the proposed model is a Cam clay type model that can also consider tensile stress conditions. Although the stress is in a tensile state, the compressive stress of the soil part is maintained by the matrix part ensuring the tensile stress.



(a) Oedometer test



(b) Undrained triaxial test

Figure 4. Comparison of observed and computed results of element tests on structured soil (tested by Adachi et al., 1995).

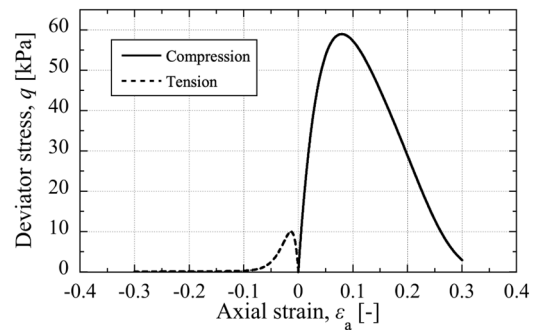


Figure 5. Simulation results of uniaxial compression and tension tests

4 VALIDATION OF PROPOSED MODEL FOR EXPANSIVE SOFT ROCK

A simulation of element test of expansive soft rock is conducted to validate the proposed model. Firstly, the one-dimensional swelling and consolidation tests on the expansive soft rock (intact sample) and its remolded one are simulated. The initial stress partition ratio ξ_0 was set to $\xi_0 \cong 1$ for the remolded sample, assuming the completely destructured state, while for the intact sample, it was set to $\xi_0 = 0.6$ through a calibration. For the simulation, suction and cation concentration were set assuming a completely dried initial condition, considering that the specimens used in the experiments were dried. During the water soaking process, suction was reduced to reach saturation according to SWCC under constant stress condition, and cation concentration was decreased to a value equivalent to that of distilled water.

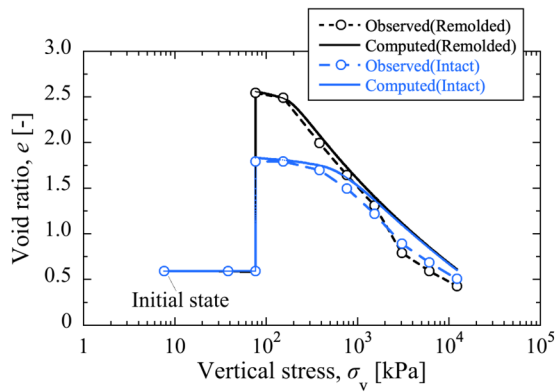


Figure 6. Comparison of observed and computed results of the swelling-consolidation tests on expansive soft rock.

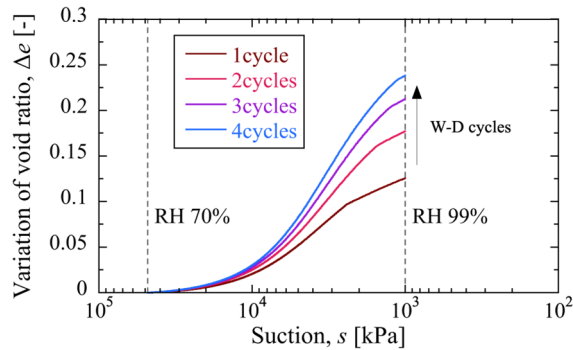


Figure 7. Variations of void ratio in each wetting process.

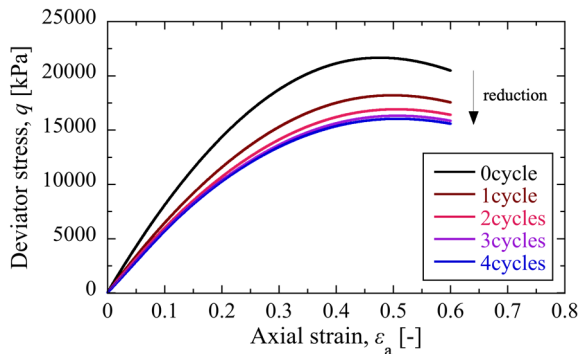


Figure 8. Stress-strain relationships in conventional triaxial compression after each drying process.

The results of the one-dimensional swelling and consolidation test shown in Figure 6 indicate that the proposed model accurately reproduces the behavior of expansive soft rock. As the soil expands during water soaking, tensile stress is induced in the matrix, while its stiffness restrains the expansion, leading to the development of compressive stress in the soil part. This results in the intact sample exhibiting less expansion, where the effective stress in the soil part is higher than that of the remolded sample. This is because the expansion of the interlaminar spacing is suppressed by the force equilibrium described in Equation (11).

Subsequently, with the aim of describing deformation of a tunnel in the ground of expansive soft rock, the performance of the proposed model on the wetting-drying cycles was investigated through parametric study. During wetting-drying cycles, suction was varied under constant cation concentration under anisotropic stress conditions, assuming a tunnel environment with a relative humidity (RH) of 70% for drying and 99% for the saturated state. Figure 7 presents variations of the void ratio during the wetting process in each cycle.

Additionally, Figure 8 shows the stress-strain relationship in conventional triaxial compression ($\sigma_r = 49$ kPa) under relative humidity of 70% after each drying process. It can be seen from Figure 7 that the swelling magnitude increases as the wetting-drying cycles progress, and from Figure 8 that the stiffness and peak strength decrease as the cycles progress. This is because the stress partition ratio ξ increases irreversibly due to the wetting-drying cycles. These results indicate that the proposed model successfully represents both the manifestation of swelling behavior and the degradation of strength induced by loss of cementation due to the deformation during wetting-drying cycles, which would be the key of deformation of a tunnel in the ground of expansive soft rock.

5 CONCLUSIONS

This study aimed to quantitatively evaluate the deformation of the expansive soft rock, especially due to the wetting-drying cycles, which is thought to be a key factor of uplift and extrusion of the inside of a tunnel. In order to consider the interaction between swelling and cementation, the parallel spring-damage model was proposed by combining the swelling nature of expansive soil and the cementation of the rock matrix. The proposed model can suitably describe the manifestation of swelling behavior, and stiffness and strength reductions due to the wetting-drying cycles. This indicates that the proposed model can reproduce the phenomenon of tunnel deformation over a long service period due to the degradation of the surrounding ground over time. In the future, this model can be used in the numerical simulation of soil/water coupling to clarify the long-term deformation mechanism of tunnels.

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