

Crushing progression and critical state surface of Ta-d-p pumice in triaxial tests

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ABSTRACT: Volcanic pumice, a highly crushable and porous granular material, is widely distributed across the globe and contributes to various geotechnical issues, including slope failures. Owing to its intrinsic intra-particle voids and high compressibility, coupled with crushing-induced grain size evolution, its mechanical behaviour is highly complex. Consequently, crushable porous granular soils are often classified as problematic soils and treated as distinct cases, rather than being systematically understood within conventional soil mechanics frameworks. This study investigates the progression of crushing and the critical state surface of Ta-d-p pumice through a series of triaxial compression tests, including isotropic consolidation and both drained and undrained tests (CD/CU). A total of 48 tests were conducted, with axial strain ranging from 0% to 70%, and the particle size distribution before and after testing was compared to evaluate the extent of crushing. The results were fitted to a previously proposed critical state surface equation for crushable porous soils. The experimental data aligned closely with the predicted surface, confirming that the model effectively represents the mechanical behaviour of Ta-d-p pumice. The results show that the path to the critical state surface differs between CU and CD tests. In CU tests, the material reached the critical state at approximately 5% axial strain, following a nearly linear trajectory whilst undergoing crushing. In contrast, CD tests exhibited pronounced contractancy, with crushing progressing alongside volumetric compression. The material gradually approached the critical state surface, reaching it at around 50% axial strain. This study underscores the importance of incorporating crushing progression in the mechanical modelling of porous granular soils. The findings provide fundamental insights into the interplay between crushing, stress, and void ratio evolution, thereby contributing to the advancement of constitutive models for crushable geomaterials.

KEYWORDS: Volcanic pumice; particle crushing; critical state surface; triaxial test

1 INTRODUCTION

Crushable soils with high void structures are distributed across many regions of the world and cause various engineering problems. Understanding the mechanical behaviour of these crushable porous granular materials is one of the key challenges in geotechnical engineering. It has been shown that crushable soils follow an undrained stress path similar to that of loose sands during shearing (Hyodo, Hyde and Aramaki, 1998), and that the broadening of particle size distribution due to crushing improves the packing efficiency of soil particles, significantly influencing the location of the critical state in the $e - \log p'$ space (Bandini and Coop, 2011). In addition, soils with intra-particle voids, such as volcanic coarse-grained soils, exhibit high compressibility and show marked changes in particle arrangement due to crushing. Consequently, compared to non-porous materials, particle crushing occurs more readily even at lower pressure levels and has a substantial impact on the compressive and shear behaviour within typical engineering pressure ranges (Sato, Kuwano and Otsubo, 2024). A pore structure model has been proposed in which the voids in porous soils are classified into two components: the inter-particle void ratio and the intra-particle void ratio (Figure 1) (Ishikawa and Miura, 2011). For porous pumice, the application of critical state theory using the state parameter has been explored, and it has been suggested that undrained mechanisms of pore water pressure rise in volcanic soils can be explained based on the state parameter obtained from static tests (de Cristofaro et al., 2022). Moreover, significant changes in the degree of crushing affect the particle packing structure and result in changes in the position of the critical state line in the $e - \log p'$ space. This, in turn, alters the basis for defining the state parameter and impacts the applicability of the critical state framework. Thus, to understand the mechanical behaviour of porous pumice, it is important to investigate the relationship between the degree of crushing and the mechanical properties. Sato, Kuwano and Otsubo (2025) proposed that the critical state of crushable porous soils can be expressed as a critical state surface in a three-dimensional space defined by stress, void ratio, and degree of particle crushing, and presented a mathematical formulation for the surface. However, in the case of crushable

porous soils, the progression of particle crushing with strain leading up to the critical state has not yet been sufficiently clarified.

This study aims to identify at what stage and to what extent particle crushing occurs in the process by which crushable porous soils approach the critical state. A total of 48 triaxial compression tests (including isotropic consolidation and CD/CU tests) were conducted using volcanic pumice, varying the axial strain at the end of the tests from 0% to 70%. The progression of particle crushing was evaluated by comparing the particle size distributions before and after the tests.

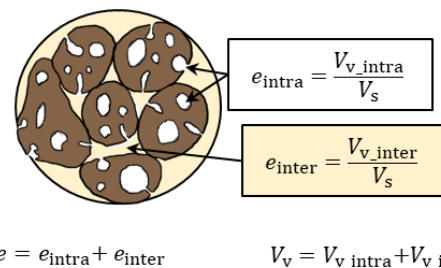


Figure 1. Conceptual diagram of intra-particle void in porous soils.

Table 1. Physical properties of Ta-d-p pumice.

	ρ_s [g/cm ³]	Consistency characteristic			Aspect ratio
		w_L [%]	w_p [%]	I_p	
Ta-d-p	2.57	99.2	94.2	5.00	1.30

2 MATERIAL

In this study, Ta-d-p pumice was used as the experimental material. The Ta-d pumice, which is deposited in the vicinity of Atsuma Town in Hokkaido, can be classified into two types: Ta-d-p, which floats when immersed in water, and Ta-d-m, which sinks. Ta-d-m contains a higher proportion of mineral components, and its single-particle crushing strength has been reported to differ from that of Ta-d-p (Sato et al., 2022). To ensure material homogeneity as much as possible, this study employed Ta-d-p pumice, which floated immediately upon immersion in water. The material was completely oven-dried at 50°C prior to testing. The Ta-d pumice used in this study was collected on 3 July 2023 during a field investigation from the

western margin of a collapsed slope (42°44'48.9"N, 141°53'52.0"E). Table 1 shows the physical properties of the experimental material. The particle density was measured after crushing the samples and thoroughly boiling them to eliminate the influence of intra-particle voids, thereby obtaining the true particle density. Figure 2 shows the intra-particle void ratio (e_{intra}) of Ta-d-p pumice plotted against particle size, measured using the sieve-count method (Sato, Kuwano and Otsubo, 2024). The intra-particle void ratio ranged from approximately 2 to 6, confirming that Ta-d-p is highly porous.

3 TEST PROCEDURE

Table 2 provides a summary of the triaxial compression tests conducted in this study. As the tests were carried out to large strains exceeding 20%, all strains were evaluated using logarithmic (true) strain. To facilitate the analysis of the effects of particle crushing during shearing, the initial particle size distribution was standardised across all tests, and only particles in the range of 2 mm to 4.75 mm were used. For specimen preparation, particles were compacted as densely as possible while minimising particle breakage. This was achieved by filling a metal mould (50 mm diameter, 100 mm height) with small amounts of material using a spoon, while lightly tapping the mould with a hammer to apply vibration. All tests were performed under saturated conditions, and saturation was confirmed by ensuring a B-value of at least 0.95. The back pressure was set to 200 kPa. During consolidation, the pressure was increased at a rate of 10 kPa/min, and the axial strain rate during triaxial compression was maintained at 0.5%/min. After each test, to avoid particle crushing, the drainage valve was kept closed while gradually reducing the cell and back pressures, after which the specimens were carefully retrieved. The retrieved samples were washed through a 75 µm sieve and then oven-dried at 50°C. Sieve analysis was conducted thereafter. The mass of fines passing the 75 µm sieve was calculated as the difference between the oven-dried weight before and after

testing. The sieve tests were carried out manually for 3 minutes, both before and after testing, in accordance with the method recommended by the Japanese Geotechnical Society for volcanic soils. Figure 3 shows the changes in particle size distribution before and after testing. In all cases, the grain size curves shifted to the left, indicating the progression of particle crushing.

To quantify the degree of particle crushing, this study employed the Grading State Index I_G requires a reference grading representing the fully crushed state, known as the limiting grading. However, it is difficult to determine the precise limiting grading for porous particles. Therefore, a one-dimensional compression test was conducted using Ta-d-p pumice with an initial particle size range of 2 mm to 4.75 mm under a vertical stress of 25,000 kPa. The resulting particle size distribution (see Figure 3) was fitted using Einav's fractal grading model (Einav, 2007). From this fitting, a fractal dimension of $\alpha = 2.99$ was obtained and adopted as the limiting grading for this study. The ratio of the minimum to maximum particle size in the fractal distribution function was assumed to be 0.0001, following the precedent of a previous study (Sato, Kuwano and Otsubo, 2025).

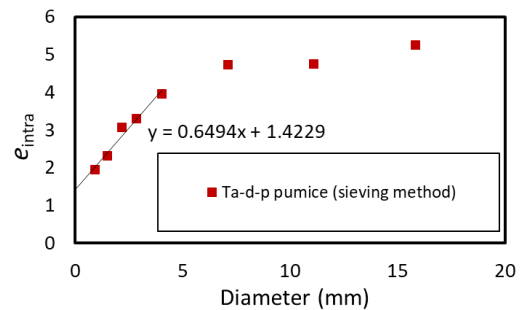


Figure 2. Relationship between intra-particle void ratio and particle size.

Table 2. Summary of triaxial compression tests.

Paricle size (mass ratio)	Test type	Confining Pressure (kPa)	Axial strain at the end of the compression stage
2–4.75 mm (100%)	C (Isotropic consolidation test)	50	0%
		100	0%
		200	0%
		600	0%
	CD	50	2, 5, 11, 22, 35, 50, 70%
		100	2, 5, 11, 22, 35, 50, 70%
		200	35, 50, 70%
	CU	50	2, 5, 11, 22, 35%
		100	0.1, 0.2, 0.4, 0.5, 1, 2, 5, 11, 22, 35%
		200	11, 22, 35%
		300	11%
		400	11%
	500	11%	
	600	0.2, 0.5, 1, 2, 5, 11, 15%	

4 TEST RESULTS AND DISCUSSION

Figure 4 shows the results of the triaxial compression tests on Ta-d-p pumice. In both CD and CU tests, tests initiated from the same initial conditions exhibited broadly similar stress–strain relationships, confirming sufficient reproducibility. In the CU tests, the stress path was similar to that of very loose sands, and the critical state was reached once the axial strain exceeded 5%. In contrast, the CD tests exhibited high contractancy, and the deviator stress reached a critical state (steady state) when the axial strain exceeded 50%. In this study, for CD tests where the stress ratio reached a critical state, the critical state was deemed to have been achieved even if the volumetric strain had not yet stabilised. This is because, in crushable materials, particle crushing may continue to occur after the critical state stress is reached, and the resulting changes in grain size distribution can cause ongoing volumetric changes. Therefore, it should be noted that the definition of the critical state in the context of crushable sands differs from that commonly applied to clays. Observations of the specimens after testing revealed that no distinct shear band formed until an axial strain of 11% in CU tests and 50% in CD tests. Beyond these strain levels, clearly visible shear bands were observed (see Figure 5).

Next, the results of this study are interpreted using the critical state surface equation proposed for crushable porous soils (Sato, Kuwano and Otsubo, 2025).

$$e = e_{\text{inter_min}}^i - e_c I_G + e_{\text{intra}}^i + (e_{\text{inter_max}}^i - e_{\text{inter_min}}^i) \exp \left[- \left(\frac{p'}{p_{cs}} \right)^{k_1} \right] \quad (1)$$

To determine the critical state surface of Ta-d-p pumice, the test results at the critical state were fitted to Equation (1). In this study, test cases where a distinct shear band had formed at the critical state were excluded from the fitting, as particle crushing tends to localise along the shear band, which could lead to either overestimation or underestimation of the overall crushing when compared to homogeneous deformation. Accordingly, only the cases with axial strains of 5% and 10% in CU tests and 50% in CD tests were considered to have reached a homogeneous critical state and were used for fitting. Figure 6 presents the critical state surface of Ta-d-p pumice obtained from the fitting. During the fitting process, the intra-particle void ratio e_{intra}^i at the maximum particle size was taken

as 4.51, based on a linear approximation for particle sizes below 5 mm shown in Figure 2, corresponding to a particle diameter of 4.75 mm. Other parameter ranges were set in accordance with a previous study (Sato, Kuwano and Otsubo, 2025).

From the right-hand perspective in Figure 6, all data points corresponding to the critical state fall near the fitted surface, indicating that the proposed critical state surface appropriately captures the relationship among particle crushing, stress, and void ratio in porous granular materials using reasonable parameter values. Sato et al. (2025) also suggested that isotropic consolidation states may lie on the same critical state surface for crushable porous soils. However, in the present study, the isotropic consolidation results deviated from the fitted critical state surface, implying that the isotropic consolidation surface may exist as a separate surface from the critical state surface in such materials.

Figure 7 shows the relationship between particle crushing and pressure. In both CD and CU tests, particle crushing increased with the progression of axial strain and approached the critical state line. In the cases where the critical state had been reached and a distinct shear band was observed (i.e. CU tests with axial strains above 15% and the CD test at 70%), the data were plotted above the red critical state line, indicating a higher degree of crushing. This is interpreted as a consequence of continued axial deformation after reaching the critical state, where localised crushing progressed within the dominant shear band. Figure 8 displays a three-dimensional plot of the data in Figure 7 together with the critical state surface. In CU tests, the stress paths moved almost linearly towards the critical state surface while accompanied by particle crushing, reaching the surface at around 5% axial strain. In contrast, CD tests exhibited increasing pressure and crushing with ongoing volumetric contraction, gradually approaching the critical state surface and reaching it at around 50% axial strain. Considering the cases where distinct shear bands were observed after reaching the critical state, the corresponding data also remained close to the surface, indicating that the data continued to move along the critical state surface even after localisation occurred.

These results confirm that, in both CD and CU tests, the stress paths initiated from the isotropic consolidation line and approached the critical state surface with the progress of axial strain in the three-dimensional space defined by crushing, stress, and void ratio. The relative position between the current state and the critical state surface thus serves as an important indicator for understanding the mechanical behaviour of crushable porous soils.

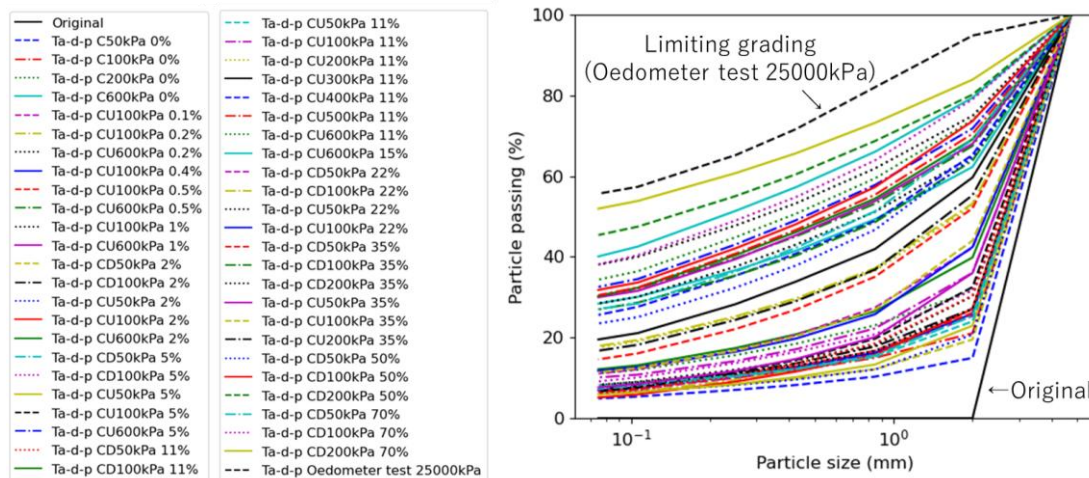


Figure 3. Particle distributions before and after testing.

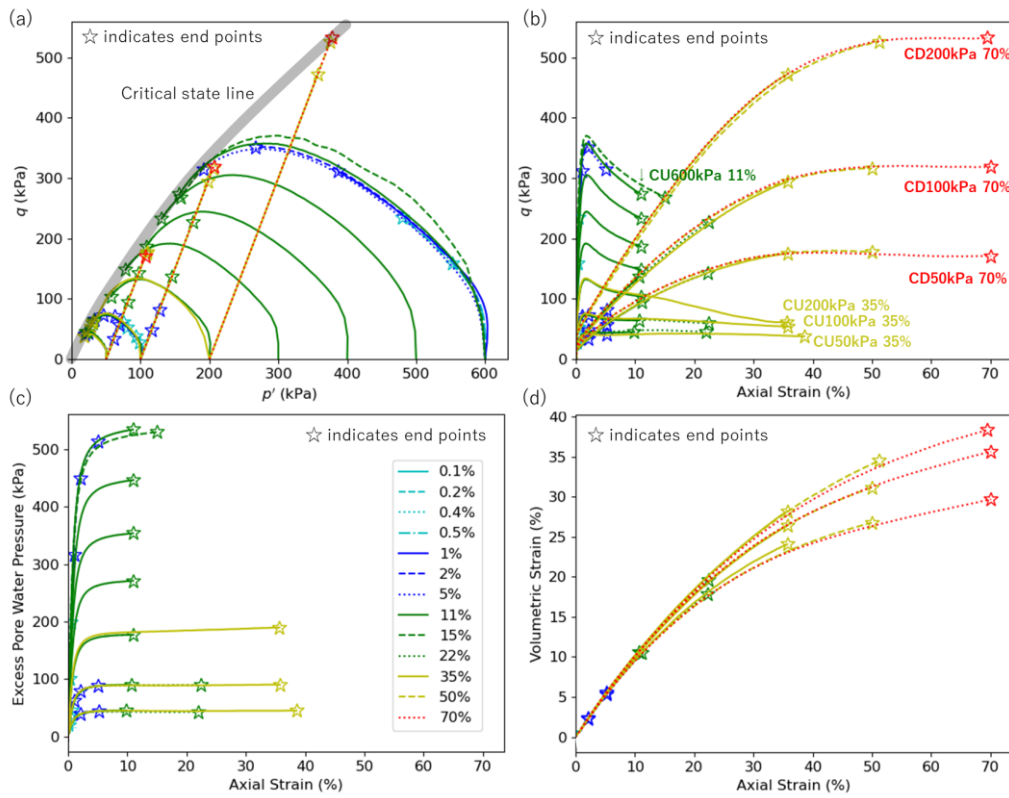


Figure 4. Results of triaxial compression tests: (a) Stress path, (b) Deviator stress vs. axial strain, (c) Excess pore water pressure vs. axial strain, (d) Volumetric strain vs. axial strain.

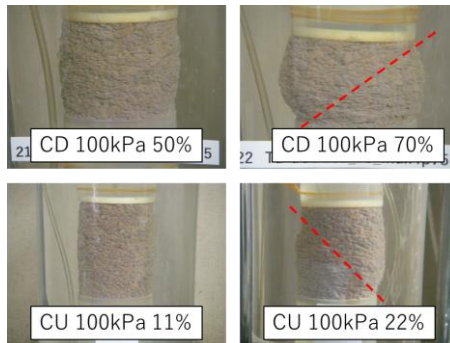


Figure 5. Examples of specimens at the critical state.

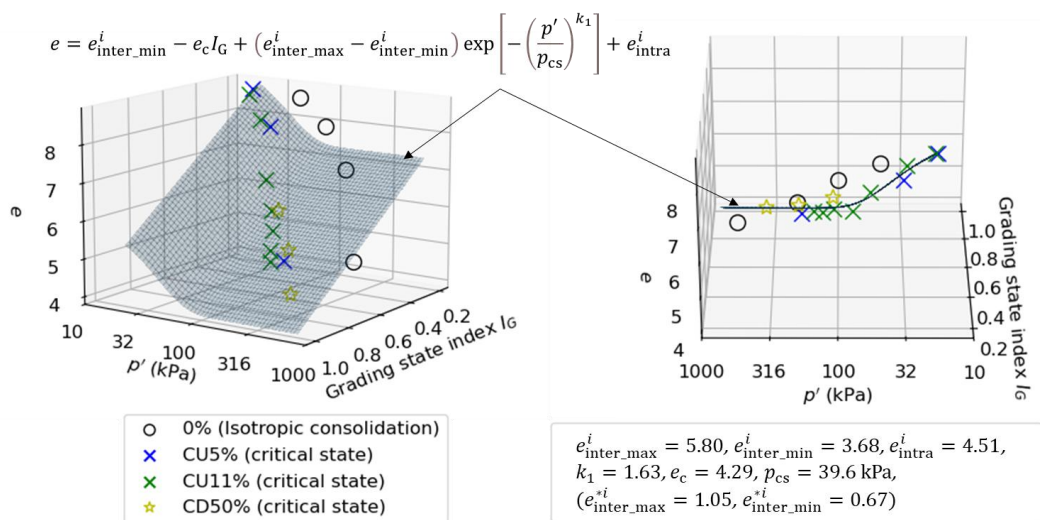


Figure 6. Critical state surface of Ta-d-p pumice.

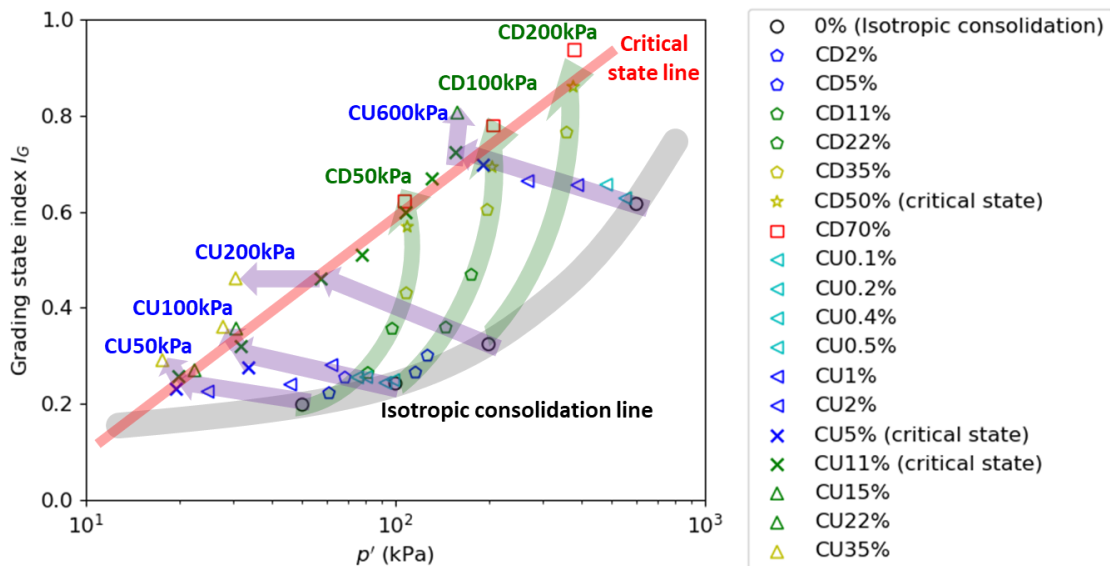


Figure 7. Relationship between particle crushing and mean effective stress.

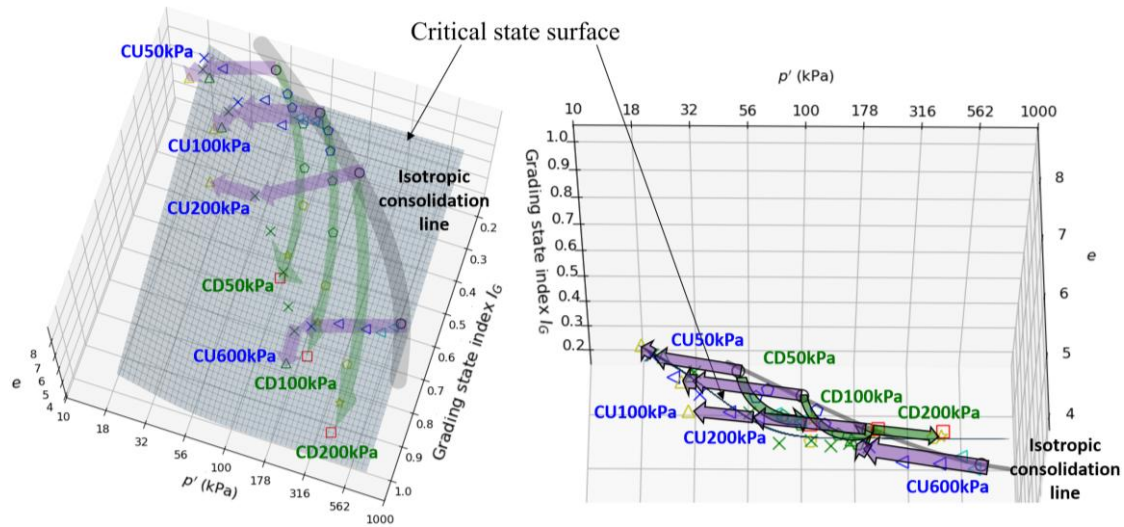


Figure 8. Critical state surface and progression of particle crushing.

5 CONCLUSIONS

This study aimed to investigate the progression of particle crushing in crushable porous soils up to the critical state. Triaxial compression tests, including isotropic consolidation and CD/CU tests, were conducted on Ta-d-p pumice while varying the axial strain at the end of each test. The extent of crushing was evaluated by comparing the particle size distribution before and after each test. The main findings are as follows:

- The critical state surface obtained from the tests that reached the critical state was in good agreement with the experimental data, demonstrating that it can appropriately represent the mechanical behaviour of crushable porous soils.
- Isotropic consolidation states did not coincide with the critical state surface, suggesting the presence of a separate surface.
- In compression tests, the material exhibited a gradual approach to the critical state surface with increasing axial strain, accompanied by ongoing particle crushing.

- In CU tests, the critical state was reached at approximately 5% axial strain, following a linear path towards the critical state surface while undergoing crushing. In contrast, CD tests showed progressive particle crushing along with volumetric contraction, reaching the critical state only at around 50% axial strain.
- In both CD and CU tests, it was observed that even after the critical state was reached, the development of localised shear bands accompanied further increases in particle crushing. However, these cases also plotted close to the critical state surface.

These findings clarify the behaviour of crushable porous soils in the three-dimensional space defined by crushing, stress, and void ratio, demonstrating that the material progressively approaches the critical state surface with increasing axial strain. The outcomes of this study contribute to a better understanding of the mechanical behaviour of crushable porous soils and provide fundamental insights for the development of constitutive models for porous granular materials.

6 ACKNOWLEDGEMENTS

This work was supported by the JDC Foundation Inc. Research Grant.

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