

# On the influence of the flow rule on earth pressures in confined excavations

Fengwen Lai, Feng Chen

College of Civil Engineering, Fuzhou University, China, laifengwen@fzu.edu.cn

Dechun Lu

Institute of Geotechnical and Underground Engineering, Beijing University of Technology, China

Franz Tschuchnigg, Helmut F. Schweiger

Institute of Soil Mechanics, Foundation Engineering and Computational Geotechnics, Graz University of Technology, Austria

**ABSTRACT:** Confined excavations are often encountered in dense urban areas. Estimation of earth pressures on retaining structures is one of the design concerns in such cases. This contribution presents numerical investigations on the influence of the flow rule on active/passive earth pressures exerted onto structures supporting confined excavations. Comparisons, in terms of failure mechanisms and earth pressures, between the Mohr-Coulomb (MC) model and the Hardening Soil Small (HSS) model with associated and non-associated flow rules, are made under various drainage conditions (drained and undrained analyses in terms of effective stresses). The results from displacement based finite element analysis (FEA) and finite element limit analysis (FELA) are compared, which in turn gives confidence in FEA results. The results clearly indicate that for certain conditions the influence of the constitutive model, flow rule, and drainage conditions may become significant.

**KEYWORDS:** Earth pressure, Flow rule, Confined excavation, Constitutive model, Drainage condition.

## 1 INTRODUCTION

Confined excavations are increasingly performed as underground excavation activities are getting closer to existing buried structures, in particular in an urban environment (Lai et al. 2024, 2025). The determination of active and passive earth pressures is essential for the design of retaining structures supporting excavations. Finite element analysis (FEA) and finite element limit analysis (FELA) are common techniques to compute earth pressures (e.g. Schmüdderich et al. 2021; Schweiger & Tschuchnigg, 2021). In such analyses, the flow rule (associated/non-associated), drainage conditions (drained/undrained), constitutive model (e.g. traditional Mohr-Coulomb (MC) model vs advanced Hardening Soil Small model (HSS)) may have an impact on the computed results. In the first part of the paper, earth pressures and failure mechanisms in confined excavations, determined by FEA and FELA, are compared under drained conditions. The influence of the flow rule and backfill width on earth pressures are also discussed. In the second part of the paper it is investigated to what extent the constitutive model in undrained analysis (in terms of effective stresses) influence the undrained response (effective stress and pore water pressure) in confined excavations.

## 2 METHODS

### 2.1 Displacement finite element analysis (FEA)

Over the past few decades, displacement finite element analysis (FEA) has become a standard tool in geotechnical engineering because it allows for a reliable assessment of displacements and stresses for complex geotechnical structures. In addition possible failure mechanisms may be identified and ULS design (Daxer et al. 2023) has become feasible. From a theoretical point of view, equilibrium, compatibility, material behaviour and boundary conditions, for both load and displacements, are fulfilled albeit in an approximate manner. In addition to the benefits above, the implementation of interface elements to approximate non-linear soil-structure interaction as well as advances in computer hardware (fast computation) have resulted in a widespread application of FEA in the solutions of geotechnical boundary value problems (Schweiger et al. 2019).

In this study, a finite element programme, Plaxis 2D (Brinkgreve et al. 2019), is used to study earth pressure problems in confined excavations.

### 2.2 Finite element limit analysis

Finite element limit analysis (FELA) is based on limit theorems of plasticity, i.e., the lower-bound (LB) and upper-bound (UB) theorems, developed by Drucker et al. (1952). FELA postulates small deformations, perfectly plastic material and an associated flow rule. The LB theorem satisfies equilibrium, the stress boundary conditions, and the yield criterion. The UB theorem satisfies the velocity boundary conditions and the plastic flow rule. As such, the exact failure load can be bracketed between UB and LB solutions. More details on FELA formulation can be found in Lyamin & Sloan (2002a; 2002b) and Sloan (2013). In this study, all the FELA are performed with the mesh adaptivity option using the code Optum G2 (Krabbenhöft, 2019) and all results presented in the following represent the average of the upper and lower bound solution, the difference between the two being in general a few percent.

### 2.3 Davis approach

Since FELA is limited to associated plasticity, Davis (1968) suggested that introducing a reduction factor ( $\beta$ ) to lower effective strength parameters ( $c'$  and  $\phi'$ ) may be an alternative to consider the non-associativity of the materials with a dilatancy angle  $\psi'$  lower than the friction angle  $\phi'$  ( $\psi' < \phi'$ ). The above approach is known as the “Davis approach”, which can be written as

$$c^* = \beta c' \quad (1)$$

$$\phi^* = \beta \tan \phi' \quad (2)$$

$$\beta = \frac{\cos \psi' \cos \phi'}{1 - \sin \psi' \sin \phi'} \quad (3)$$

Previous studies (Tschuchnigg et al. 2015a; Oberhollenzer et al. 2018; Schmüdderich et al. 2021) have demonstrated that in some cases the computed earth pressures (or the factors of safety) adopting an associated flow rule may lead to unrealistic, non-conservative, results, but the use of the Davis approach may produce overly conservative results. The feasibility of the Davis approach to estimate the active/passive earth pressures in

confined excavations under drained conditions will be discussed in the following.

### 3 PROBLEM DEFINITION AND MODEL DETAILS

The active and passive responses of a normally consolidated soil are studied by the problem defined in Fig. 1(a). A rigid wall with a height of  $H$  is used to retain a narrow column of soil with a width of  $B$ . The aspect ratio  $B/H$  varies from 0.4 to 0.8 in both the active and passive problems. The groundwater table is assumed at the surface of the model.

Figures 1(b) and 1(c) present typical FEA and FELA models respectively. For FEA the finite element domain is discretised with  $\sim 10,000$  15-noded triangular elements. For FELA, a LB mesh after failure is shown. In combination with adaptive mesh refinement the number of elements is increasing from 5,000 to 10,000 to obtain the solutions. As shown in Fig. 1(a), the wall is actively pulled away or passively pushed into the soil by means of prescribed displacements. Other than the wall, vertical displacements are allowed for the lateral boundaries while full fixities are imposed at the bottom boundary.

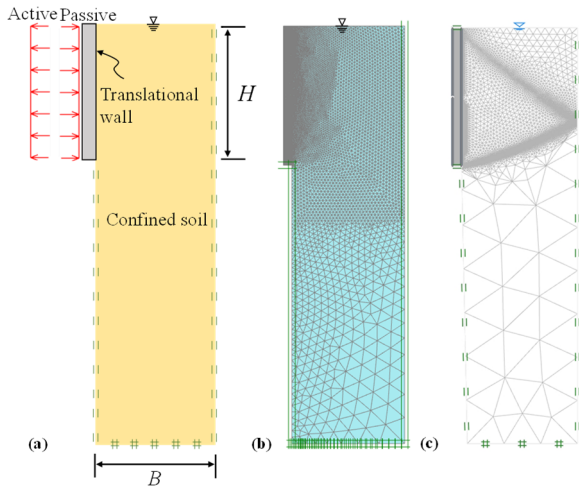


Figure 1. Problem definition and numerical models: (a) Geometry; (b) FEA mesh and (c) FELA mesh (LB mesh after failure).

Two different constitutive models, namely the elastic-perfectly plastic Mohr-Coulomb (MC) model and the advanced Hardening Soil Small (HSS) model (Benz, 2007), are used to model the soil behaviour. The parameters for two models are listed in Table 1. Note that, for the MC model, two different values of the dilatancy angle ( $\psi' = 0^\circ$  and  $\phi'$ ) are defined to represent non-associated and associated flow rules, respectively.

Table 1. Material parameters for the MC model.

MC-Drained/Undrained		HSS-Undrained	
$\gamma$	20 kN/m <sup>3</sup>	$\gamma$	20 kN/m <sup>3</sup>
$E$	12 MPa	$E_{50,ref}$	25 MPa
$\nu$	0.3	$E_{oed,ref}$	20 MPa
$\phi'$	35°	$E_{ur,ref}$	100 MPa
$\psi'$	0°/35° *	$p_{ref}$	100 kPa
$c'$	0 kPa	$\nu_{ur}$	0.2
$K_0$	0.426	$\phi'$	35°
$R_{inter}$	0.67	$\psi'$	0°
		$c$	0 kPa
		$K_0$	0.426
		$G_{0,ref}$	150 MPa
		$\gamma^{0.7}$	2.00E-04
		$m$	0.8
		$R_{inter}$	0.67

\*only used under drained conditions

## 4 RESULTS AND DISCUSSION

### 4.1 Drained earth pressure problem

This section discusses earth pressure problems under drained conditions. Two questions are answered by means of FEA, FELA, and Davis approach: (i) to what extent does the flow rule influence active/passive responses of confined excavations? (ii) how feasible is the Davis approach with reduced strength parameters to model the non-associativity for this type of problems? For this purpose, only the MC model is used.

#### 4.1.1 Drained active responses

Figure 2 shows the effective active pressure with depth for associated and non-associated plasticity, obtained by means of FEA and FELA. It follows that FEA solutions show almost perfect agreement with FELA solutions for a drained material with associated plasticity, giving confidence that FEA solutions are reliable. The active earth pressure increases with a decrease in dilatancy angle and/or with increasing in aspect ratio  $B/H$ . As previously reported by Lai et al. (2024), the FEA results with non-associativity exhibit numerical instability due to the non-uniqueness of the failure mechanisms, leading to the significant oscillations of the numerical solutions such as earth pressure distribution over wall depth, in particular for low  $B/H$  ratios, e.g. 0.4. In addition, the active pressure for non-associativity ( $\phi' = 35^\circ, \psi' = 0^\circ$ ) oscillates above and below that for associativity ( $\phi' = 35^\circ, \psi' = 35^\circ$ ) and the Davis solution ( $\phi' = 29.8^\circ, \psi' = 29.8^\circ$ ), which shows that the Davis approach may be an alternative to model the non-associativity. However, the results indicate that the effect of the dilatancy angle on the active response can be clearly identified but is not very pronounced.

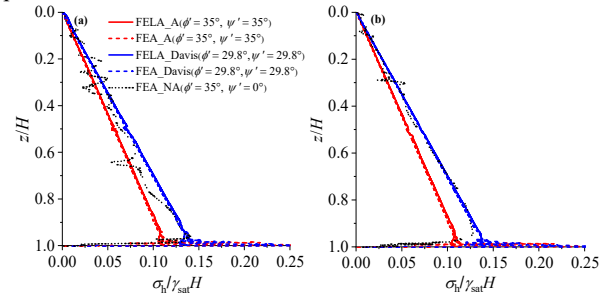


Figure 2. Influence of flow rule on active earth pressure under drained conditions: (a)  $B/H = 0.4$  and (b)  $B/H = 0.8$ .

The active failure mechanisms for  $B/H = 0.4$  in the cases mentioned above are compared in Fig. 3. The comparison between Figs. 3(a) and 3(c) or between Figs. 3(b) and 3(d) clearly show that almost the same failure mechanisms are obtained from FEA and FELA. Fig. 3(e) indicates numerical instability for a non-associated flow rule as a consequence of a non-unique failure mechanism as highlighted e.g. by Nordal (2008) and Tschuchnigg et al. (2015b, 2015c).

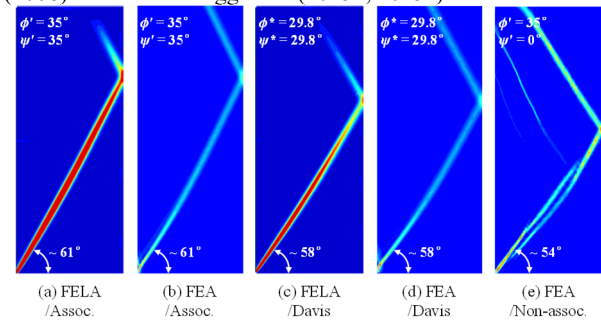


Figure 3. Active failure mechanisms under drained conditions ( $B/H = 0.4$ ).

#### 4.1.2 Drained passive responses

The influence of the flow rule on passive pressure for various aspect ratios  $B/H$  is depicted in Fig. 4. Again good agreement between FEA and FELA is observed. The lateral pressure increases significantly with increasing dilatancy angle and/or decreasing  $B/H$  ratios. These results strongly suggest that careful considerations have to be given to the selection of the dilatancy angle in this type of problems. The Davis approach will improve results but may still be not realistic, in particular for low  $B/H$  ratios. Oscillations for the non-associated FE analysis are again apparent. Fig. 5 confirms that basically identical failure mechanisms are obtained for FEA and FELA for associated cases (including the Davis approach) but a different pattern follows from the non-associated FEA. Moreover, a trapezoid wedge rupture body around wall top under passive thrust can be observed in Figs. 6(b) and 6(d).

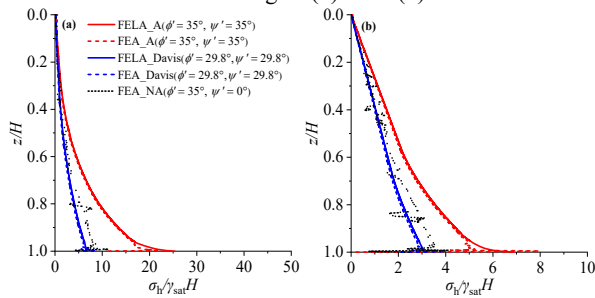


Figure 4. Influence of flow rule on passive earth pressure under drained conditions: (a)  $B/H = 0.4$  and (b)  $B/H = 0.8$ .

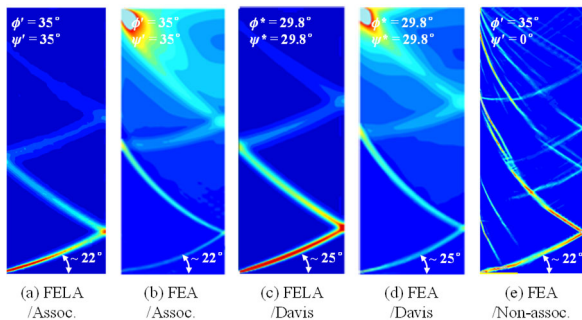


Figure 5. Passive failure mechanisms under drained conditions ( $B/H = 0.4$ ).

#### 4.2 Undrained earth pressure problem

This section studies the earth pressure problem under undrained conditions. A key focus is examining how constitutive models (i.e. MC and HSS) influence active/passive responses—including earth pressure and (excess) pore water pressure—in confined excavations, using FEA. Note that all the undrained analyses are performed in terms of effective stresses, and therefore the undrained shear strength is not an input parameter but rather a result of the constitutive model.

##### 4.2.1 Undrained active responses

Figure 6 presents the influence of constitutive model on active responses under various aspect ratios performing undrained analysis. It can be found that, compared to the HSS model, the MC model yields higher earth pressure and (negative) excess pore water pressure ( $u_e$ ), resulting in lower total pore water pressure ( $u_p$ ). This difference arises because the MC model follows a vertical effective stress path, whereas the HSS model exhibits a curved path, leading to greater undrained strength in the MC model. This can be confirmed by Fig. 7 where stress paths in  $p'$ - $q$  space for a stress point close to the wall in 2.5 m depth below surface are provided. Interestingly, the active failure points fall between compression and extension for  $K_0 = 1 - \sin \phi'$ , as evidenced by Schweiger et al. (2021). Fig. 8 further

reveals the difference in the sliding surface (inclination angle) between the MC and HSS models, particularly for wider aspect ratios.

##### 4.2.2 Undrained passive responses

Figure 9 demonstrates the effect of constitutive models on passive response under various aspect ratios in undrained analysis. Similar to the active response, the MC model yields higher effective passive pressures than the HSS model, attributable to its greater undrained strength (Fig. 10). Therefore applying the MC model to calculate the passive earth pressure at failure may be on the unsafe side. Notably, aspect ratio variations show negligible influence on these results. The evolution of excess pore water pressure ( $u_e$ ) closely follows the effective earth pressure trends, while the influence of constitutive models on the pore water pressure appears to be moderate in this case. Furthermore, Fig. 11 reveals distinct failure mechanisms between the two constitutive models, highlighting their fundamentally different undrained material behaviours under passive loading conditions.

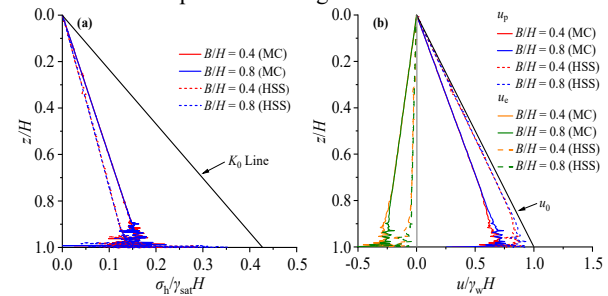


Figure 6. Influence of constitutive model on active response with undrained analysis in terms of effective stresses: (a) effective earth stress and (b) pore water pressure.

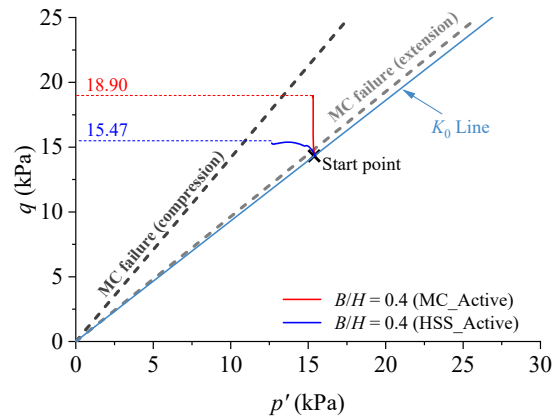


Figure 7. Stress paths in active earth pressure problem obtained by undrained analysis in terms of effective stresses (stress points in 2.5 m depth below surface).

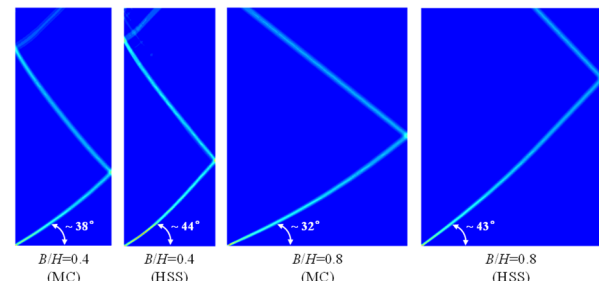


Figure 8. Active failure mechanisms obtained by undrained analysis in terms of effective stresses.

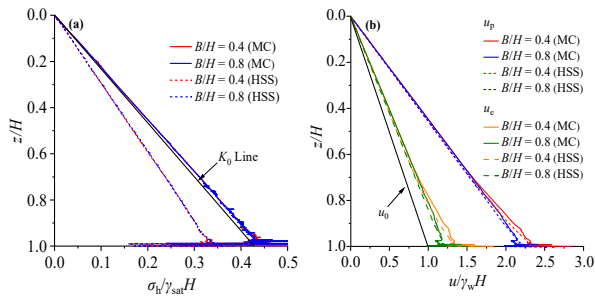


Figure 9. Influence of constitutive model on passive response with undrained analysis in terms of effective stresses: (a) effective earth pressure and (b) pore water pressure.

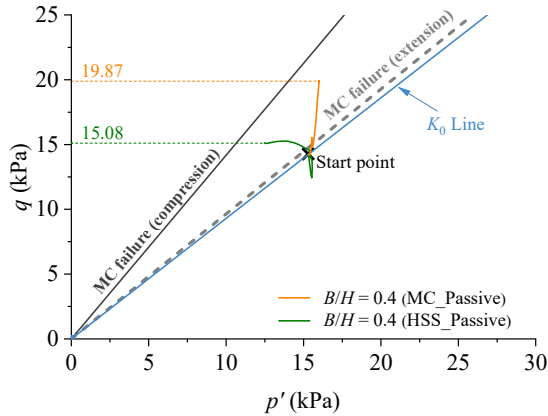


Figure 10. Stress paths in passive earth pressure problem obtained by undrained analysis in terms of effective stresses (stress points in 2.5 m depth below surface).

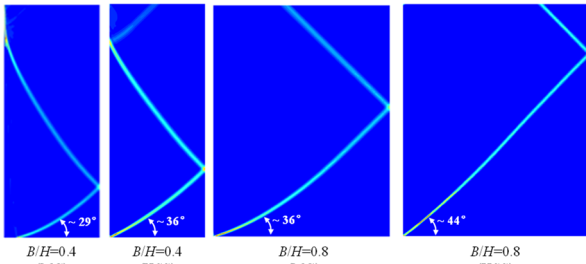


Figure 11. Passive failure mechanisms obtained from undrained analysis in terms of effective stresses.

## 5 CONCLUSIONS

The results presented in this paper have confirmed that FEA solutions and FELA solutions are in good agreement for a drained material with associated plasticity. It was shown by a drained active problem that for confined excavations, although the influence of the flow rule on the magnitude of earth pressure is moderate, numerical instability due to the non-associativity of materials is still significant. The use of the Davis approach to address this drawback is confirmed as an alternative for active responses of confined excavations. The effect of the flow rule on drained passive problems is significant, particularly for low aspect ratios. For the undrained earth pressure problems, the analyses performed in terms of effective stress indicated that, the MC model may obtain higher effective pressure and (excess) pore water pressure than the HSS model, along with the distinct failure mechanisms.

## 6 ACKNOWLEDGEMENTS

This work is supported from the National Natural Science Foundation of China (Grant Nos. 52025084 and 52408356),

and the China Postdoctoral Science Foundation (Grant Nos. 2025T180886 and 2024M760176).

## 7 REFERENCES

- Benz, T. 2007. *Small-strain stiffness of soils and its numerical consequences*. Ph. D Thesis, University of Stuttgart, Stuttgart, Germany.
- Brinkgreve, R.B.J., Kumaraswamy, S., Swolfs, W.M., Zampich, L. & Manoj, N.R. 2019. *Plaxis 2D Material Models Manual*. Delft, The Netherlands, Plaxis bv.
- Davis, E. 1968. *Theories of plasticity and failures of soil masses*. In: Lee IK (ed) *Soil mechanics: selected topics*. Elsevier, New York, USA, 341–354.
- Daxer, H-P., Schweiger, H. & Tschuchnigg, F. 2023. Ultimate limit state design of deep excavation problems according to EC7 using numerical methods. *Numerical Methods in Geotechnical Engineering 2023 (NUMGE 2023)*, London, UK, 1-6.
- Drucker, D.C., Prager, W. & Greenberg, H.J., 1952. Extend limit design theorems for continuous media. *Quarterly of Applied Mathematics* 9, 381–389.
- Krabbenhoft, K. 2019. *OptumG2: Theory. Optum Computational Engineering*.
- Lai, F., Zhang, N., Liu, S. & Yang, D. 2024. A generalized analytical framework for active earth pressure on retaining walls with narrow soil. *Geotechnique* 74(11), 1127-1142.
- Lai, F., Tschuchnigg, F., Schweiger, F. H., Liu S., Shiao J. & Cai G. 2025. A numerical study of deep excavations adjacent to existing tunnels: integrating CPTU and SDMT to calibrate soil constitutive model. *Canadian Geotechnical Journal* 62, 1-23.
- Lyamin A.V. & Sloan SW (2002a). Lower bound limit analysis using nonlinear programming. *International Journal for Numerical Methods in Engineering* 55(5), 573–611.
- Lyamin A.V. & Sloan SW (2002b). Upper bound limit analysis using linear finite elements and nonlinear programming. *International Journal for Numerical and Analytical Methods in Geomechanics* 26(2), 181–216.
- Oberhollenzer, S., Tschuchnigg, F. & Schweiger, H., 2018. Finite element analyses of slope stability problems using non-associated plasticity. *Journal of Rock Mechanics and Geotechnical Engineering* 10, 1091-1101.
- Nordal S. Can we trust numerical collapse load simulations using nonassociated flow rules? In: Singh, editor, *Proceedings of 12th International Conference of the International Association of Computer Methods and Advances in Geomechanics (IACMAG)*, Goa, India, 755–762, 2008.
- Schmüdderich, C., Tschuchnigg, F. & Schweiger, H.F. 2022. Significance of flow rule for the passive earth pressure problem. *Acta Geotechnica* 17(1), 81-92.
- Schweiger, H.F., Fabris, C., Ausweger, G., & Hauser, L. 2019. Examples of successful numerical modelling of complex geotechnical problems. *Innovative Infrastructure Solutions*, 4(1), 1-10.
- Schweiger, H.F. & Tschuchnigg, F. 2021. A numerical study on undrained passive earth pressure. *Computers and Geotechnics* 140, 104441.
- Sloan, S.W. 2013. Geotechnical stability analysis. *Geotechnique* 63(7), 531-572.
- Tschuchnigg, F., Schweiger, H.F. & Sloan, S.W. 2015a. Slope stability analysis by means of finite element limit analysis and finite element strength reduction techniques. Part II: Back analyses of a case history. *Computers and Geotechnics* 70, 178-189.
- Tschuchnigg, F., Schweiger, H.F. & Sloan, S.W. 2015b. Slope stability analysis by means of finite element limit analysis and finite element strength reduction techniques. Part I: Numerical studies considering non-associated plasticity. *Computers and Geotechnics* 70, 169-177.
- Tschuchnigg, F., Schweiger, H., Sloan, S. W., Lyamin, A.V. & Raissakis, I. 2015c. Comparison of finite-element limit analysis and strength reduction techniques. *Geotechnique* 65(4), 249-257.