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3D Stochastic analysis of the vertical displacements of a tunnel (bubble effect) considering the spatial variability of the soil elasticity modulus

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ABSTRACT: This paper presents the application of the stochastic finite element method to the assessment of short-term vertical displacements of a tunnel built in the lacustrine subsoil of the Valley of Mexico. In a deterministic analysis, displacements are obtained considering the different layers of the subsoil as homogeneous strata. The variability of the undrained elasticity modulus *(E)* of the soil is then incorporated in the analysis. The values of modulus *E* in the first clay layer (FCL) of the site are represented by random fields and Monte Carlo simulations are performed. In a first stochastic analysis, the variability of modulus *E* with depth is represented by a one-dimensional random field. In a second scenario, the modulus variability in the three directions of the domain is considered. From these analyses, the short-term displacements of the tunnel and surrounding soil of the first clay formation are assessed and the uncertainty associated to the results is quantified. Results of the numerical deterministic and stochastic analyses are compared

KEYWORDS: Tunneling, soft soil, soil variability, stochastic finite element method, Monte Carlo simulation.

1 INTRODUCTION

Tunnel construction is a geotechnical engineering challenge, particularly when faced with soils with complex or unpredictable behavior (Liu *et al.,* 2025). Subsoil displacements in urban areas are among the main factors to consider during tunnel construction (Zhang *et al*., 2015). In soft soils such as those of the Valley of Mexico, surface displacements occur during tunnel construction. These displacements are generated during the excavation and construction stages. These displacements are the result of the following three effects: 1) the closure of the excavation, 2) the change in the shape of the tunnel, and 3) the emergence generated by the effect of soil unloading. This last effect occurs when the tunnel lining is placed immediately after the excavation is carried out ("bubble" effect) (Zaldívar *et al*., 2012).

In lacustrine soils such as those of the Valley of Mexico, the variability of geotechnical properties can be affected by several factors. One of these is the formation process; these types of soils generally present a stratified structure with large vertical variations and smooth horizontal variations. Another factor that can influence the variability of subsurface properties is the natural anomalies present in soft soil deposits. These factors are due to interference with other geological formations and anthropogenic factors, especially in urban areas (Auvinet, 2019).

The spatial variability is often modeled using random field theory (Auvinet, 2002; Huang *et al*., 2017). Studies about the influence of soil variability on geotechnical systems using random fields have been performed for many years since the works by Vanmarcke (1977) and Auvinet & Abaziou (1993). Currently, the random fields analyses are performed for different topics such as the bearing capacity of footing, slope stability and foundation settlement (Griffiths & Fenton, 2007; Liu *et al*., 2015). In the tunneling area, the effect of the soil spatial variability on structural behavior has received attention from Mollon *et al*., (2011), Huang *et al.,* (2017) and Yuan *et al*., (2017).

The stochastic finite element method (SFEM) is a generalization of the finite element method that incorporates random fluctuations of the model parameters or geometry. The main stages of the SFEM are the appropriate choice of probabilistic models for the simulation of uncertainties within the system, the discretization of the random field in the system, the representation of the problem by means of finite elements and, finally, the estimation of the probabilistic characteristics of the system response (Auvinet *et al.* 1996; Mousavi *et al*., 2011).

In this work, the stochastic finite element method with a Monte Carlo non-intrusive approach is used to analyze the uncertainty on vertical displacements during the excavation and placement stage of the primary lining of a tunnel in the lacustrine subsoil of the Valley of Mexico.

In the stochastic analyses, two scenarios are considered. First, a random field with isotropy in the horizontal direction is considered to represent the variability of elastic modulus *E* in the vertical direction. In the second scenario, an anisotropic 3D random field is used to represent the variability of this modulus in the vertical and horizontal directions. The results of both analyses, are used to assess the uncertainty associated to the results of the numerical analysis. Displacement values obtained by the deterministic and probabilistic approaches are compared.

2 RANDOM FIELDS

Random fields have become indispensable tools for the modeling and analysis of various natural and engineered spatially extended material properties fields that are characterized by complex variability and uncertainty. A random field can be viewed as a collection of random variables that are distributed in space (Hristopulos, 2020; Vanmarke, 2010; Auvinet, 2025).

In geotechnical engineering, if $V(X)$ is a property or parameter value of either physical (e.g. water content), mechanical (e.g. undrained shear strength), or geometric type (e.g. thickness of a certain stratum), defined at points X of a certain domain Ω^P (P = 1, 2, or 3) and if at each point of the domain this property or parameter is regarded as random, the set of these random variables constitutes a random field (Auvinet, 2002).

Geotechnical properties of the subsoil may vary from one point to another and in the vertical and horizontal directions within the soil mass. Mathematically, the spatial variability of a soil properties can generally be modelled as the sum of the two components $T(X)$ and $R(X)$ (Vanmarcke, 1977; Phoon & Kulhawy, 1999) by means of the following expression:

$$V(X) = T(X) + R(X) \qquad (1)$$

where $V(X)$ is the value of geotechnical property at a given location, $T(X)$ is a deterministic trend and $R(X)$ is the residual field.

The characteristics of the random field (expected value, variance and autocorrelation functions) are obtained by statistical methods from the available data (boreholes results, correlations with index properties or expert assessments).

2.1 *Simulation of random field*

To evaluate the effect of the spatial variability of the undrained elastic modulus (*E*) in the First Clay Layer of the subsoil, the *LU* decomposition matrix technique (Alabert, 1987; Davies, 1987) is used. With this simulation technique, possible configurations of the *E* modulus distribution are generated.

The autocorrelation matrix (ρ_E) associated to the coordinates $X_1, X_2, \ldots X_n$ of the nodes of a given mesh (centers of finite elements for example) presents the following form:

$$\rho_E = \begin{bmatrix} 1 & \rho(X_1 - X_2) & \cdots & \rho(X_1 - X_n) \\ \rho(X_2 - X_1) & 1 & \cdots & \rho(X_2 - X_n) \\ \vdots & \vdots & \ddots & \rho(X_3 - X_n) \\ \rho(X_n - X_1) & \rho(X_n - X_2) & \cdots & 1 \end{bmatrix} \quad (2)$$

For a 3D random field (Auvinet, 2002), the autocorrelation function can generally be defined by the following equation:

$$\rho_E = exp\left(-\sqrt{\left(\frac{\Delta_x}{\delta_x}\right)^2 + \left(\frac{\Delta_y}{\delta_y}\right)^2 + \left(\frac{\Delta_z}{\delta z_z}\right)^2}\right) \quad (3)$$

where δ_x, δ_y, δ_z are the correlation distances in the horizontal (*x*, *y*) and vertical (*z*) directions, Δ_x, Δ_y, Δ_z are the distances between the coordinates of the pairs of points. The autocorrelation matrix ρ_E is decomposed into the product of a lower triangular matrix *L* and the transpose matrix (L^T) by means of the Cholesky matrix decomposition as follows:

$$L \cdot L^T = \rho_E \quad (4)$$

Given the matrix *L*, it is possible to obtain the values of a normalized gaussian random field *G*, based on the following expression:

$$G = L \cdot Z \quad (5)$$

where *Z* is a sequence of independent normal random variables. The normalized random field *G* can be transformed into values of the residual field of parameter *E* (Zhu *et al*., 2017).

2.2 *Monte Carlo Simulation*

There are two main variants of SFEM: i) the perturbation approach, which is based on a Taylor series expansion of the response vector and, ii) the spectral stochastic finite element method where each response quantity is represented using a series of random Hermite polynomials. Monte Carlo simulation can also be added to these two variants (Stefanou, 2009).

Monte Carlo Simulation (MCS) is the most general and direct approach for the Stochastic Finite Element Method (Stefanou, 2009). In this method, *NSIM* samples of the stochastic system matrix are generated using a random number generator and the final equilibrium is solved *NSIM* times, leading to a population (sample) of the response vector. Based on this population, the response variability of the system is calculated using simple relationships of statistics. For example, if u_i is the displacement, the unbiased estimates of the mean value and variance of the sample are:

$$E(u_i) = \frac{1}{NSIM}\sum_{j=1}^{NSIM} u_i(j) \quad (6)$$

$$\sigma^2 = \frac{1}{NSIM - 1}\left[\sum_{j=1}^{NSIM} u_i(j) - NSIM \cdot E^2(u_i)\right] \quad (7)$$

3 METHODOLOGY

The methodology for the stochastic analysis of the mechanical behavior of the tunnel considering the variability of the undrained elastic modulus of the upper clayey formation is divided into two parts, as shown in Figure 1.

In the first part, the possible configurations of the parameter considered as a random field are obtained. Initially, the geotechnical parameter to be considered as a random variable is defined. In the next stage, the exploratory analysis is performed and a statistical description is made to determine the variation range and detect the presence of outliers in the measurements that could affect the following stages. The parameter probability distribution is also determined. Subsequently, the trend analysis is performed, and the experimental field of the parameter is transformed into a residual field (without trend). Following this transformation, the residual field values are sought to adjust the distribution to a gaussian model and make it possible to apply the simulation technique. Considering the residual field data, the correlation distances are obtained in the preferential directions depending on the size of the study domain. To conclude the first part of the methodology, the matrix decomposition simulation technique is applied to obtain the different configurations of the variability of the Young´s modulus in the mesh or volume discretization considered in the numerical analysis.

The second part of the methodology incorporates the elastic modulus configurations into the numerical analysis using the stochastic finite element method. In the first stage, a deterministic model of the tunnel is defined, assigning a single value per stratum. In the next stage, the area of interest is discretized with volume elements (prisms) where the values of the random variable (modulus) will be simulated point by point; the boundaries of the deterministic model must be respected. Subsequently, the mesh of the numerical model is generated, which will be used in the deterministic and stochastic analyses. In the next stage, the numerical analysis of the tunnel is performed iteratively, considering the possible configurations with the variability of *E* obtained in the simulation stage. Each result of the numerical analysis is stored in a database for processing and interpretation. Finally, the uncertainty associated with the results of the numerical analysis is quantified and expressed as a standard deviation or coefficient of variation.

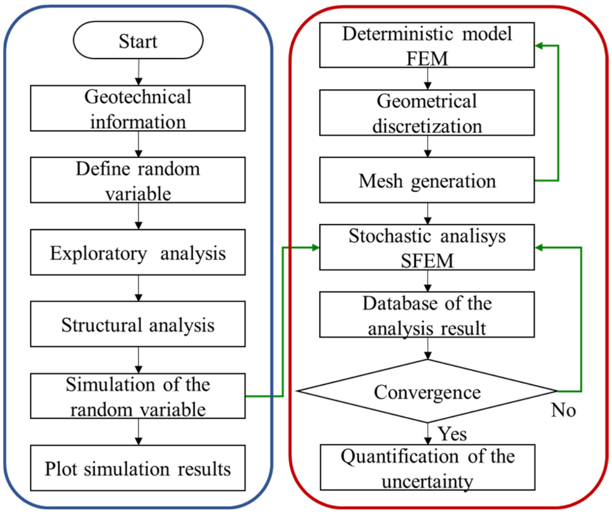


Figure 1. Workchart of the methodology.

4 STUDY AREA

The study area is located in the eastern part of the State of Mexico, in the municipality of Chimalhuacán. Based on the geotechnical zoning of Mexico City complementary technical standards for foundation design, the tunnel axis is located in Zone III (or lake zone), which is characterized by stratigraphy composed of thick, highly compressible clay deposits.

Figure 2 shows water content and stratigraphic profiles of the study area (lake zone), where five layers can be distinguished. The superficial Dry Crust (DC) is a thin layer with low water content values. Under the DC, a thicker stratum can be seen with water content values ranging from 200 to 400%, corresponding to the First Clay Layer (FCL), a stratum with an approximate thickness of 35 m. This is where the excavation and placement of the tunnel lining will be carried out. Below the FCL a hard layer is found, which is composed of rigid soil with water contents of the order of 50%. Then, an increase in the magnitude of the water content can be observed again, which is associated with the Second Clay Formation (SCL). Finally, in the lowest part, the Deep Deposits (DD), which are the most rigid material in the stratigraphic profile, are found.

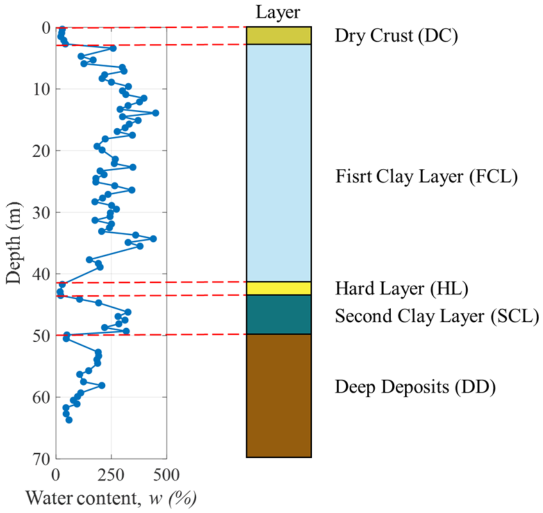


Figure 2. Typical profile of water content of the lake zone in Mexico valley.

Figure 3 shows the location of the eight mixed boreholes carried out in the study area along the tunnel axis. The maximum exploration depth is approximately 60 m, thereby covering the entire stratigraphic profile of the subsoil.

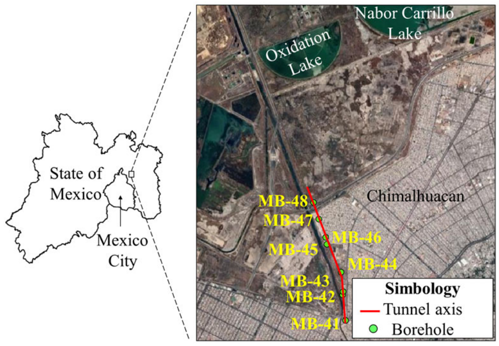


Figure 3. Location of the study area and boreholes distribution.

5 RESULTS

5.1 *Random field of modulus E.*

Undrained elasticity modulus (*E*) is considered to be a random field *E(X)* distributed within R^P, with P=3 (3D case). The *E* dataset measured within domain R^3 (Figure 4), from the unconsolidated undrained (*UU*) triaxial test constitutes a sample from that random field. The variability of the modulus in the stochastic analysis is considered only within the First Clay Layer, therefore, only measured values within the depth range of 2 to 35 m are used.

5.1.1 *Exploratory and trend analysis*

In this stage, an exploratory analysis of the experimental data of the *E* modulus is carried out. A summary of the statistical parameters of this modulus is shown in Table 1. The statistical parameters indicate that the modulus has a variation range from 1600 to 5200 kPa and a mean value of 3187 kPa, these values are typical of clay layers of the Valley of Mexico according to what was indicated in the works of Auvinet, (2002), Auvinet *et al*., (2017) and Auvinet, (2019). During the statistical analysis, no atypical values were detected that could affect the following stages.

Table 1. Statistical parameters of the undrained modulus

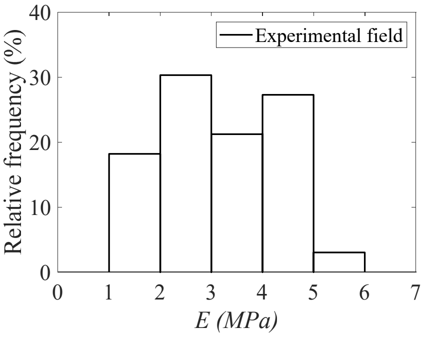
Parameter	Value	Unit
Mean	3186.9	kPa
Median	3030.4	kPa
Mode	2500.0	kPa
Standard deviation	1091.7	kPa
Kurtosis	-1.28	-
Maximum value	1667.9	kPa
Minimum value	5170.48	kPa

The trend of the random fields within the study domain Ω^3, as is the case, was represented by a hyperplane with the linear regression equation $T(X)=ax+by+cz+d$, where a, b, c and d are regression coefficients (Phoon & Kulhawy, 1999). The values of the coefficients of the linear regression of the elastic modulus are shown in Table 2

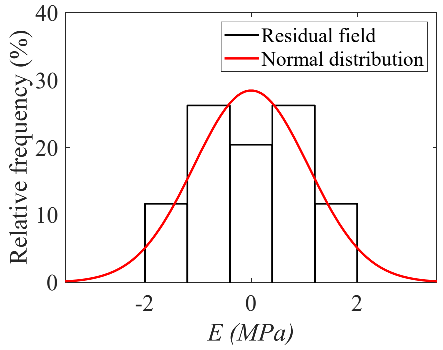
Table 2. Elastic modulus linear regression coefficients

Coefficient	Value	Unit
a	0.08	m^{-1}
b	-0.12	m^{-1}
c	15.48	m^{-1}
d	213015.87	kPa

Removing the trend from the experimental data also modifies the distribution of values. In this case, the residuals of *E* fit adequately to a normal distribution (Figure 4b). The normal distribution in the residuals allows the application of the simulation technique for the analysis of the spatial variability of the *E* modulus.



4a. Distribution of the experimental field



4b. Distribution of the residual field

Figure 4. Distribution of the experimental and residual profiles of undrained modulus.

5.1.2 *Correlation analysis*

Considering the values of the residual field of the undrained elastic modulus and using the simple exponential correlation model as indicated, the correlograms were defined in the horizontal and vertical directions, and the correlation distances indicated in Table 3 were obtained, which will be used in the prediction stage.

Table 3. Correlation distances of the elastic modulus

Directional correlogram	Value	Unit
Vertical	5.0	m
Horizontal	700.0	m

The values in the table above indicate that the undrained modulus of the First Clay Layer shows greater variability in the vertical direction (depth) than in the horizontal directions, which is associated with the processes of formation of the lake soils, which were deposited in horizontal layers. Figure 5. shows the directional correlograms of the undrained modulus.

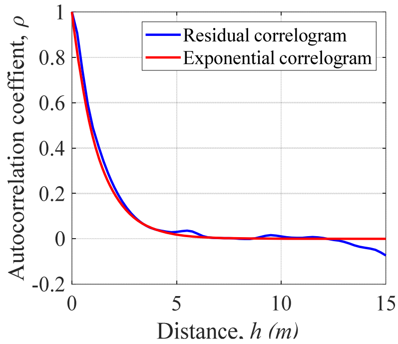


Figure 5a. Vertical residual correlogram.

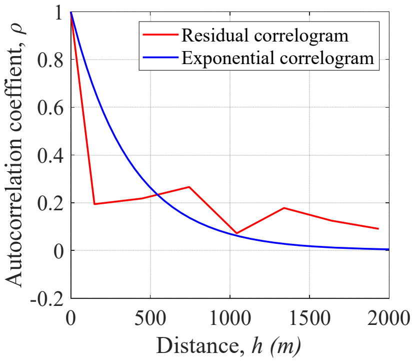


Figure 5b. Horizontal residual correlogram.

Figure 5. Directional residuals correlograms of undrained modulus.

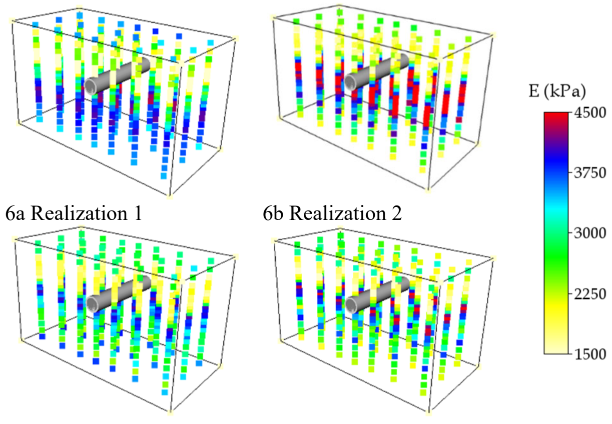
Based on these distances, two models of variability of *E* are proposed for the numerical analyses of the tunnel, one with variability only with depth and another with 3D variability.

5.1.3 *Conditional simulation*

In the prediction stage, the *LU* matrix decomposition technique is applied to obtain the possible values of *E* at points where there is no measurement in the FCL. After performing the simulation with the residual data, the coefficients of Table 2 are used to return the trend to the actual *E* field. The simulation domain or mesh is defined based on the discretization in the numerical model and the soil properties vary from unit to unit to reflect the spatial variability of soil.

In the first scenario, where only the variability of *E* with depth is considered, the domain of the FCL is discretized in the numerical model considering 36 layers with a thickness of 1 m. In this scenario the realizations follow the same trend as the measured E values and are in the range of 1600-5200 kPa.

In the second scenario, the variability of the *E* modulus in vertical and horizontal directions is considered (Figure 6), the FCL domain was discretized into 483 elements with thicknesses in the vertical direction of 0.5, 1 and 1.5 m.



6a Realization 1

6b Realization 2

6c Realization 3

6d Realization 4

Figure 6. Results of 3D conditional simulations of *E* in the FCL.

6 NUMERICAL ANALYSIS OF THE TUNNEL BEHAVIOR

6.1 *Stratigraphic conditions in the tunnel construction*

Figure 7a illustrates the stratigraphic conditions used in this study. The water table was assumed to be 2 m deep, at the lower border of Dry Crust. The tunnel crown is situated at a depth three times the internal diameter (5.6 m).

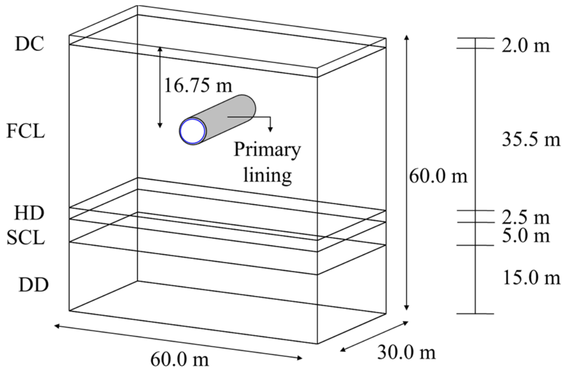


Figure 7a. Stratigraphic conditions in tunnel construction area.

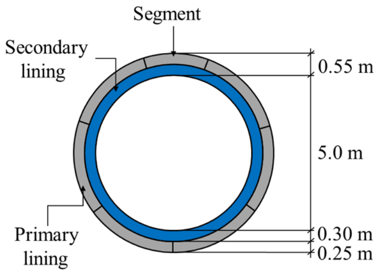


Figure 7b. Geometry of primary lining

Figure 7. Stratigraphic conditions and geometry of the lining.

The constitutive models employed in the numerical simulation were as follows: i) Linear Elastic (LE) model to characterize the mechanical response of the primary lining, and ii) Mohr-Coulomb (MC) to replicate the behavior in shorth term of the subsoil layers.

6.2 *Stages of the analysis*

The analysis stages were as follows:

Initial Stage: Initial stresses are generated in the medium using the K_0 procedure.

Stage 1: The excavation is carried out in the subsoil and the primary lining is placed.

The stage described above are used in both approaches to numerical analysis, deterministic and stochastic

6.3 *Deterministic analysis*

The short-term deterministic analysis of the tunnel, which aims to simulate the excavation and placement of the primary lining, is carried out using "*traditional*" discretization of subsoil stratigraphy (Figure 10a), where an average value is assigned to each layer. The geotechnical model consists of seven layers with the properties indicated in Figure 4

Table 4. Geotechnical model for short-term deterministic analysis.

Layer	γ (kN/m^3)	E (MPa)	e	ν	c (kPa)	φ (°)	K_0
DC	13	5	4	0.39	30	2	0.49
FCL1	11.6	2.2	7.7	0.49	22.5	1	0.69
FCL2	11.5	4.7	7.8	0.49	19.5	1	0.69
FCL3	11.4	2.3	5.9	0.49	26.5	1	0.69
HD	14.7	10	1.8	0.43	15	20	0.61
SCL	12.2	3.8	3.8	0.40	38.5	1	0.58
DD	15.2	10	5	0.35	20	17	0.24

γ = unit weight, E = undrained elastic modulus, c = cohesion; φ =friction angle, ν = Poisson´s ratio and K_o= coefficient of earth pressure.

The parameters of the constitutive model of the primary lining of the tunnel are indicated in Table 5. To account for the effect of the joints between the segments on the ring's bending stiffness. Rodríguez *et al*., (2024) propose the use of a stiffness reduction factor denoted as α, on the order of 0.2.

Table 5. Parameters of the tunnel lining.

Element	Thickness (m)	E (GPa)	γ (kN/m^3)	υ	α
Segment	0.25	5.2	24	0.16	0.2

γ= unit weight, E= Young´s modulus, υ= Poisson´s ratio, α=reduction factor for bending stiffness.

Two analyses of tunnel behavior were performed using the deterministic approach. One analysis was performed for each of the meshes generated in the scenarios considered using the stochastic approach.

The results of the deterministic finite element method using the stochastic model mesh, which considers the vertical variability of the E-module, and in a 3D domain, are the same. In both analyses, an upward vertical displacement of 0.0533 m was obtained (Figure 11and Figure 12). This is because in both cases, the mesh was refined in the lining zone.

6.4 *Stochastic analysis*

In the three-dimensional numerical analysis of the mechanical behavior of the tunnel using the stochastic finite element method, where the excavation and placement of the primary lining are represented and the variability in the First Clay Layer of the E modulus is incorporated, two scenarios are presented.

In the first, as mentioned above, only the variability of E with depth is considered, therefore, the FCL is discretized into 36 layers with a constant thickness of 1 m (Figure 10b). In the second, the variability of Elastic modulus in the vertical and horizontal directions is considered; in this scenario, the FCL was discretized into 483 elements with thicknesses of 0.5, 1, and 1.5 m (Figure 10c). The thinnest elements are concentrated in the primary lining area, where the displacement generated by the excavation is analyzed.

In both scenarios considered in the numerical analysis of the tunnel, a deterministic approach is performed for each stochastic analysis, as indicated in the methodology. Therefore, to compare the displacement results of both approaches, the mesh generated in the stochastic analysis is used, respecting the strata boundaries from the deterministic analysis.

In the stochastic finite element method using Monte Carlo simulation, results were obtained from 1000 simulations for each of the two scenarios of the variability of the E module.

The expected value of the maximum vertical displacement in both cases was 0.0526 m. However, as can be seen in the graph in Figure 8, there is a greater dispersion in the results of the stochastic analysis between simulations 100 and 600, where only the vertical variability of the E module is considered.

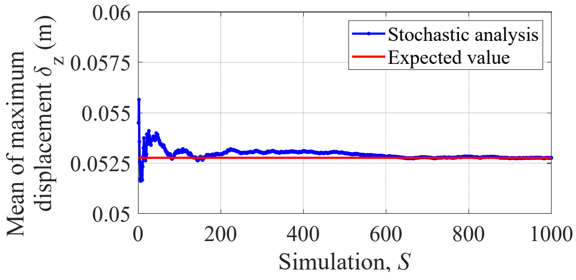


Figure 8. Mean of maximum vertical displacement (vertical variability of E).

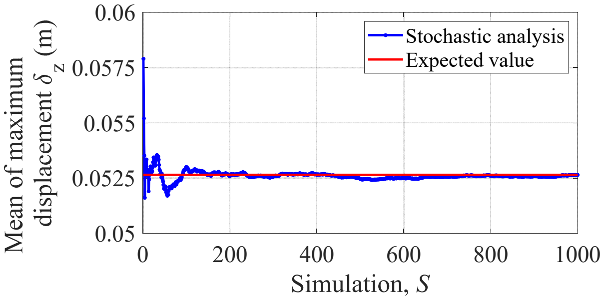
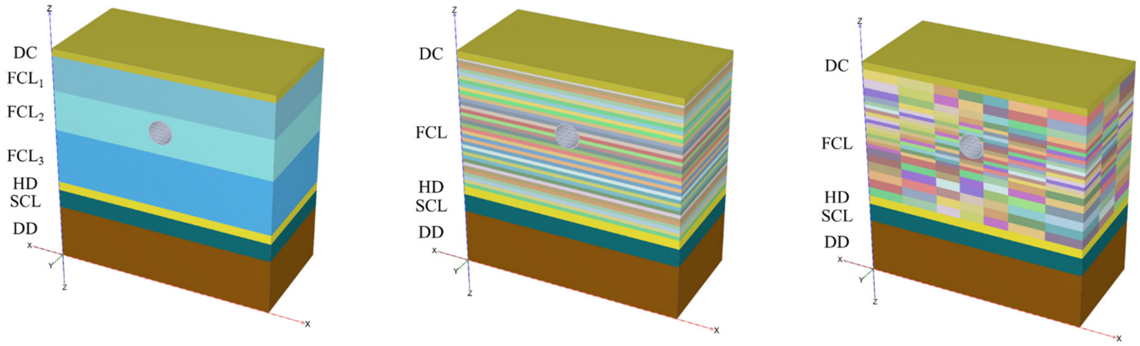


Figure 9. Mean of maximum vertical displacement (3D variability of E).



10a. Deterministic model 10b. Stochastic model (isotropy) 10c. Stochastic model (anisotropy)

Figure 10. Geometry of the numerical models of the tunnel.

For the 3D random field of *E*, displacements with a lower dispersion in the displacement results are obtained, as observed in the graph of Figure 11and in the distribution of PDF in the Figure 12. This dispersion is reflected in lower values of standard deviation for the 3D random field of *E* compared to the results of the first scenario.

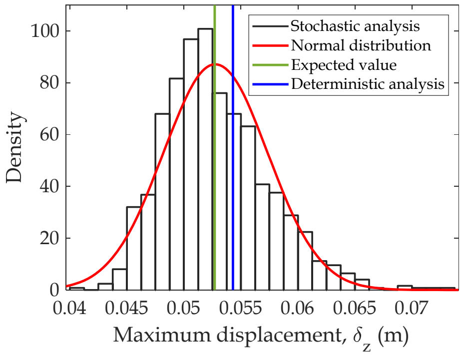


Figure 11. Probability density of the maximum vertical displacement (vertical variability of undrained modulus).

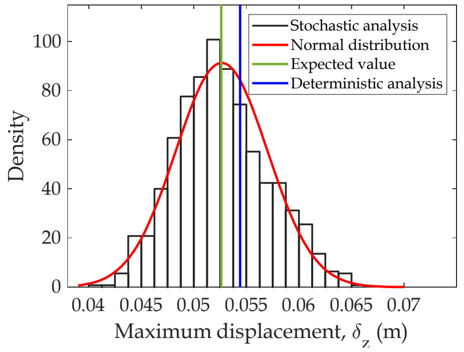


Figure 12. Probability density of the maximum vertical displacement (3D variability of undrained modulus).

7 CONCLUSIONS

The stochastic finite element method allows the variability of geotechnical properties to be considered in order to quantify the uncertainty on the results of numerical analyses.

The results of the stochastic analysis presented in this paper regarding vertical displacements of a tunnel in soft soil indicate that when considering the vertical variability of the undrained elastic modulus, the uncertainty on the displacements is slightly higher than in the case of the three-dimensional random field. Additionally, a greater dispersion in the displacement values can be observed in the simulation values, indicating that this type of analysis may require a greater number of simulations to reach convergence.

Finally, in the stochastic analysis where the three-dimensional random field of the *E* module is considered, it can be seen that the uncertainty values of the displacements are lower compared to the previous case. This can be attributed to a statistical compensation effect (Auvinet *et al*., 1996).

The methodology described in this work was applied to study the mechanical behavior of a tunnel; however, this methodology is "flexible" and can be generally applied to different geotechnical analyses.

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