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Advanced numerical multiphase simulations of triaxial tests

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ABSTRACT: The numerical simulation of soil behavior under static and cyclic conditions requires careful consideration of soil skeleton behavior, as the combination of non-linearity and pore water pressure present significant challenges. This study implements hypoplastic constitutive equations through ANSYS User Programmable Features to model soil skeleton non-linearity in fully saturated soil conditions. The flow conditions in the multiphase soil medium are analyzed and compared with experimental results from the Laboratory for soil dynamics at the Institute of Earthquake Engineering and Engineering Seismology in Skopje, N. Macedonia. The results demonstrate that appropriate non-linear material models accurately predict the observed soil behavior, while the minor differences can be adjusted with calibration coefficients during using the numerical modelling.

KEYWORDS: Numerical simulations, triaxial apparatus, drained tests, undrained cyclic tests, ANSYS

1 INTRODUCTION

Soil as a porous medium is a composition of the solid skeleton and pores which are filled with water and/or air. In describing porous medium it is important to consider the interaction between the solid, water and air phases. Especially in dynamic conditions, such as earthquake excitations, this interaction among the phases gains importance. Modeling based on the porous media theory in which an averaged macroscopic continuum is considered, has proved to be useful to better understand coupled phenomena such as flow and deformation processes. The modern formulations based on multiphase mixture theories developed in the last decade are in detail described in the monograph of de Boer[1]. These were based on the concept of volume fractions by Morland[2], Goodman and Cowin[3] andSampaio[4]. On the other hand, the averaging theories were developed by Whitaker [5, 6] and Hassanizadeh and Gary[7]. A simple extension on two phase formulation of porous media considering the air pressure to be constant and equal to the atmospheric pressure was proposed by Zienkiewicz[8]. Recent contributions to this topic include the one of Lewis Sukirman [9]where a two phase flow model is used leading a two pressure variables in addition to the displacement field to be approximated by suitable finite element spaces.Most recently, in the work of Oettl et al. [10] a coupled multiphase formulation is applied to geotechnical problems considering different water retention relations. The work of Edip [11] follows the Oettl's for multiphase modelling including hypoplastic model in for solid state simulations in the multiphase frame.

2 THE PHYSICAL MODEL – DEVELOPMENT OF MULTIPHASE MODEL

The physical model containing unsaturated or its simpler form as saturated deformable porous media behavior is based on the Theory of Porous Media. In porous media the mechanical behavior of multiphase deformable porous media and the interaction among phases are considered through mass conservation, equilibrium and from the solid skeleton's constitutive behavior. The constitutive law of solid phase is written in terms of the effective stress can be written as follows:

$$\sigma_{xy}{}' = \sigma_{xy} - p_w \qquad (1)$$

In defining the constitutive relations if only small deformations are considered, the stress-strain relationship for the porous medium can be written in incremental form as follows:

$$d\sigma_{xy}{}'' = D_{T,xykl}(d\varepsilon_{kl} - d\varepsilon_{kl}^{0}) \qquad (2)$$

where$D_{T,xykl}$is the fourth order tangential stiffness tensor of the material,$d\varepsilon_{kl}$is the second order tensor describing the total strain increment, while $d\varepsilon_{kl}^{0}$is the fraction of the strain increment from the previous step. The discretization follows from the weak form of the continuity equation and results in an equation of a matrix form as given below [12] :

$$\begin{pmatrix} M & 0 \\ M_w & 0 \end{pmatrix}\begin{pmatrix} \ddot{x} \\ 0 \end{pmatrix} + \begin{pmatrix} C & 0 \\ C_{sw} & P_w \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{p_w} \end{pmatrix} + \begin{pmatrix} 0 & C_{sw} \\ 0 & H_w \end{pmatrix}\begin{pmatrix} x \\ p_w \end{pmatrix} = \begin{pmatrix} f_x \\ f_w \end{pmatrix} \qquad (3)$$

The developed multiphase model is implemented in the finite element software ANSYS by using the programmable features of the software[12-14].

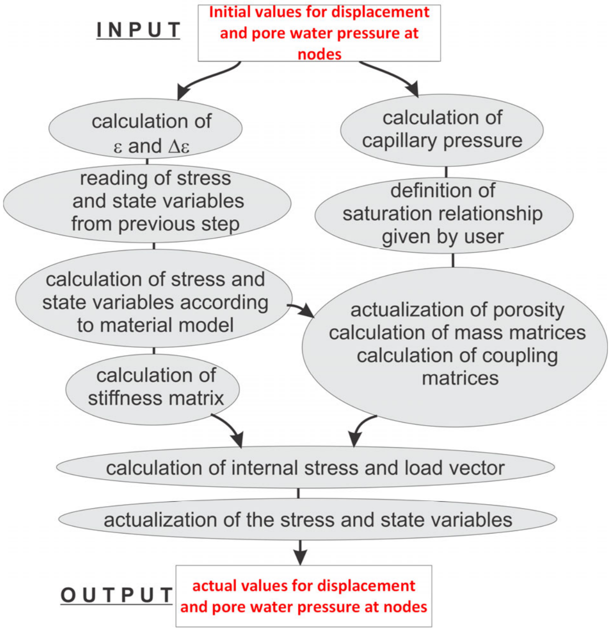


Figure 1. Calculation of actual values in ANSYS software [15]

The investigations have been performed in the Laboratory for soil dynamics at the Institute of earthquake engineering and engineering seismology – IZIIS where a cyclic triaxial apparatus has been used as shown in the Figure 2 below.

Figure 2. The cyclic triaxial apparatus in IZIIS

The investigations of triaxial testing in the laboratory has long history beginning from early '90s[16-21].The soil specimen is prepared by tamping material in layers at consistent density and has the dimensions of 70x140mm. Once prepared, the triaxial cell top is placed on the base with the ram clamped to prevent sample contact. After tightening the cell top with screws, the ram is lowered until it touches the rubber suction ring, with the actuator positioned at mid-travel. The phases of saturation and consolidation are done according to well established methodology before the tests of static and cyclic tests are done.

3 NUMERICAL SIMULATIONS

The numerical simulations contain various effective stress calculations to account for different parameter sets and their respective influences on the results. Although numerous investigations have been conducted on various sand types from Skopje [21], the selection of Toyoura sand for this research provides distinct advantages due to its well-documented characteristics and extensive use as a standard reference material in geotechnical testing. The numerical parameters for Toyoura sand were determined as follows:

Table 1. Parameters for Toyoura sand

Parameter	Symbol	Value	Unit
Angle of friction	φ	30	°
Granulate hardness	h_s	2600	MPa
Exponent	n	0.269	-
Void ratio-dense	e_{d0}	0.61	-
Void ratio-critical	e_{c0}	0.98	-
Void ratio-maximum	e_{i0}	1.09	-
Parameter	α	0.2	-
Parameter	β	1.00	-
Density of solid phase	ρ_s	2.7	ton/m^3
Permeability	k	$1.0*10^{(-7)}$	m/s
Compression modulus of water phase	K_w	$2.10^{(4)}$	kPa
Compression modulus of solid phase	K_s	$10^{(9)}$	kPa

The obtained parameters at the laboratory conditions are in good correlation with the parameters given in the work of other authors[22-24].

3.1 *Monotonic tests simulations*

Drained and undrained triaxial compression tests are carried out by using the triaxial machine at constant confining pressure of 100 kPa, 200 kPa and 250 kPa at constant strain rate of 0.3% per minute. The average stress strain curve comparison between experimental and numerical values for drained and undrained conditions is given in figures 3 and 4 below.

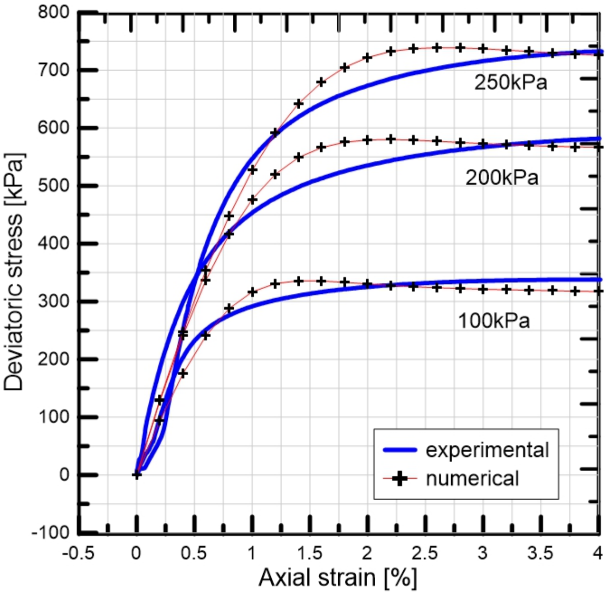


Figure 3. Comparison of stress results in drained conditions

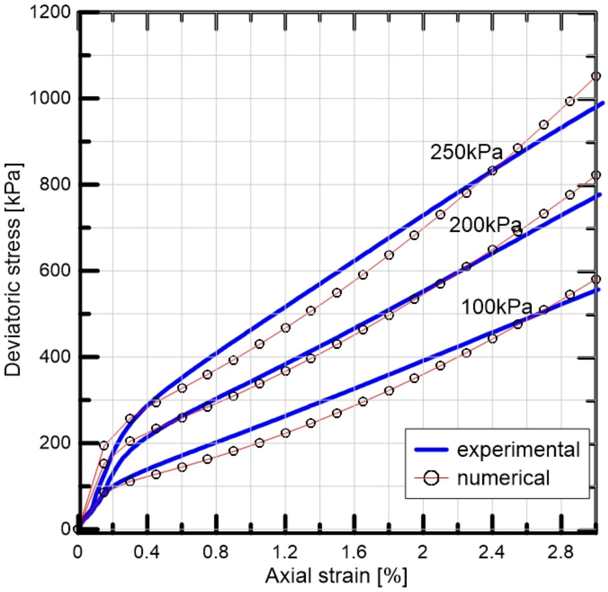


Figure 4. Comparison of stress results in undrained conditions

As can be seen in the results above, the figures clearly demonstrate the influence of confining pressure on the stress development in the samples for both drained and undrained tests. As expected, higher confining pressures result in increased peak deviatoric stress values which are well predicted by the numerical model.

Next, the volume change comparison for drained case and excess pore water pressure for undrained case are shown below.

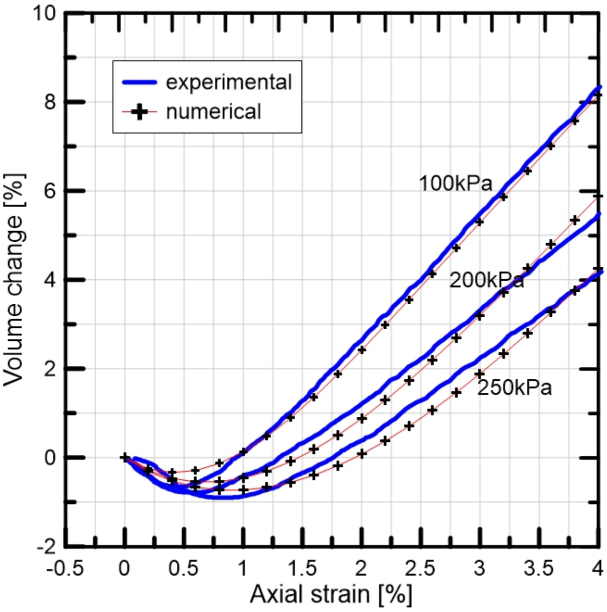


Figure 5. Comparison of volume change results in drained conditions

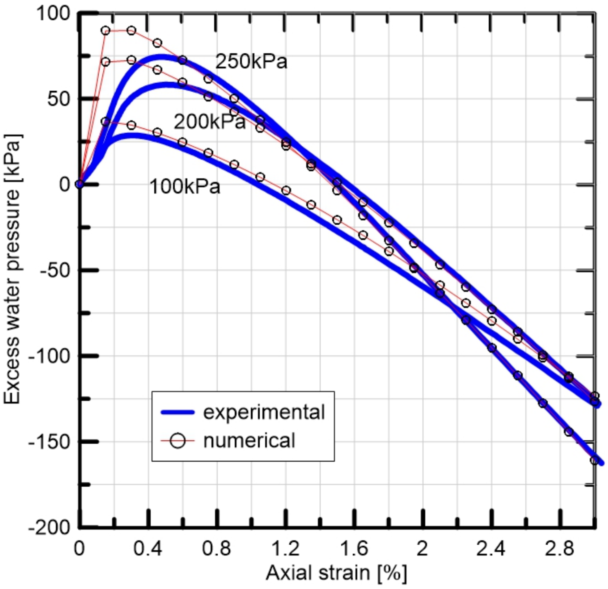


Figure 6. Comparison of excess pore pressure results in undrained conditions

As it can be seen from the figures 5 and 6, the results demonstrate solid agreement between the hypoplastic model simulations and laboratory tests for all three confining pressure levels. Figure 5 illustrates the volumetric strain evolution under drained conditions where the characteristic initial compression followed by dilatancy is accurately captured by the numerical model, with higher confining pressures producing reduced dilatant behavior as expected for dense sand. On the other hand, the Figure 6 shows the excess pore water pressure generation during undrained loading, where the model successfully predicts the progressive pressure buildup and the influence of confining pressure on pore pressure development patterns, insignificant deviations can be seenbut only in the very initial phases of the tests, until 0,5% axial strain.

3.2 *Cyclic test simulations*

The cyclic triaxial tests are carried out to investigate the stress- strain relations in different cycles with the effect of pore pressure development through cycles shown as pore pressure ratio, which approaching the value of one presents the liquefaction initiation. In order to compare the experimental results,the number of cycles of experiments is compared with the results given in the work of Tatsuoka et al. [25].

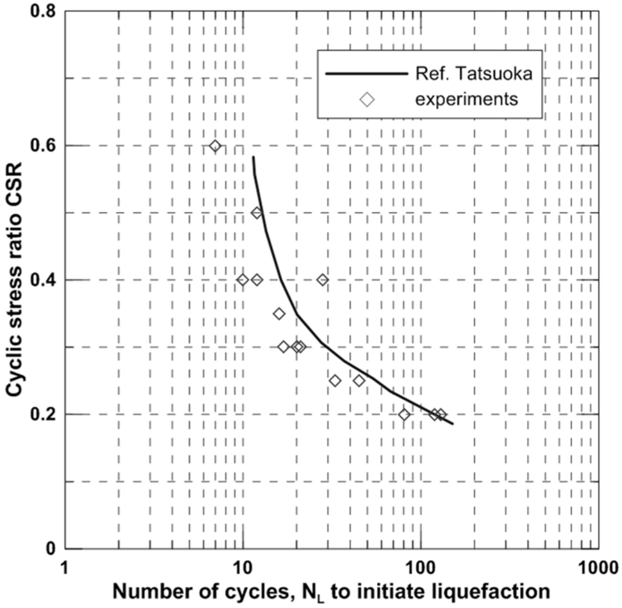


Figure 7. Comparison of experiments with the reference of Tatsuoka

As can be seen from Figure 7, the results from the performed tests show similar patterns which prove the correctness of the cyclic tests. After having completed the experimental comparison, in order to secure the correctness of the experiment performance, the results have been simulated numerically. The cyclic triaxial tests were performed on Toyoura sand samples that were saturated and consolidated as explained above. The loading frequency was 0.5 Hz, thus each cycle took two seconds. In this study the density of each layer was controlled by adjusting the number of tamping blows with a constant free fall of 2cm. This method was adopted as a simulation of field compaction of moist sand in layers by vertical tamping on the ground surface. The cyclic undrained triaxial strength was defined as 5% double amplitude axial strain values or the pore pressure ratio not greater than 0.9. The samples were reconstituted in dense state with an initial density of D_R=50%. The samples have an effective mean pressure of p'=100kPa. Since the effect of ratcheting (strain accumulation caused by ground vibrations) is to be considered, the numerical simulation ofhypoplasticity model has been improved with intergranular strain effects according to Niemunis and Herle [26]. The parameters for intergranular strain are obtained as follows: R=0.00001, m_r=5.0,m_t=2.0,β_r=0.5 and χ=6.0.The results obtained from the cycles simulations are shown below. For the sake of completeness, two levels of cyclic stress ratios(CSR) of 0.2 and CSR of 0.3 are compared. First, the CSR of 0.2 are shown below.

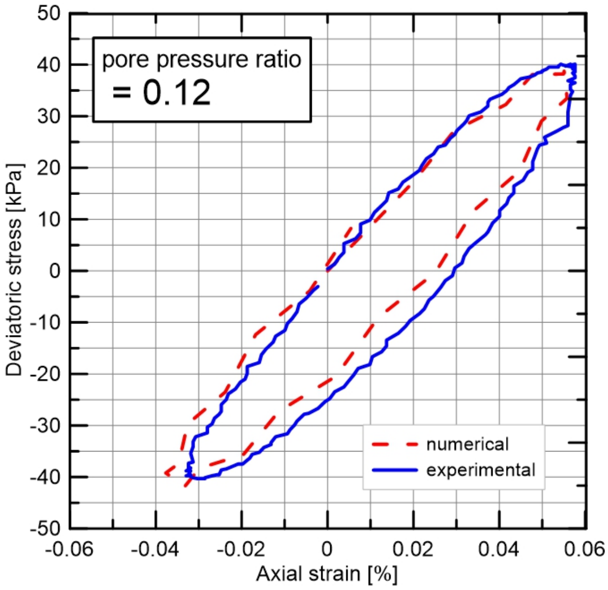


Figure 8. Comparison of results for CSR 0.2 – cycle 2

As can be seen from the Figure 8 the stress-strain hysteresis loops obtained from undrained cyclic triaxial testing under a pore pressure ratio of 0.12, demonstrate excellent agreement between experimental results (solid line) and numerical simulations (dashed line). The numerical model successfully captures the characteristic butterfly-shaped hysteresis pattern and accurately reproduces both the loading and unloading phases of the cyclic response and plastic strain development.

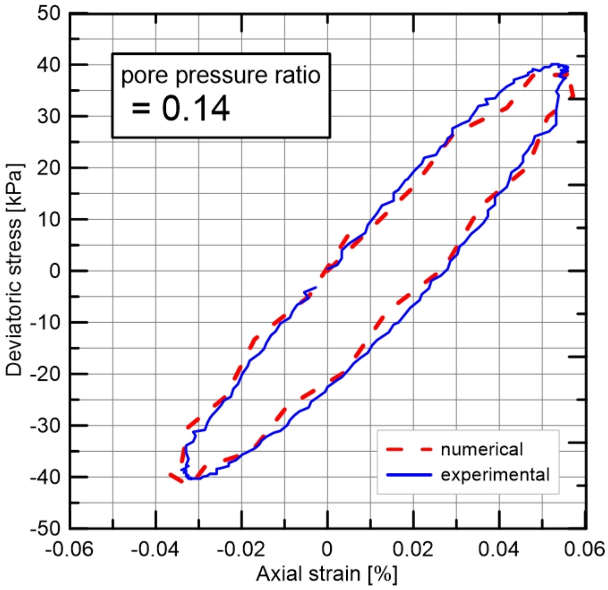


Figure 9. Comparison of results for CSR 0.2 – cycle 3

In Figure 9 the values of numerical and experimental results are in good correlation for the slightly higher pore pressure ratio of 0.14 which confirm the correctness of the used numerical model.

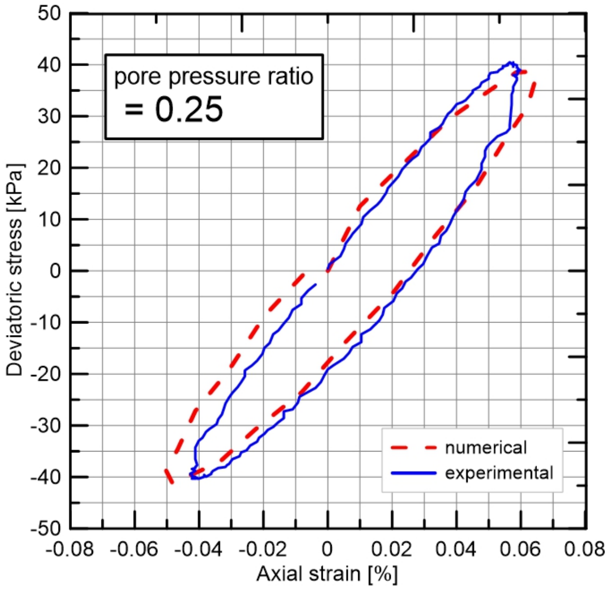


Figure 10. Comparison of results for CSR 0.2 – cycle 60

As number of cycles increase as shown in the Figure 10 the hysteresis loops become more elongated and exhibit increased axial strain amplitude under the higher pore pressure ratio of 0.25, while the numerical model maintains still good agreement.

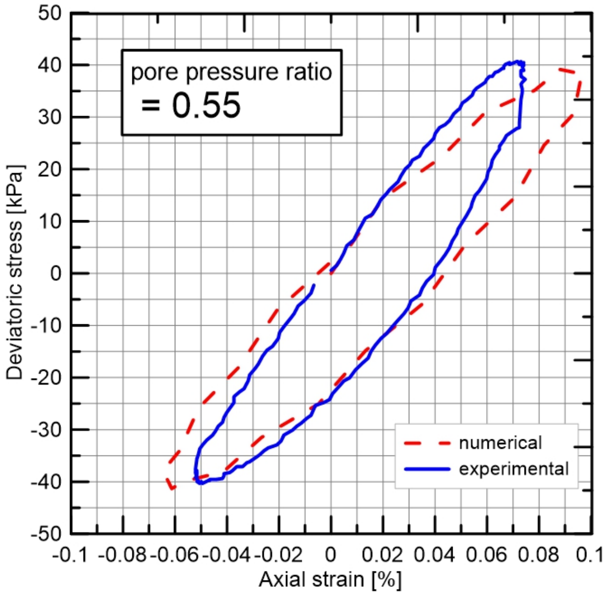


Figure 11. Comparison of results for CSR 0.2 – cycle 100

Figure 11 shows the further widening and increased strain accumulation while the numerical model results begin slightly to deviate from the experimental results although there is good correlation between the hysteresis loops. As the loading continues, the deviation of the results increases as is given in Figure 12.

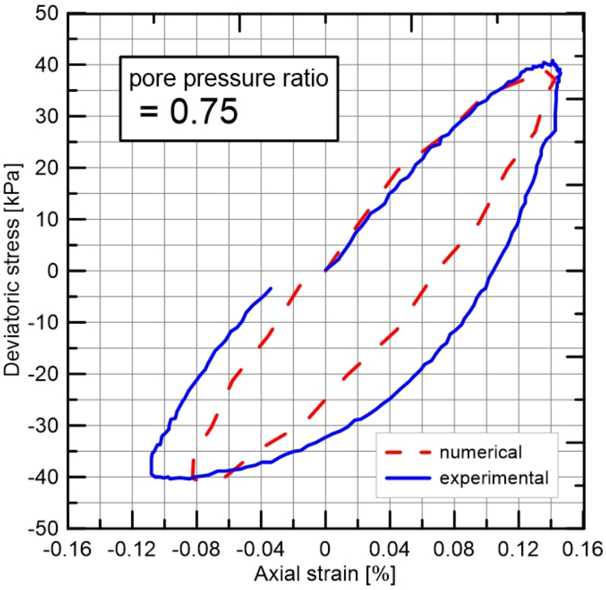
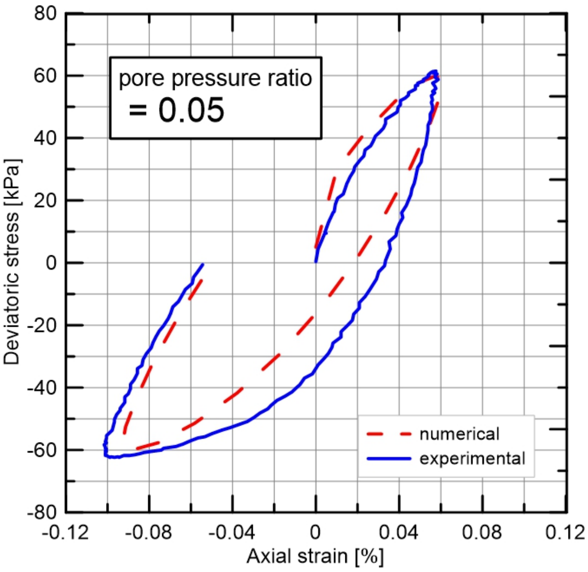


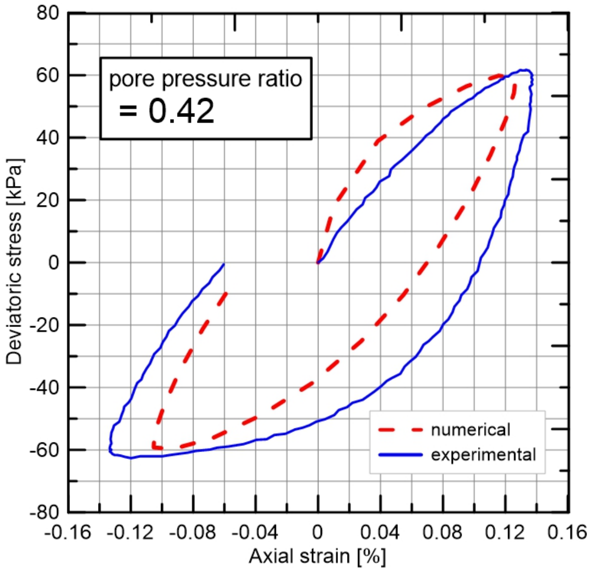
Figure 12. Comparison of results for CSR 0.2 – cycle 115

As can be seen from Figure 12, at the highest pore pressure ratio of 0.75 approaching liquefaction, the numerical model accurately predicts the overall cyclic behavior, though deviations increase due to the complex nonlinear response characteristic of near-liquefaction conditions.

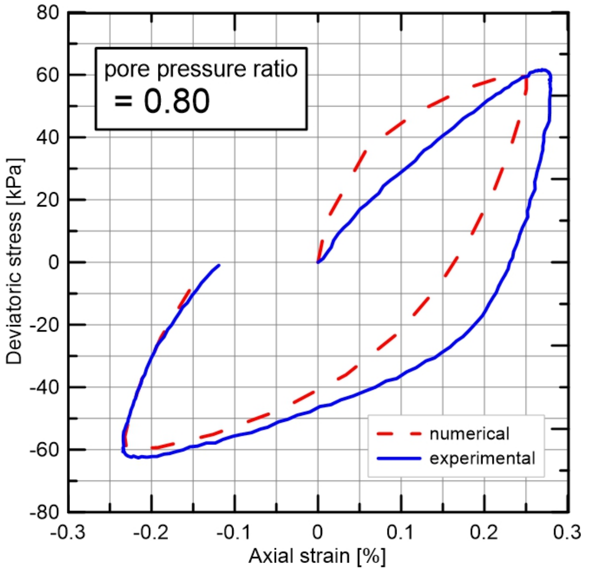
Next, for the sake of completeness, the experimental cyclic results have been compared by using the CSR of 0.3.As it is expected, the number of cycles is decreased and shown in Figure 13 below.



a) Cycle 1



b) Cycle 4



c) Cycle 6

Figure 13. Comparison of results for CSR 0.3 – cycles 1, 4 and 6

As can be seen from the Figure 13, progressive evolution of cyclic behavior across a range of pore pressure ratios (0.05, 0.42, and 0.80) demonstrate increasing strain accumulation and hysteresis loop widening as the pore pressure ratio approaches unity.The numerical model accurately predicts experimental behavior across all pore pressure ratios, validating the hypoplastic framework for cyclic soil response under varying state conditions. It should be mentioned that the numerical simulation is quite successful in considering the end values.

4 CONCLUSIONS

This study presents both experimental investigations conducted at the Laboratory for soil dynamics, Institute of Earthquake Engineering and Engineering Seismology in Skopje, N. Macedonia (IZIIS-Skopje) and implementation of hypoplastic constitutive equations through ANSYS User Programmable Features to model the complex non-linear behavior of soil skeleton in fully saturated conditions. The numerical simulations demonstrate that the hypoplastic material model programmed in general finite element program such as ANSYS can accurately predict experimental soil behavior under both drained and undrained static conditions as well as complex cyclic loadingtest conditions with varying pore pressure ratios.The cyclic behavior of soil testing has given correct simulated results up to high values of pore pressure above which initiation of liquefaction begins, where, due to big deformations, the numerical simulation can not be done correctly. The validated modelling approach using the programmable features of ANSYS establishes a solid base for further simulations using other software in order to facilitate further simulations in the field of geotechnical earthquake engineering.

5 ACKNOWLEDGEMENTS

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